AN EQUIVALENT LINEARIZATION SOLUTION SEQUENCE FOR MSC/NASTRAN

J.H. Robinson
Aerospace Engineer
Structural Acoustics Branch
NASA Langley Research Center
Hampton, VA 23681-0001

C.K. Chiang
Postdoctoral Research Associate
Department of Mechanical Engineering and Mechanics
Old Dominion University
Norfolk, VA 23508

ABSTRACT

A classical equivalent linearization solution procedure for the geometric nonlinear random response of structures is incorporated into MSC/NASTRAN by Direct Matrix Abstraction Programming (DMAP). The equivalent linearization solution sequence was derived from the existing Super Element Modal Frequency (SEMFREQ) response solution sequence. The definition of the equivalent linear stiffness matrix in terms of the MSC/NASTRAN differential stiffness for Gaussian random loads is presented. The required modification and inclusions to the SEMFREQ solution sequence are discussed. Results are presented for the nonlinear random response of a simple and a complex panel.

An Equivalent Linearization Solution Sequence for MSC/NASTRAN

J.H. Robinson
Aerospace Engineer
Structural Acoustics Branch
NASA Langley Research Center
Hampton, VA 23681-0001

C.K. Chiang
Postdoctoral Research Associate
Department of Mechanical Engineering and Mechanics
Old Dominion University
Norfolk, VA 23508

Nomenclature

{a}, {A}	element and system displacement vector
[Bl]	linear strain matrix
[Bn]	nonlinear strain matrix
[C]	damping matrix
[K]	stiffness matrix
K1	first-order nonlinear stiffness
K2	second-order nonlinear stiffness
$[K_e]$	equivalent linear stiffness matrix
[M]	mass matrix
{P}	load vector
{Q}	modal response vector
u, v, w	displacements
x, y, z	rectangular coordinates
$\{\Gamma\}$	stiffness vector
$\{\epsilon\}$	strain vector
$[\Phi]$	matrix of eigenvectors
ω	frequency

Introduction

The current trends in advanced vehicle development show a need for lighter, more economical structural components. This trend coupled with increasing propulsion and environmental loads associated with these vehicles has renewed interest in nonlinear structural response. This is most evident in, but not necessarily limited to, the aerospace industry with such proposed vehicles as the NASP and the high speed civil transport. The surface panels, particularly those that are exposed to the engine noise and jet exhaust and those in the region of shock boundary layer interactions, are anticipated to respond nonlinearily in at least part of the flight regime as shown in figure 1. Other intense random loads may be transmitted through the structure from engine mounts or other hard points. To effectively and economically evaluate these structural components, a practical method of predicting their large deflection random response is required.

There are several methods currently in use to predict the large deflection random response of structures. A perturbation method, Crandall 1963, based on classical perturbation theory for nonlinear deterministic motion, can be used to obtain approximate solutions to weakly nonlinear systems. A stochastic averaging method, Stratonowich 1967, yields approximate solutions when the damping is light and the excitation is broadband. This method has been applied principally to single-degree-of-freedom systems. The Fokker-Plank-Kolmogorov (FPK) approach, Caughey 1971, is the only method that yields an exact solution, but solutions are only available for a few restricted classes of problems. The numerical simulation technique, also referred to as the Monte Carlo method, Shinozuka 1975, is the most general method and yields the best results of all the approximate methods. A substantial drawback to the Monte Carlo method is the computational time required to solve realistic structural problems. The most widely used and commercially viable method is equivalent linearization (EL) , Caughy 1963. It yields good approximate solutions for the statistics of the random response of simple and complex structures and lends itself to an incremental solution procedure similar to the methods employed in static nonlinear problems.

AEROTHERMAL-ACOUSTIC LOADS

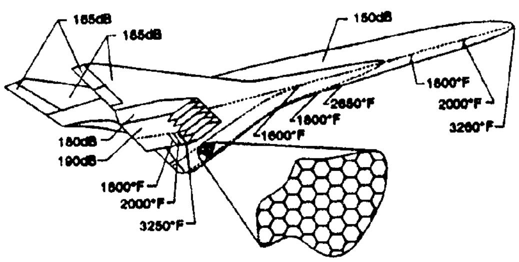

Figure 1. Generic design environment for a hypersonic vehicle

From the list of available methods, the EL method of obtaining nonlinear random responses was an obvious choice for implementation in a commercial package. The technique has been used, refined, and validated by many authors , Roberts and Spanos 1990, Locke and Mei 1989, Mei and Chiang 1987, Atalik and Utku 1976, Lin 1967. The validation of the method is well documented by many authors for beams, plates, and other nonlinear dynamic structures. The refinements include methods for solving structural problems with thermal and acoustic loads, initial stresses and imperfections. Techniques have been developed for the random response of pre- and post- thermally and mechanically buckled plates, linear and nonlinear statically deflected panels, and various combinations of concentrated and distributed random loads. However, the EL procedure has been applied primarily in research or special purpose codes, a general purpose finite element code incorporating this procedure is not yet available.

