A Comparison of Three Adaptive Remeshing Techniques

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Abstract

Adaptive Mesh Refinement and Adaptive Remeshing of finite element meshes based on results to ensure result accuracy is a growing field of interest in the analysis modeling community. This paper will discuss and compare three methods of Adaptive Remeshing used in conjunction with the FAM Analysis Modeling System.

Introduction

The use of finite element methods for analysis is an approximation technique that introduces numerical errors based on the approximation used. The ability to achieve a specified level of error for a given analysis is a major priority for the analysis community. This priority is driven by many factors which include: a need to implement analysis much earlier in the design cycle, a need to broaden the use of finite element analysis to designers and design engineers, a need to take advantage of automatic mesh generation algorithms, and a broadening of application of finite element techniques to new areas such as process simulation and geometric shape optimization. Adaptive Mesh Refinement and Adaptive Remeshing are two (2) current methods under investigation to achieve a specified level of numerical error.

Adaptive Mesh Refinement involves the improvement of an existing mesh based on error estimates for that mesh. There are three (3) methods of Adaptive Mesh Refinement as follows: 1.) 'r'-method - refines locations of nodes, 2.) 'h'-method - refines an element into multiple elements, and 3.) 'p'-method - refines polynomial order of element shape functions. There is a great deal of research activity in all three methods and in coupling these methods. Adaptive Mesh Refinement is a mesh based activity and is commonly performed as an integral portion of the analysis solution algorithm. This paper will not discuss Adaptive Mesh Refinement.

Adaptive Remeshing involves the adjustment of mesh control parameters based on error estimates or attributes, deleting the existing mesh, and regenerating a new mesh. These mesh control parameters may include mesh sizing functions, mesh flow control, and mesh density control. Adaptive Remeshing is a geometry/topology based activity and is commonly performed as a pre/post-processing function.

This paper will discuss three methods of Adaptive Remeshing used in conjunction with the FAM Analysis Modeling System.

Error Estimate

Errors in a finite element analysis may be caused by a variety of factors. It should be noted clearly that this paper and most activity on mesh adaptivity is dealing only with numerical error. Other sources of error such as boundary conditions and abstraction error are separate areas of research in their own right.

The effectiveness of any Adaptive Remeshing or Adaptive Mesh Refinement scheme is a function of the error estimator used to indicate numerical error in the mesh. The subject of error estimators is an extensive research topic far beyond the scope of this paper. Additional research efforts to obtain accurate error estimates are required and are ongoing at several locations.

For the purposes of this paper an error estimator was used which was a derivative of the error estimator presented by Zienkiewicz and Zhu in 1987. The same error estimator was used for all three methods in order to evaluate the difference between the methods. Both global error estimators and element error estimators were calculated. Smoothed nodal error estimates were calculated by simple nodal averaging of element error estimates. This method of error estimate calculation was only for the purpose of establishing a baseline error estimate and no comment is made on its validity other than for the model investigated the error estimate used produced error estimate percentage predictions consistently within 60% of the actual error in VonMises stress at the peak stress location.

Analysis Modeling Framework

The FAM Analysis Modeling System from FEGS provides an analysis modeling framework architecture involving an analysis interface tool kit and a user interface tool kit. These tool kits include: a process executive to run executables or system functions external to FAM, parametric symbols, and extensive macro functionality. These FAM based tools were used to provide and receive data from external FORTRAN programs and to perform operations on the model based on the data received. This architecture allowed for modular development of external FORTRAN programs to adaptively modify the basic FAM program as illustrated in Figure 1.

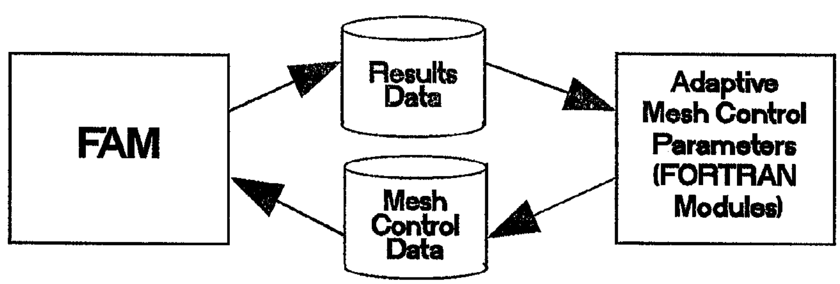


Figure 1: External FORTRAN Code

Target Element Size

A target element size is calculated at the start and end of each curve or line segment used in the model based on the equation below:

Target element size = Current element size * (Estimated error / Target error) ** 0.5

The target error of 1.0% was selected for this comparison to actively exercise the Adaptive Remeshing methods under investigation.

Element Type

It was decided that for the purposes of this paper that only 4 noded quadrilateral elements would be used. Most adaptivity efforts deal with triangular elements. The quadrilateral element type was chosen to indicate that Adaptive Remeshing does not require the use of triangular elements.

The Initial Model

The model under investigation is one quarter of a plate with a hole. The model is shown below in Figure 2. The length and width dimensions of the quarter of the plate are 10 inches with a hole radius of 1 inch. Symmetry boundary conditions are applied to the left and bottom surface and a tensile pressure loading of 1000/psi is applied to the top edge. Steel material properties are used with Youngs Modulus of 30E6 and Poisson's ratio of 0.3.