The MacNeal-Schwendler Corporation version of NASTRAN (MSC/NASTRAN) was selected for this work due to its extensive use in the aerospace and automotive industries where nonlinear random phenomena are most prevalent. The equivalent linearization procedure is being programed as a "stand alone" solution sequence for version 67 using the Direct Matrix Abstraction Programing (DMAP) language. It was found that all the necessary components of the EL procedure already existed as DMAP modules. The essence of the new solution sequence therefore consists of incorporating the necessary modules and iterative procedures into a standard MSC/NASTRAN solution sequence for linear random analysis. The two available solution sequences from which to work are the Super Element Modal Frequency Response (SEMFREQ) and Super Element Direct Frequency response (SEDFREQ) solution sequences.

In this paper, the large deflection finite element formulation is first reviewed to establish the general nonlinear equations of motion. The theory of equivalent linearization is then presented and the expression for the equivalent linear stiffness is derived. An overview of the iterative implementation of the equivalent linearization procedure is presented in flow chart form with consideration to the various methods of solving dynamic systems. The ease with which the expression for the equivalent linear stiffness is evaluated in multi-degree-of-freedom systems is somewhat dependent on the method used to form and solve the equations of motion. The evaluation of the equivalent linear stiffness and the particulars of the programing of the new solution sequence are presented for broad band Gaussian loads and modal equations of motion. In the last section of this paper, a simple plate example is used to compare the MSC/NASTRAN EL solution sequence with published results. A second example demonstrates the ability of the solution sequence to efficiently solve complex structural systems.

Finite Element Formulation

The nonlinear strain-displacement relationships taken from classical elasticity, [Love, 1944], are:

$$\epsilon_x(x,y,z,t) = \frac{\partial u}{\partial x} + \frac{1}{2}[(\frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial y})^2 + (\frac{\partial w}{\partial z})^2]$$
$$\gamma_{xy}(x,y,z,t) = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \tag{1}$$

where u(x,y,z,t), v(x,y,z,t) and w(x,y,z,t) are the displacements. The other strain-displacement relations are written similarly.

Equation (1) is typically expressed in matrix notation

$$\{\epsilon\} = ([Bl] + [Bn])\{a\} \tag{2}$$

where $\{\epsilon\}$ is the strain vector and the matrices [Bl] and [Bn] are the linear and nonlinear components of strain. The vector {a} is the vector of nodal degrees-of-freedom and is related to the displacements by the finite element interpolation functions and the assumptions about the displacements. (i.e. from the classical plate theory, $u(x,y,z,t) = u(x,y,t) - z\frac{\partial w}{\partial x}$.)

Assemblage of the elements and application of the boundary conditions follow the usual procedures. The equations of motion based on the nonlinear strain-displacement relations are thus

$$\begin{gathered}[M]\{\ddot{A}\} + [C]\{\dot{A}\} + \\ \Big[[K] + [K1\{A\}] + \big[K2\{A\}\{A\}^T\big]\Big]\{A\} = \{P\}\end{gathered} \tag{3}$$

or in more general form with

$$\{\Gamma\} = \Big[[K] + [K1\{A\}] + \Big[K2\{A\}\{A\}^T\Big]\Big]\{A\} \qquad (4)$$

as

$$[M]\Big\{\ddot{A}\Big\} + [C]\Big\{\dot{A}\Big\} + \{\Gamma(A,\ A^2,A^3)\} = \{P\} \qquad (5)$$

where the matrices [M], [C], and [K] are the system linear mass, damping, and stiffness matrices. The vector {P} is the time dependent load and K1 and K2 are the system first and second order nonlinear stiffnesses.

Equation (5) has no general solution when the excitation is random. An approximate solution to these equations is obtained by seeking an equivalent linear system, [Roberts and Spanos, 1990], of the form

$$[M]\{\ddot{A}\} + [C]\{\dot{A}\} + [K_e]\{A\} = \{P\} \qquad (6)$$

where $[K_e]$ is an equivalent linear stiffness matrix.

Equivalent Linear Stiffness

The equivalent linear stiffness matrix, $[K_e]$, is to be defined such that the difference between the actual nonlinear system and the approximate linear system is minimized. The approach may be thought of as a statistical version of a classical least square minimization. The error in obtaining the approximate system is defined as

$$\{\Delta\} = \{\Gamma\} - [K_e]\{A\} \qquad (7)$$

Since the error is a random function of time, the required condition is that the ensemble averaged, expectation, of the mean square error be a minimum. This is expressed as

$$E\Big[\{\Delta\}\{\Delta\}^T\Big] \rightarrow \text{minimum} \qquad (8)$$

where $E[\cdot]$ denotes the expectation operator. As in the cases of classical least square minimization, the necessary condition for equation (8) to hold true is

$$\frac{\partial E\Big[\{\Delta\}\{\Delta\}^T\Big]}{\partial [K_e]} = 0 \qquad (9)$$

Substituting equation (7) into equation (8) and interchanging the expectation and differentiation operators yields

$$E\Big[\{\Gamma\}\{A\}^T\Big] = E\Big[\{A\}\{A\}^T\Big][K_e]^T \qquad (10)$$

Using the fact that the matrix $E[\{A\}\{A\}^T]$ is symmetric and positive definite, the equivalent linear matrix is defined as

$$[K_e] = E\Big[\{A\}\{A\}^T\Big]^{-1} E\Big[\{\Gamma\}\{A\}^T\Big] \qquad (11)$$

The equivalent linear stiffness $[K_e]$ defined in equation (11) can be directly programed in a finite element code if the stiffnesses K1 and K2 are available and the expectation operator can be evaluated. Neither of these two conditions is generally true. The nonlinear stiffnesses are generally formed in tangential or differential form and the expectation operator requires knowledge of the joint probability density function of the response vector which is the unknown. Therefore, the equivalent linearization solution procedure is programmed in an iterative method and some assumptions regarding the expectations of the response vector are required. It should be noted that, if K1 and K2 are available, the mean square response can be obtained directly, [Locke and Mei, 1989], with appropriate assumptions for the expectation operator. In all instances cited above, assumptions regarding the expectations of the response vector are required. These assumptions are usually based on a knowledge of the excitation and the solution method used. Therefore, a discussion of the general iterative equivalent linear solution procedures used is presented.

Iterative EL Solution Methods

There are two basic means to solve linear dynamic equations of motion, one is using the physical degrees-of-freedom and the other is to use modal degrees-of-freedom. The first method is generally referred to as the direct frequency response method and requires the solving of a complex coupled system of equations in the nodal degrees-of-freedom at each frequency of interest. The second method is generally referred to as the modal frequency response method. It involves solving for the linear eigenvectors first and transforming the equations of motion into modal coordinates. The resulting system of equations is uncoupled and can be easily solved at each frequency of interest.

The primary consideration as to which method to be used for a particular linear system is based on the computational time required. This can be phrased as a simple question: Which will take more time, solving one eigenvalue problem (approximately an N cubed process) or solving some number of linear systems of equations (approximately an N squared process)? The decision as to which method to use in an EL solution procedure is further complicated by the iterative nature of the problem and the evaluation of the expectations. The choice of method can greatly simplify or complicate the process.

The direct method would seem to be the easiest and most straight forward to implement and the computational time required would be simple to compute. The difficulty in the direct method arises in the assumptions regarding the expectation operator in the expression for the equivalent linear stiffness and the implementation of these assumptions in a general sense. Accurate approximations of the expectation operator require assumptions regarding the full set of moments up to the fourth moments (mean, standard deviation, skewness, and kurtosis) of the response vector in nodal degrees-of-freedom. It should be noted that in physical coordinates, the correlations between all the degrees-of-freedom are necessary and must be determined.

As a simple example of the direct method, assume a beam of length L with ten nodes and three degrees-of-freedom, u, w, and θ, at each node. The evaluation of equation (11) for the equivalent linear stiffness requires the evaluation of the complete set of expectations of all the nodal degrees-of-freedom to the fourth moments. The EL solution relies on determining expressions for the third and fourth moments in terms of the first and second moments. These may be obtained by assuming appropriate probability distributions for the nodal displacements. In the beam example, if the excitation is broadband, Gaussian distributed, and spatially correlated over the beam, it can be assumed that the responses w and θ are Gaussian and u is Chi-square distributed. From these assumptions, an expression for the equivalent linear stiffness in terms of the first and second moments of the response can be found. However each entry in the thirty by thirty equivalent linear stiffness matrix of this problem could have a different coefficient representative of the degrees-of-freedom, correlation coefficients between the degrees-of-freedom and the order of the expectations involved. The complexity in using physical degrees-of-freedom can be deduced from this simple problem when it is noted that it is terms such as the square of the slope and the in-plane displacement that are strongly correlated. This entire process is programmable but it is not easily done in a general sense. The selection of modal coordinates will be seen to make the evaluation of equation (11) simpler.