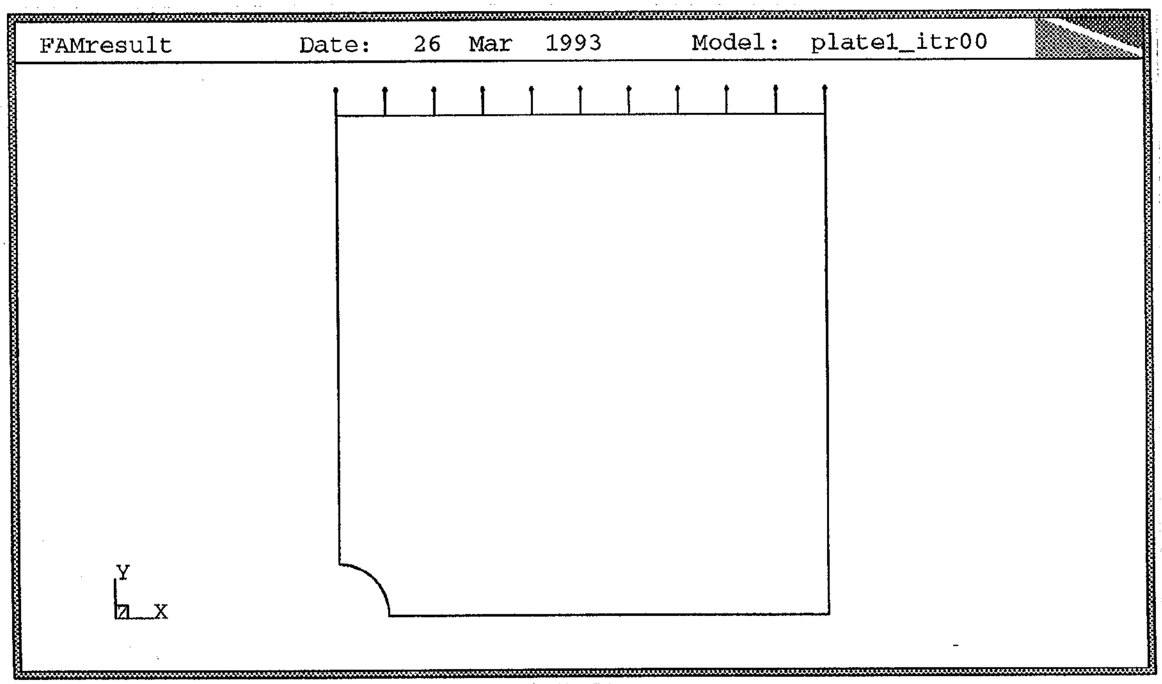


Figure 2: Plate with Hole

The initial mesh consisted of 99 elements and 120 nodes as illustrated in Figure 3. The mesh is distributed in a rectilinear pattern with element spacing of approximately one (1) inch. Twelve (12) mesh regions were used to obtain this initial mesh and to provide locations for mesh control data adjustments.

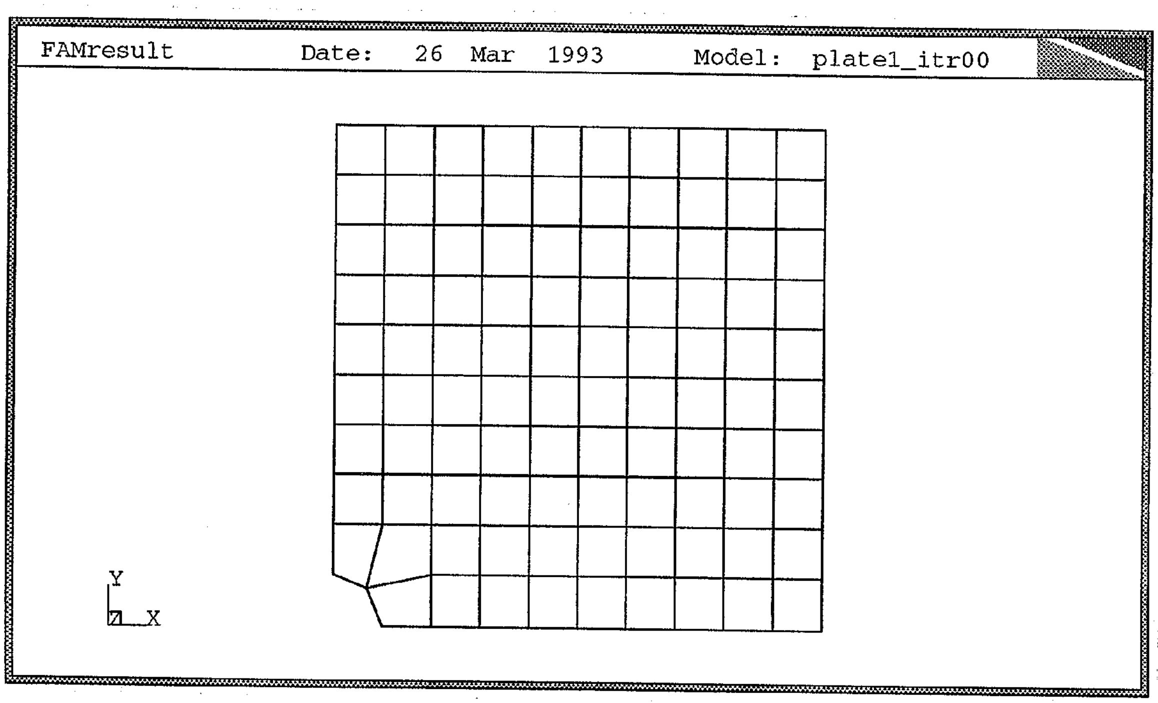


Figure 3: Initial Mesh

Convergence Criteria

The ratio of estimated error to target error was calculated for each node in the model and an average value was calculated. The convergence criteria was when this average error ratio was equal to or less than the average error ratio of the previous iteration and that a minimum of three iterations were completed. A maximum iteration limit of 12 iterations was used.