The modal solution method of the EL procedure is simpler to implement than the direct method because reasonable assumptions regarding the correlation of the modal degrees-of-freedom as well as their joint distribution are possible. This is not to say that the modal approach is without deficiencies or difficulties. To illustrate the advantages and difficulties with the iterative modal solution procedure, the simple beam problem discussed in the direct method is used. The first difficulty arises immediately from the linear eigenvalue problem. The extracted eigenvectors for the simple isotropic case are functions of either the out-of-plane nodal displacement, 'bending modes', or the in-plane nodal displacement, 'membrane modes', and not both. This is because the bending motion of the beam is coupled to the membrane motion through the nonlinear terms.

There are three ways to handle the decoupling of the membrane and bending motion induced by the use of the linear eigenvectors. The first way is to simply exclude the membrane modes from the modal response. This is easy but not particularly accurate. A popular corollary to this solution is used for one-, and two- dimensional structures, Mei 1989. This procedure assumes the in-plane inertia and damping to be negligible. It is then possible to solve for the membrane modes in terms of the bending modes and thus account for the in-plane stiffness. This procedure is efficient but highly specialized and difficult to include in a general finite element code.

The second method is to select particular bending modes and membrane modes to include in the formulation. The difficulties that arise from this solution are similar to those encountered in the direct method when trying to evaluate the expectations and solving the system of equations. In the beam problem, it is again assumed that the bending is Gaussian and the membrane is Chi-square distributed when the excitation is Gaussian. The bending modes can be assumed uncorrelated with respect to each other as can be the membrane modes but the membrane modes are strongly correlated to the square of the bending modes. The resulting system of equations is coupled and the expression for the equivalent linear stiffness matrix is only marginally simplified with respect to the direct method. Another difficulty with the linear modal solution procedure is that the type of modes, bending, membrane, or otherwise are not always readily identifiable or available. Many current finite element programs use Lancsoz type eigenvalue solvers in which only the lowest modes or modes within a certain range are computed. It is difficult to construct a general program using this method that will extract the particular eigenvectors needed for an accurate solution.

The third modal solution method for the equivalent linearization procedure uses updated or 'equivalent linear' modes. The obvious drawback to this method is that it requires the eigenvalue problem be solved at each iteration. The advantages of this method are that the system of equations that are solved at each frequency are uncoupled and that simple assumptions regarding the moments and correlation of the modal responses are adequate for accurate solutions. The simple beam problem discussed in the previous solution methods could be solved with only a small number of updated modes. These modes would be assumed to be Gaussian distributed if the load were Gaussian and they could also be assumed uncorrelated. Although the means of the equivalent linear modal amplitudes are also assumed to be zero, this does not require that all the nodal displacements comprising the mode shape have zero means. The relationship between the mean of the in-plane displacement, u, and the mean square of the slopes, θ, in the simple beam problem, is implicitly maintained in the equivalent linear modal approach.

Implementation

The relationship between the steps involved in the direct, linear modal, and equivalent linear modal approaches to implementing the equivalent linearization solution procedure are outlined in the flowchart in figure 2. for a general finite element program. The solution procedure is iterative as discussed before since the nonlinear stiffness is only available in a differential form. The convergence of the iterative procedure is based on the Euclidian norm of the vector of the variance of the responses. From the discussion of the various methods available, it was determined that the equivalent linear modal method of solving the iterative EL procedure would be the simplest and most versatile to implement in MSC/NASTRAN.

The equivalent linear stiffness matrix in equation (11) must first be expressed in equivalent linear modal coordinates in order to evaluate the expectation operator. The stiffness vector $\{\Gamma(A, A^2, A^3)\}$ in equivalent linear modal coordinates has the form $\{\overline{\Gamma}(Q, Q^3)\}$ where the bar indicates a quantity transformed into modal coordinates. The expression for the equivalent linear stiffness with the Gaussian, zero mean, and uncorrelated modal response assumptions reduces to

$$\left[\overline{K_e}\right] = E\left[\frac{\partial\{\overline{\Gamma}\}}{\partial\{Q\}}\right] \tag{12}$$

EQUIVALENT LINEARIZATION FLOWCHART

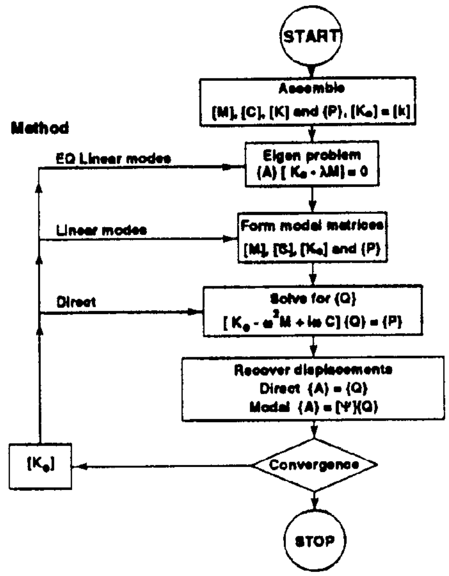

Figure 2. Flowchart for equivalent linearization solution procedure

The partial derivatives are easily taken and yield a linear modal stiffness matrix and a differential modal stiffness matrix which is based on the mean square of the modal response, Caughy 1963. The modal representation of the equivalent linear stiffness is then

$$\left[\overline{K_e}\right] = \left[\overline{K}\right] + 3\left[\overline{K2}(E[Q^2])\right] \tag{13}$$

This expression is not directly programmable in MSC/NASTRAN. It must be expressed in physical coordinates for two reasons: The first is that the eigenvalue problem in MSC/NASTRAN is solved in physical coordinates and the second reason is that differential stiffness matrix in MSC/NASTRAN is formed using the physical displacements. The linear stiffness matrix in the expression for the equivalent linear stiffness in physical coordinates is simply the linear stiffness matrix as assembled and computed in the MSC/NASTRAN program. After much consideration as to how MSC/NASTRAN computes the differential stiffness matrix in a nonlinear static sense, it turned out that the differential stiffness matrix expression in physical coordinate for the equivalent linear system is the MSC/NASTRAN differential stiffness matrix formed using an equivalent linear displacement vector. This equivalent linear displacement vector is given by

$$\{A\} = \left[\overline{\Phi}\right]\{\sigma_Q\} + \{A_o\} \tag{14}$$

where $\{\sigma_Q\}$ is a vector of the standard deviations of the equivalent linear modal amplitudes and $\left[\overline{\Phi}\right]$ is the matrix of normalized eigenvectors. The standard deviation of the modal amplitudes is always positive. The sign convention of the physical displacement is determined by the eigenvectors. The vector $\{A_o\}$ is the mean displacement obtained from a static solution sequence. The matrix of eigenvectors is normalized such that the amplitude of each eigenvector in the matrix is unity. The final expression for the equivalent linear stiffness is then

$$[K_e] = [K] + 3[K_R] \tag{15}$$

where $[K_R]$ is the standard MSC/NASTRAN differential stiffness matrix.

ELMFREQ DMAP

The Equivalent Linearization solution sequence was written by significantly rewriting the MSC /NASTRAN delivered SOL 111 - Superelement Modal Frequency main subdmap sequence. Minor alterations in the subdmaps of Phase 1, preprocessing procedures, and Phase 3, post processing procedures sections of the MSC/NASTRAN program section were also required. The modifications to Phase 1 of the program are presented first and the substantial rewriting of the main subdmap along with the Phase 3 modifications are discussed concurrently. A discussion of new user defined parameters for both convergence control and output requests is then presented.

To incorporate this new capability into MSC/ NASTRAN, the pre-processing sequence, Phase 1 has to be altered to generate the nonlinear element summary tables. This was done by setting the logical parameter NONLNR to TRUE in the call to SUPER1. NONLNR was set TRUE for Phase 1 only and not Phase 2 or 3, because linear data recovery is required in SUPER3. Due to this modification, it was required that the element summary table, ESTL, for linear analysis, not generated when NONLNR is TRUE, be equivalent to the element summary table, EST, generated when NONLNR is TRUE. This equivalence was programmed in subdmap SEMG.

The alterations to the main SEMFREQ subdmap consist of writing an iterative procedure around the frequency response solution modules, Phase 2 and post-processing subdmap SUPER3. Subdmap SUPER3 is included in the iteration loop because the rms values, which are necessary as input to the differential stiffness modules, are obtained from module RANDOM in Phase 3.

The maximum rms displacement with nonlinear stiffness effects can be extracted from the data base of PSDF in module RANDOM in subdmap SEDRCVR. The updated displacement vector is formed by multiplying the maximum rms displacement by the updated mode shape. In order to do so, one deflection point number has to be obtained first by asking for XYPRINT (or XYPLOT) in the control deck of the MSC/NASTRAN data cards.