It should be noted that all three (3) methods exhibited "near" convergence with about 33% fewer iterations. Further research to capture "near" convergence is ongoing.

Performance Tracking

The performance of Adaptive Meshing techniques is often illustrated by tracking global error estimates and estimated or actual error at peak stress location. It was determined that tracking these values did not provide adequate information related to performance. The estimated error at nodes closest to five evenly spaced positions along the radial arc defining the hole and the global error estimate will be used to evaluate performance. The five positions are: 1.) 0 degrees, 2.) 22.5 degrees, 3.) 45 degrees, 4.) 67.5 degrees, and 5.) 90 degrees. The estimated error at each of the five points is divided by the target error at each point to obtain an error ratio. The desired error ratio is 1.0.

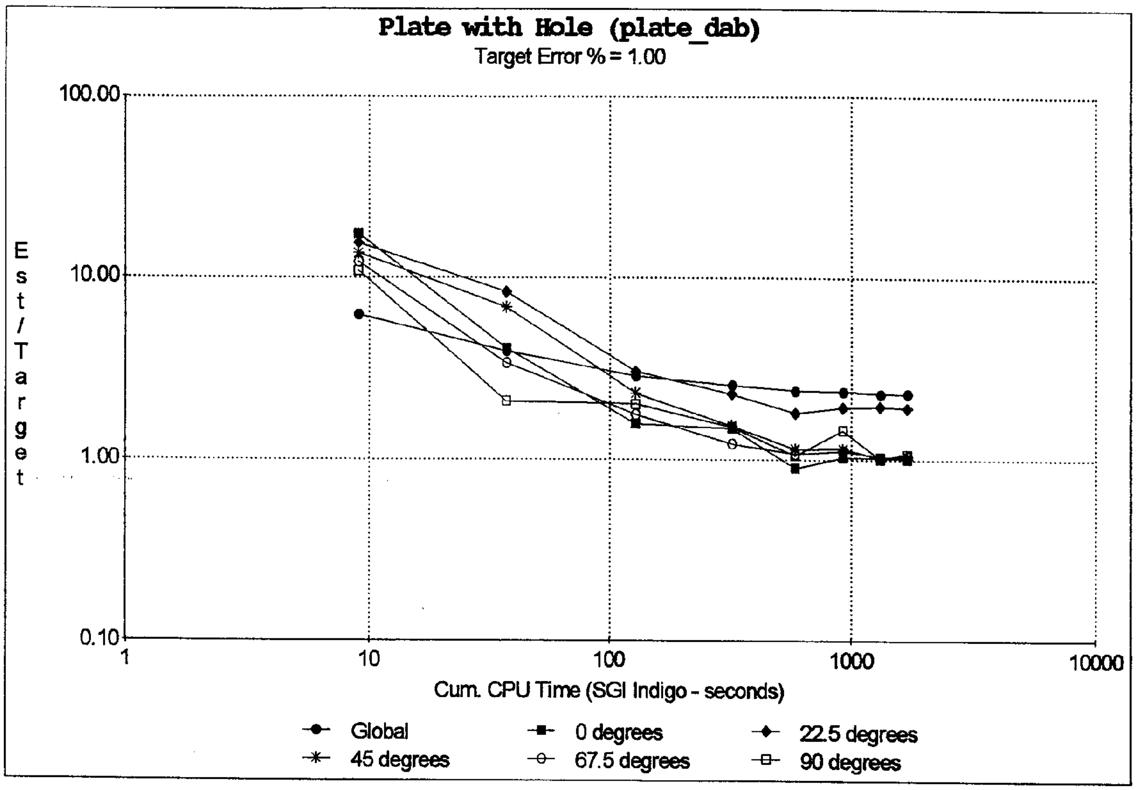
Method 1: Adjusting DIV - Constant Target Error %

FAM controls mesh by specifying mesh division (DIV) and bias for each curve. The first method investigated was to modify the division and bias settings for each curve to achieve target element sizes at each end of the curve. The target error used was a constant percentage of 1.00% error. This method of Adaptive Remeshing converged in seven (7) iterations. Table 1 indicates the mesh, global error, estimate, and actual peak stress for each iteration. This method was fully automated.

Table 1: Iteration results for Method 1

Iteration	No. of Nodes	No. of Elements	Global Error est. (%)	Peak stress Actual error (%)	Convergence Criteria
0	120	99	6.20	37.36	1.435
1	296	272	3.91	1.65	1.588
2	663	628	2.85	2.02	1.334
3	1050	1006	2.55	1.22	1.198
4	1269	1220	2.38	1.82	1.135
5	1439	1390	2.36	1.82	1.119
6	1576	1522	2.29	1.22	1.073
7	1556	1505	2.30	1.22	1.088

Figure 4 illustrates the final mesh used and provides a graph of error ratios versus cumulative CPU time in seconds on an SGI Indigo Workstation.



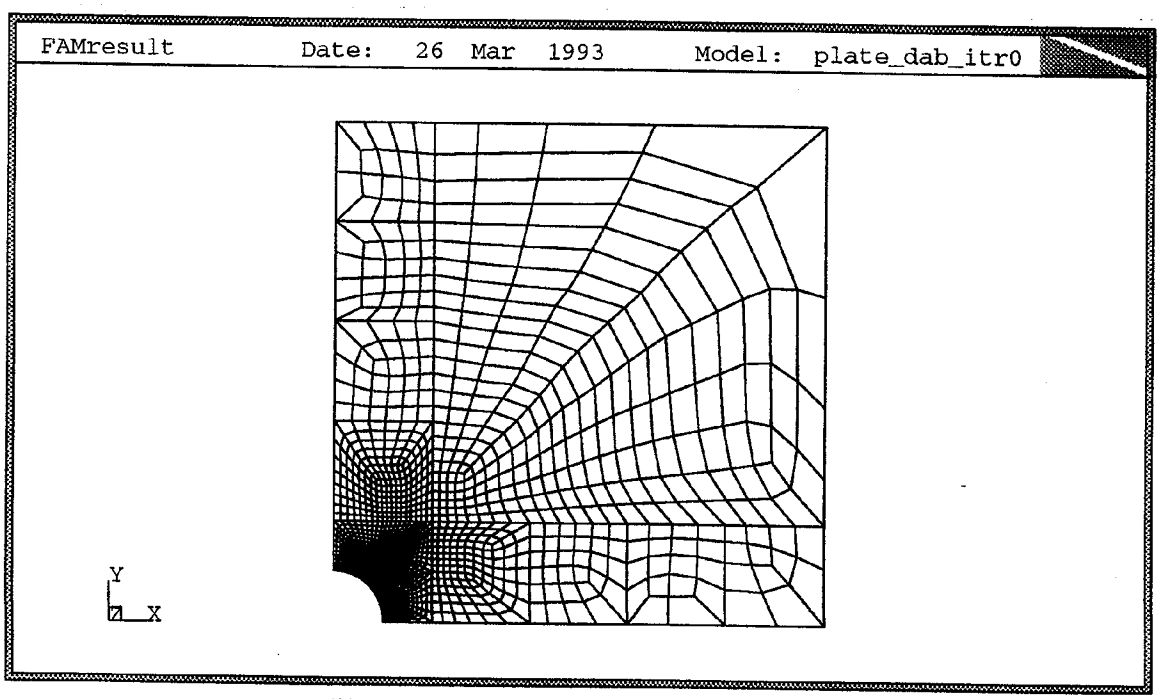


Figure 4: Method 1 (1556 nodes, 1505 elements)

The error ratio at all points except 22.5 degrees achieved values between 1.01 and 1.07. The final error ratio at 22.5 degrees was 1.91 and the global error ratio achieved was 2.30.

Method 2: Adjusting DIV - Variable Error %

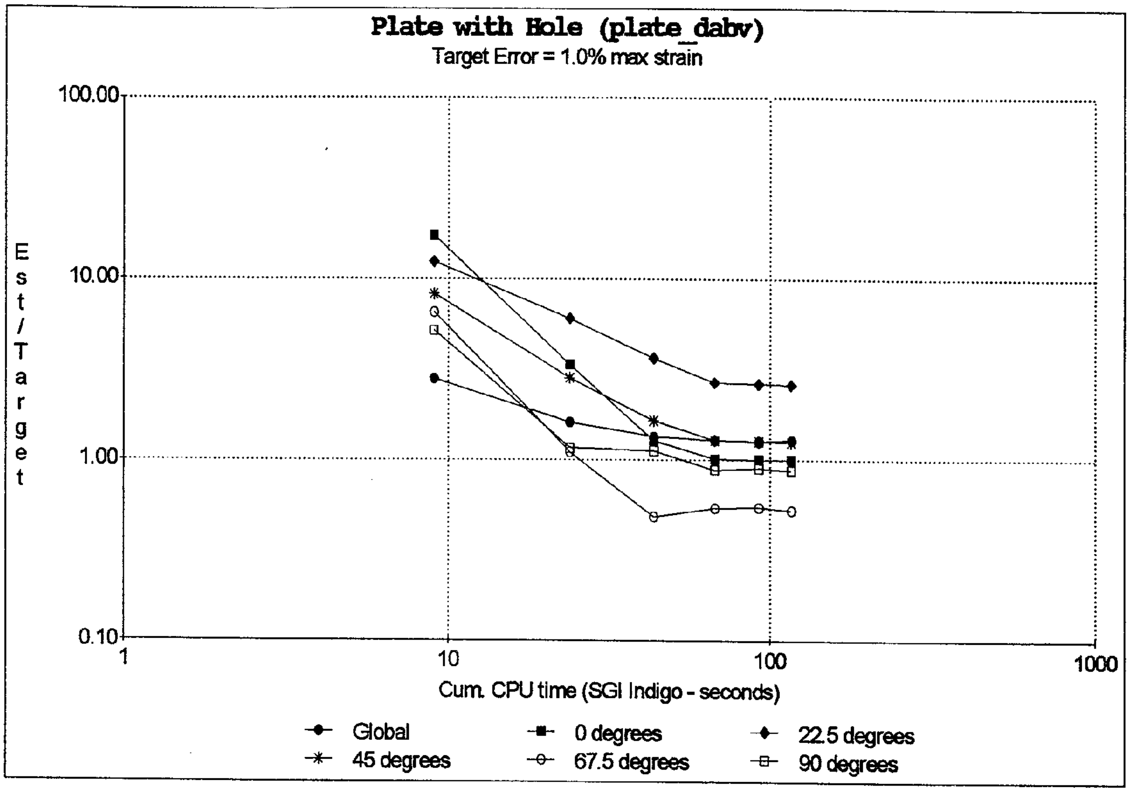
The second method investigated was also to modify division and bias settings for each curve based on target element size. This method, however, utilized a variable target error percentage to maintain a constant target error value equal to the target error percentage of the peak location (1.0% of Max VonMises Strain) The global target error percentage was determined in a similar manner using the nominal stress (final global error target = 3.03%)

This method converged in five (5) iterations. Table 2 illustrates the behavior of this method. This method was also fully automated.