If the separate modal responses are needed, the individual modes can be extracted after the rms response of the structures is calculated. Each mode is then normalized to unity for the largest component of the eigenvector. The actual rms response of each mode is obtained by multiplying the rms response by the normalized eigenvector. The updated response of the structure can be calculated by using superposition of the modes. This procedure entails the assumption that the modes and modal responses are independent.

To implement the iterative procedure, some of the files needed for the next iteration have to be saved. The module FILE to save or overwrite files is used. The linear equations of motion are solved first and the linear displacement vector $\{A\}$ is obtained. If the parameter LGDISP is greater than -1, the geometric nonlinear stiffness matrix $[K_R]$ (KDJJ in the DMAP) is calculated by applying this linear displacement vector and reduce to KDDD. (If the parameter LGDISP equals -1, only the linear frequency response is calculated.) The equivalent linear stiffness matrix $[K_e]$ in equation 11 now consists of two matrices: a linear stiffness matrix [K] and a differential stiffness matrix $[K_R]$. This equivalent linear stiffness matrix is then used to solve for the linearized frequency and updated mode shape. By enclosing subdmap SUPER3 inside the iterative loop, the iterative process can now be repeated with this updated displacement vector. With each iteration, the updated physical coordinate can be obtained by multiplying maximum rms displacement by the normalized mode shape. The iterative procedure will terminate if the rms displacement norm is achieved.

This iteration method can be used to determine the rms displacements; however, it is slow to converge. An improved method for speeding up the convergence is to use an underrelaxation approach where displacements are not updated to their full values but instead the scale of the full values after each iteration. The user defined parameter BETA is introduced to scale the updated displacements. If the nonlinearity is mild to moderate, the convergence of the iteration procedure is faster for $0.5 \leq \text{BETA} \leq 1.0$. If the nonlinearity is severe, the convergence of the iteration procedure is faster for $0.0 < \text{BETA} \leq 0.5$.

There are two user defined parameters for convergence control within the iteration loop. MAXITER defines the allowable maximum number of iteration and MAXNORM defines the maximum allowable displacement norm. If the iteration count exceeds MAXITER or if the error norm of the displacements is less than MAXNORM, the iteration procedure will stop. There is a warning message if the solution is not converged after the MAXITER iterations and the job will be stopped. There two ways to handle convergence errors, first, by increasing the MAXITER number or choosing the different BETA which is less than the previous chosen BETA.

There is no rms strain response obtained from frequency random analysis of SOL 111. If rms element strain is required, user defined parameter RMSTRAIN has to be 1. For this case, the STRAIN=ALL is needed in the control deck and the strains will be calculated.

Examples

The capabilities of the equivalent linearization solution sequence implemented in MSC/NASTRAN are demonstrated by two examples. In the first example, the equivalent linear response of a simple beam computed by the new DMAP is compared with published results. In the second example, the thermal-acoustic response of a generic thermal-acoustic protection panel is computed. This type of panel configuration has been under investigation for possible use in hypersonic aircraft. The solution procedure employed requires the equivalent linearization solution sequence to be restarted using the results and data base from a linear thermal analysis.

Example 1:

A 12 in. × 2 in. × 0.064 in. aluminum beam with first clamped and then simply supported boundary conditions is subjected to uniformly distributed acoustic excitation. The equivalent linear rms center beam displacements are predicted for excitation levels of 90 dB to 130 dB using the ELMFREQ solution sequence. Figure 3 shows the comparison of these predictions to those published by Prasad, 1987, denoted EL in the figure, and Locke, 1989, denoted FE in the figure. Excellent agreement between the ELMFREQ solution sequence and the published results is indicated.

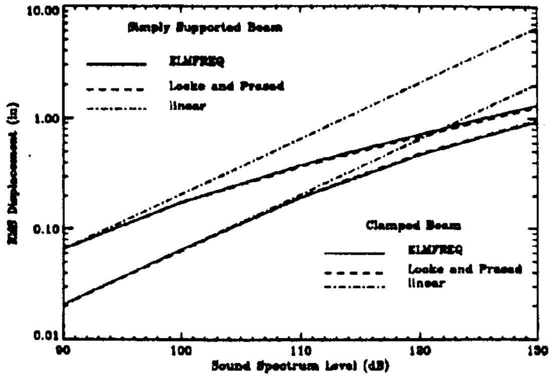

Figure 3. Effect of acoustic excitation level on maximum deflection for beams.