Table 2: Iterations results for Method 2

Iteration	No. of Nodes	No. of Elements	Global Error est. (%)	Peak stress Actual error (%)	Convergence Criteria
0	120	99	6.03	37.36	0.724
1	181	160	4.87	0.92	0.669
2	222	200	4.06	0.85	0.639
3	259	236	3.87	1.42	0.633
4	265	242	3.80	1.49	0.624
5	259	236	3.87	1.55	0.637

Figure 5 illustrates the final mesh used and provides a graph of error ratios versus CPU time. The error ratios for all points except 22.5 degrees were less than 1.24. The final error ratio at 22.5 degrees was 2.57 while the final global error ratio was 1.28. The method converged quickly and efficiently but did not achieve target errors.



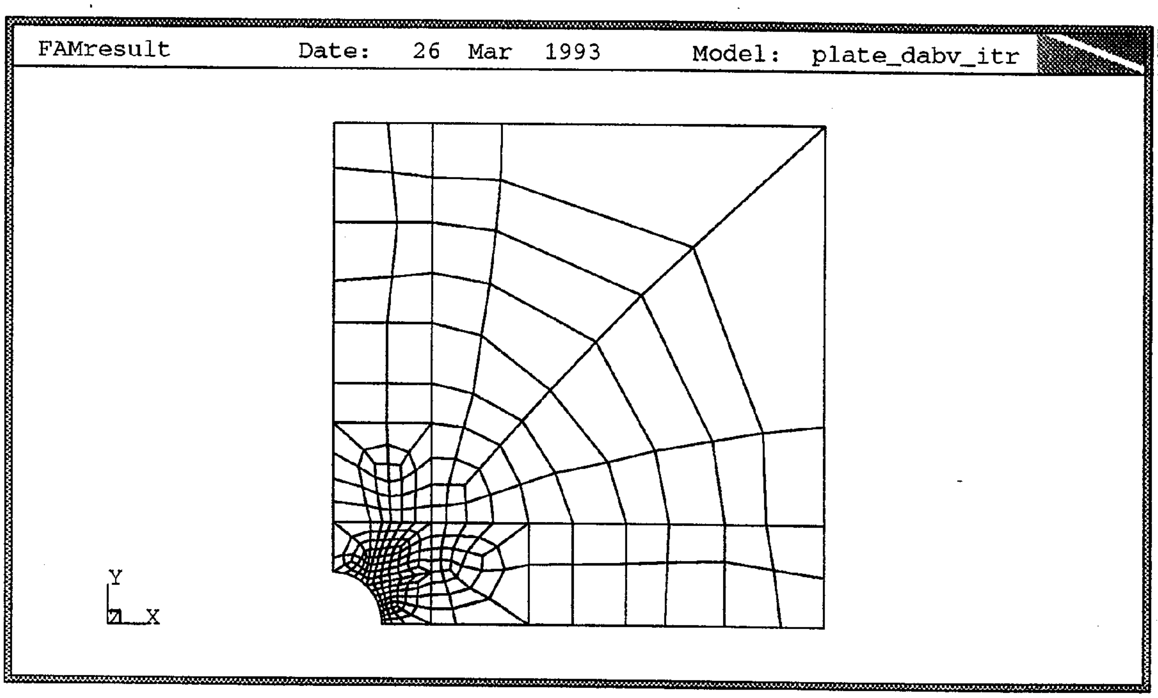


Figure 5: Method 2 (259 nodes, 236 elements)

Method 3: IsoValue Based Preconditioning

The final method investigated involved the creation of geometry for mesh flow control based on results. Mesh flow control curves were created at five (5) IsoValue locations of VonMises Strain. The regions between IsoValues were subdivided manually to obtain meshable geometry. It is anticipated that the automatic subdivision of geometry under development at FEGS would accomplish the subdivision required to make the entire process automatic. This method converged in five (5) iterations with results illustrated in Table 3. The target error was a variable percentage as outlined in Method 2. The global target error was determined as per method 2 resulting in a global target of 3.05%.

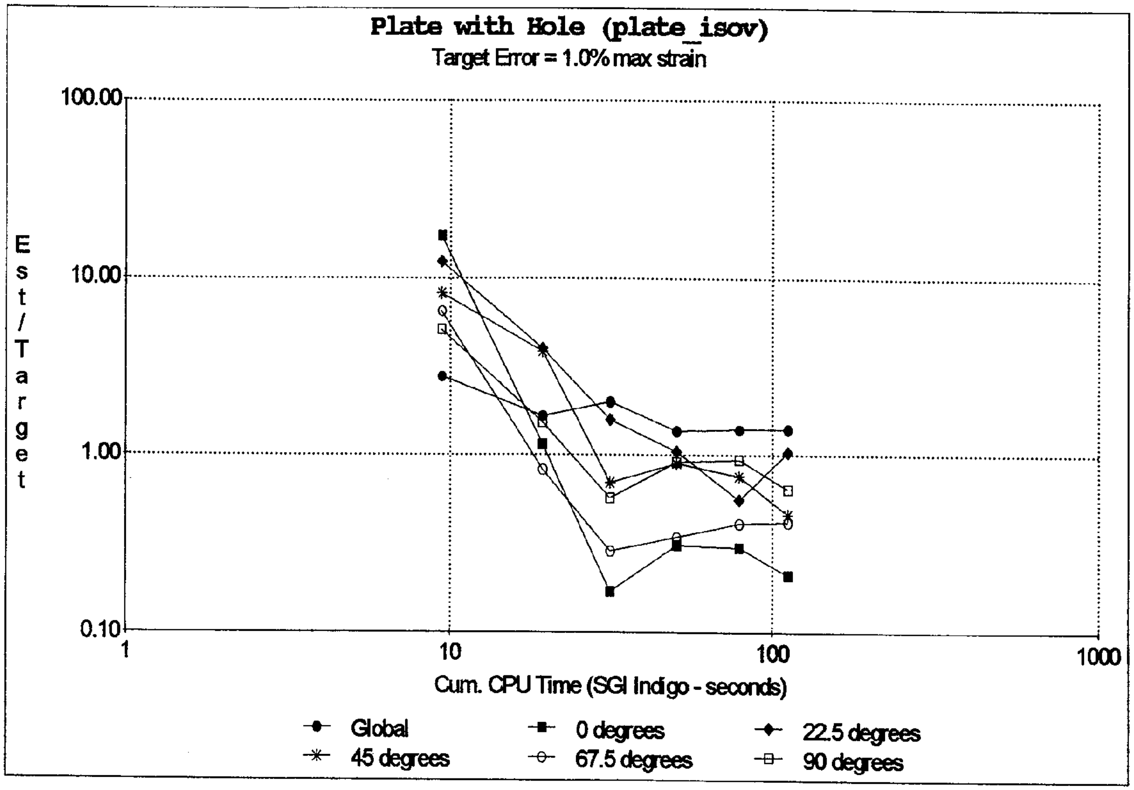
Table 3: Iteration Results for Method 3

Iteration	No. of Nodes	No. of Elements	Global Error est. (%)	Peak stress Actual error (%)	Convergence Criteria
0	120	99	6.20	37.18	0.724
1	131	113	5.31	4.65	0.745
2	159	135	5.83	4.81	0.801
3	227	200	4.14	0.85	0.664
4	305	276	4.24	0.36	0.595
5	344	314	4.24	0.92	0.648

Figure 6 indicates the final mesh and provides a graph of error ratios versus CPU time. The error ratios for all points of interest except at 22.5 degrees ranged from 0.2 to 0.64. The error ratio at 22.5 degrees was 1.03. The resultant global error ratio was 1.39.

Results Evaluation

The behavior of the estimated errors and error ratios through iterations will be used to compare the three methods. These results are graphed versus iteration number in Figure 7 for points at 0 degrees, and 22.5 degrees and in Figure 8 for points at 45 degrees, 67.5 degrees and 90 degrees.



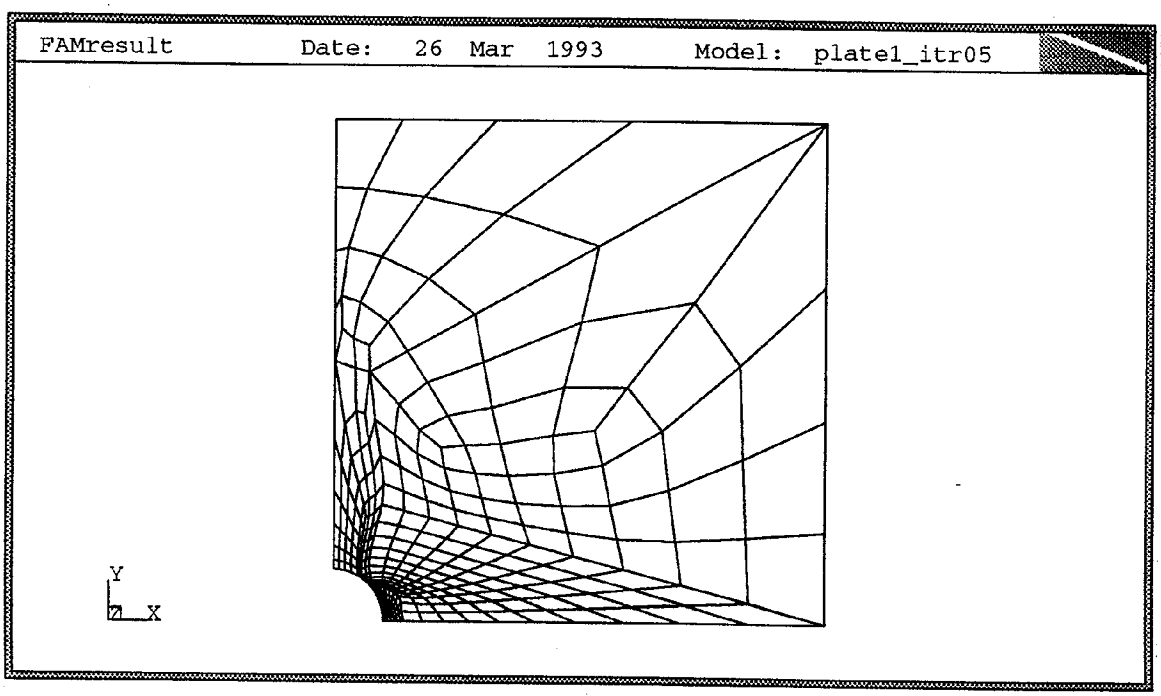


Figure 6: Method 3 (344 nodes, 314 elements)

The first method ran for seven (7) iterations and a large amount of CPU time. This method did approach the target error at all points except 22.5 degrees. The method was unable to resolve the 22.5 degree point location and the global error but did resolve the global error within 2.5 times the specified target. The initial model did not have an adequate number of mesh control curves to obtain better results. The actual peak stress error was within 1.01 versus a target value of 1.00.