Example 2:

A hexagonal thermal protection system panel similar to the cutout in figure 1 was subjected to both thermal and acoustic loads. The panel is composed of an eight-ply carbon-carbon stand-off panel and an aluminum, graphite/epoxy honeycomb substructure connected by seven titanium rods (posts). The substructure has an aluminum core sandwiched between an aluminum and a graphite/epoxy face sheet. The dimensions of the panel are given in Table 1, and the finite element mesh is shown in figure 4. The finite element model is comprised of 804 triangular elements and seven bar elements with a total of 622 nodes.

The boundary conditions imposed on the panel were designed to minimize thermal stresses and are summarized for each component. The edges of the carbon-carbon panel are constrained in the perpendicular and tangential directions, the out-of-plane and radial displacements, only. The actual boundaries at these edges are more complex and difficult to model. The choice of "simply-supported" is a compromise for simplicity. The edges of the substructure are constrained in all rotations and translations. This boundary condition is more indicative of the boundaries imposed in an experimental setup. The two panels are connected by the seven posts. The post connections to the carbon-carbon panel were modeled as pin joints using MPC bulk data cards. The three translations at the top of the posts were equivalent to the three translations at adjoining locations on the carbon-carbon panel. The connections between the posts and the substructure were also modeled using MPCs. The center post connection was modeled as a rigid link, i.e. all three translations and the two rotations at the lower end node of the post were equivalent to the translation and rotations of the adjoining node of the substructure. The remaining post connections to the substructure were also modeled as pin joints.

Table 1 Panel dimensions

Radius	13.0 in.
Overall height	2.5 in.
Radius to posts	8.0 in.
Carbon-carbon thickness	0.091 in.
Substructure thickness	0.375 in.
Center post radius	0.1875 in.
Outter post radii	0.125 in.

A 2000°F temperature load was applied to the carbon-carbon panel and 200°F load was applied to the posts and substructure. The thermal displacements and stresses were predicted using SOL 101 and are plotted in figures 5 and 6. The carbon-carbon panel results are essentially those of a stress free thermal expansion while the substructure shows a moderate compressive thermal stress with little thermal displacement. The equivalent linearization solution sequence was restarted using the data base from the static thermal solution with the initial stresses and displacements. The rms thermal-acoustic displacements and stresses were predicted for a broadband acoustic excitation of 150 dB uniformly distributed over the carbon-carbon panel. These rms displacements and stresses are plotted in figures 7 and 8. The solution sequence converged in four iterations with the convergence enhancement parameter BETA set to 0.5 and the default convergence criteria.

The level of nonlinearity in the response is typically measured in several ways. The two most common are the ratio of the equivalent linear fundamental frequency to the linear fundamental frequency, frequency ratio, and the ratio of the equivalent linear maximum rms displacement to the linear maximum displacement, amplitude ratio. For this particular problem, these ratios were 1.19 and 0.414 respectively. These ratios are typical of moderate to extreme geometric nonlinearity.

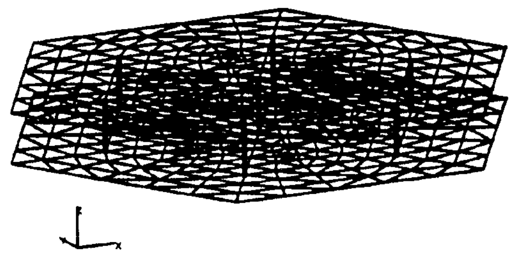

Figure 4. Finite element model

Conclusions

An equivalent linearization procedure is incorporated into MSC/NASTRAN to predict the nonlinear random response of structures. An iterative process is used to determine rms displacements. Numerical results obtained for simple plates and beams are in good agreement with existing solutions in both linear and linearized cases. The versatility of this implementation of equivalent linearization procedure in MSC/NASTRAN enables the analyst to determine the nonlinear multiple mode random responses for complex structures.