The second method ran for five (5) iterations and approximately 7 % of the CPU time used in Method 1. This method was extremely efficient. The worst error ratio achieved was 2.57 at the 22.5 degree point. The error ratios achieved in Method 2 were equal to or lower than the error ratios obtained in Method 1 at 0 degrees, 67.5 degrees , and 90 degrees. The actual peak stress error obtained was equal to the target value. Method 1 and method 2 are limited by the initial geometry definition.

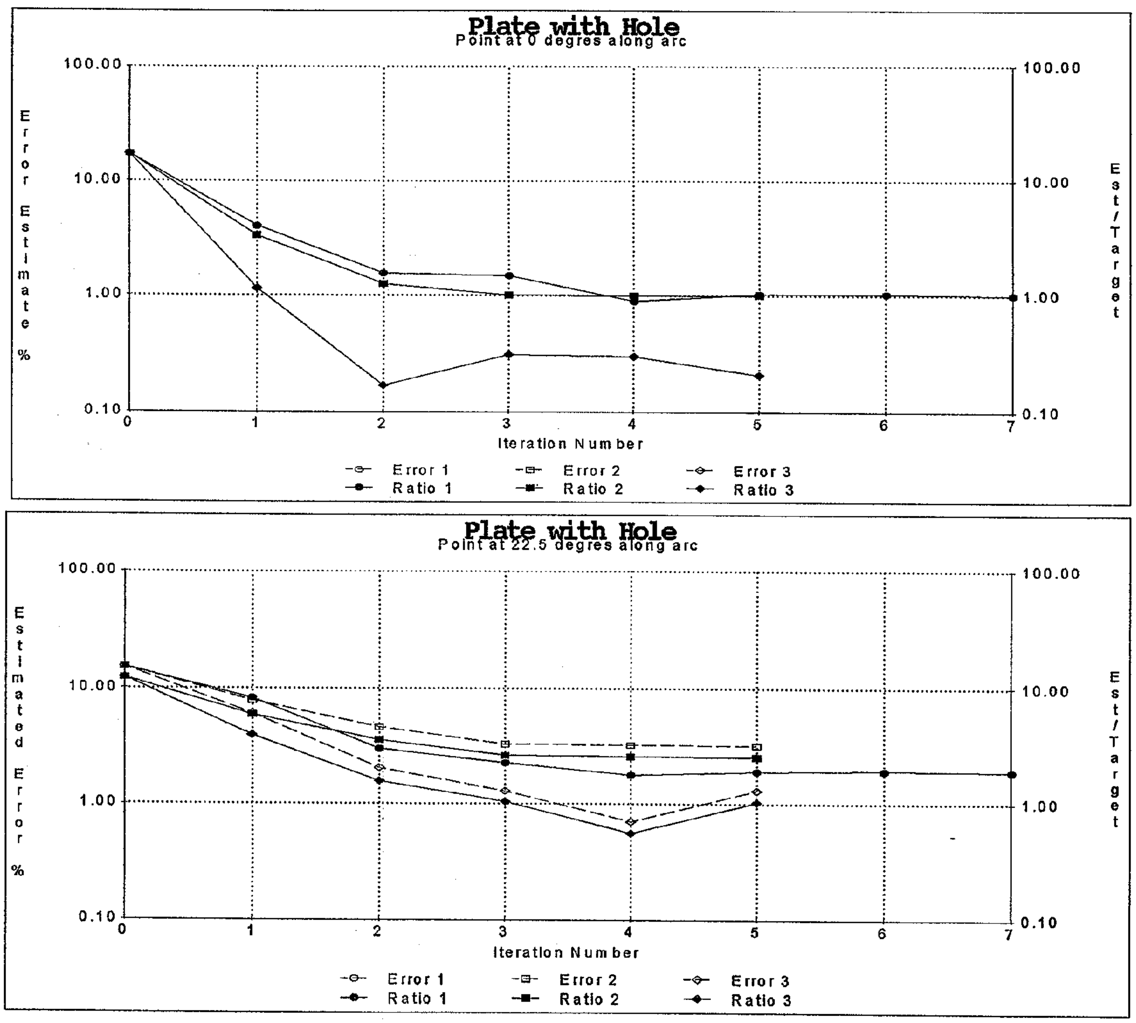


Figure 7

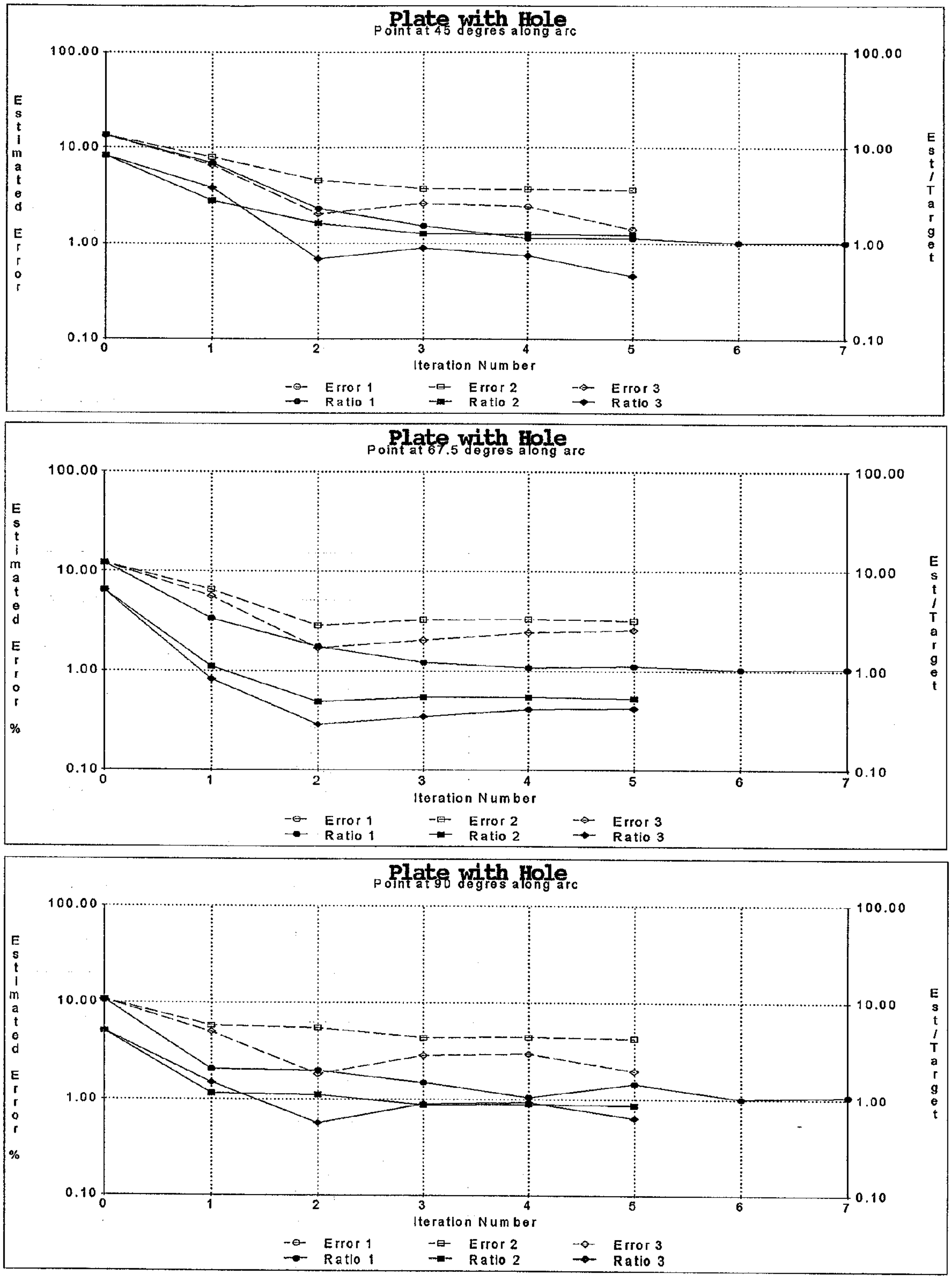


Plate with Hole
Point at 45 degres along arc
Estimated Error
Est/Target
100.00
10.00
1.00
0.10
0 1 2 3 4 5 6 7
Iteration Number
Error 1
Error 2
Error 3
Ratio 1
Ratio 2
Ratio 3
Plate with Hole
Point at 67.5 degres along arc
Estimated Error %
Est/Target
100.00
10.00
1.00
0.10
0 1 2 3 4 5 6 7
Iteration Number
Error 1
Error 2
Error 3
Ratio 1
Ratio 2
Ratio 3
Plate with Hole
Point at 90 degres along arc
Estimated Error %
Est/Target
100.00
10.00
1.00
0.10
0 1 2 3 4 5 6 7
Iteration Number
Error 1
Error 2
Error 3
Ratio 1
Ratio 2
Ratio 3

Figure 8

The third method ran for five (5) iterations and approximately 7 % of the CPU time used in Method 1. The estimated errors and error ratios achieved were consistently lower than either Method 1 or Method 2. The IsoValue based preconditioning allowed this method to achieve error estimates less than or equal to target values at all points along the hole and a global error ratio 1.39, while still retaining a high level of efficiency. The actual peak stress error obtained was 0.92 % with a target value of 1.0%. The actual peak stress error reached the target error after just three (3) iterations with the estimated error reaching target value within two (2) iterations. This method resolved the problem of limitations due to initial geometry definition. The results of this method are a function of the IsoValues used for geometry creation.

Conclusions

The following conclusions can be made from this research activity:

1.) Adaptive Remeshing can be used to approach a target error.
2.) Adjusting the division control parameters to achieve target errors is limited in as a function of the initial geometry.
3.) Adjusting the division control parameters can be fully automated.
4.) Using a variable percentage target error dramatically improves efficiency but has a small detrimental affect on accuracy.
5.) IsoValue Based Preconditioning for mesh flow control dramatically increases the accuracy of results while retaining efficiency of solution.
6.) IsoValue Based Preconditioning achieved target values for all errors and is not limited by initial geometry.
7.) IsoValue Based Preconditioning controls the element orientation to align elements with the field of interest and dramatically reduces the effect of element size and shape distortion as related to element error.
7.) Additional research is required related to the application and automation of IsoValue based preconditioning as an Adaptive Remeshing method.
8.) Additional efficiency can be obtained by recognizing "near" convergence.

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