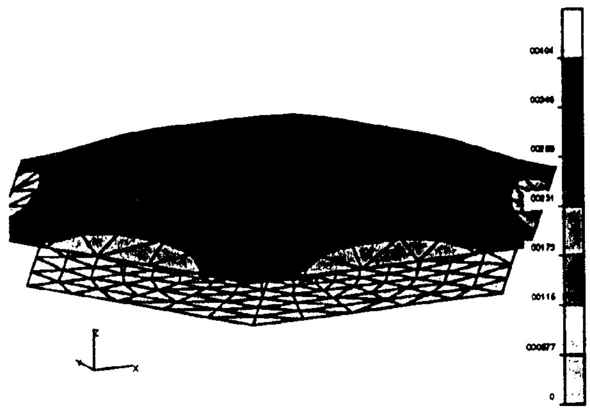

Figure 5. Deformed plot of the thermal displacement vector

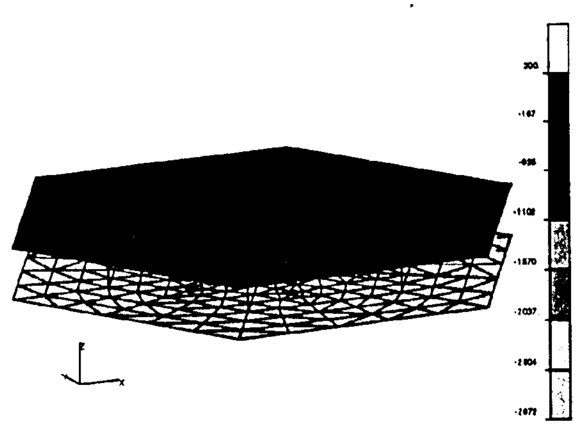

Figure 6. Thermal stresses in the radial direction ϵ_r

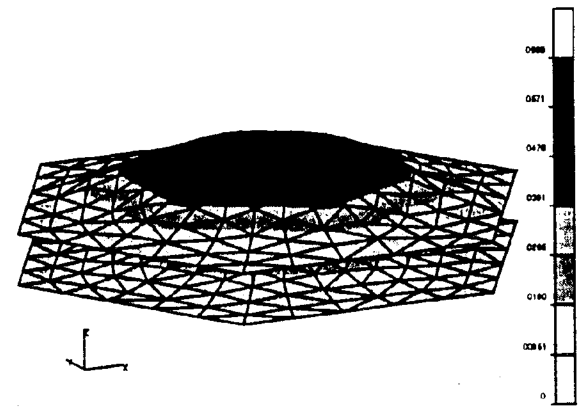

Figure 7. Deformed plot of the root-mean-square thermal-acoustic displacement vector

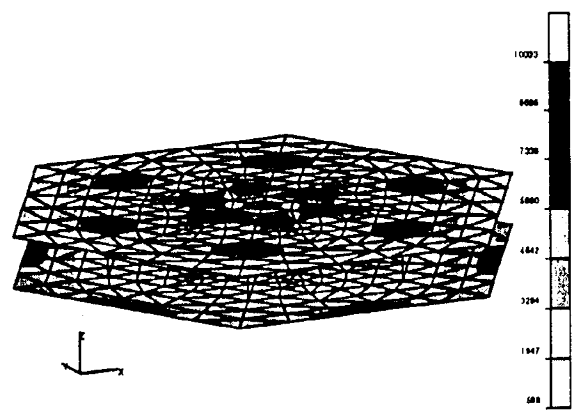

Figure 8. Root-mean-square thermal-acoustic stresses in the radial direction ϵ_r

References

Atalik, T.S. and Utku, S., 1976, Stochastic linearization of multi-degree-of-freedom nonlinear systems, *Earthquake Engineering and Structural Dynamics*, Vol. 4, pp. 411–420.

Caughey, T.K., 1971, *Nonlinear Theory of Random Vibrations*. Academic Press.

Caughy, T.K., 1963, Equivalent linearization techniques, *Journal of the Acoustical Society of America*, Vol. 35, pp. 1706–1711.

Crandall, S.H., Nov. 1963, Perturbation techniques for random vibration of nonlinear systems, *Journal of the Acoustical Society of America*, Vol. 35, pp. 1700–1705.

Lin, Y.K., 1967, Probabilistic theory of structural dynamics, *McGraw-Hill Inc. New York, NY*.

Locke, J. and Mei, C., 1989, A finite element formulation for the large deflection random response of thermally buckled plates, *AIAA paper 89-110*.

Love, A.E.H., 1944, *A Treatise on The Mathematical Theory of Elasticity*, 4th ed. ed. Dover, New York.

Mei, C. and Chiang, C.K., 1987, A finite element large deflection random response analysis of beams and plates subjected to acoustic loading, *AIAA paper 87-2713*.

Prasad, C.B. and Mei, C., October 1987, Multiple mode large deflection random response of beams with nonlinear damping subjected to acoustic excitation, AIAA 11th Aeroacoustics Conference, Paper 87-2712. Sunnyvale, CA.

Roberts, J.B. and Spanos, P.D., 1990, Random vibration and statistical linearization, *John Wiley & Sons, New York, NY*.

Shinozuka, M., 1975, Monte Carlo solution of structural dynamics, *Computers and Structures*, Vol. 2, pp. 874–885.

Stratonowich, R.L., 1967, *Topics in the Theory of Random Noise*, Vol. 2. Gordon and Breach, N.Y.