THREE DIMENSIONAL SLIDELINE CONTACT

Rakesh Allahabadi, Ph.D., P.E.
The MacNeal-Schwendler Corporation
815 Colorado Boulevard
Los Angeles, California

ABSTRACT

Slideline contacts in MSC/NASTRAN model the separation and sliding of finite amplitude between three dimensional deforming bodies. The modeling of contact requires the user to specify slideline planes in which the interaction can occur. The sliding between bodies occur along lines, specified by lists of grid points, within the slideline planes. The bodies can have large relative motions within the slideline planes. However, relative motions outside the slideline planes are ignored; therefore, they must be small compared to a typical slideline element.

The finite element formulation for both contact and friction is based on the penalty method. But, the user need not specify the penalty values as the program automatically calculates them. The slideline element force vector and the stiffness matrix are derived from a variational principle using a consistent linearization procedure. The formulation is capable of modeling nonlinear contact geometry and inelastic material behavior including large deformation.

The three dimensional slideline contact is a standard feature in Version 68 for quasi-static analyses (SOL 106). However, a special DMAP is required for Version 67.5.

INTRODUCTION:

Prior to MSC/NASTRAN Version 67.5, the only way to model contact between bodies was through the gap element. The modeling of contact using gap elements is restrictive in the sense that the gap element models contact between two discrete grid points known a priori, and large relative motion between two deforming bodies can not be modeled. With the introduction of three dimensional (3–D) slideline contact in Version 67.5, with a DMAP alter called **contact.v675**, a user may model contact between two surfaces that undergo large relative motions. The new contact capability is useful for structural problems where two or more separate components may independently come in contact, separate or slide, such as rubber seals in disc brakes, O–rings and rubber springs, etc.

The contacting components may be three dimensional. However, the contact between the components, i.e., separation or sliding, must occur in a plane called the slideline plane. Hence, the name 3–D slideline contact. A typical finite element slideline contact region is shown in Figure 1. In the figure shown, the slideline plane is the X–Y plane. The relative motion between components can occur only in the X–Y plane. In general, the slideline plane can be any plane a user desires. As shown in the figure, a slideline contact region consists of piecewise linear slave and master lines. The grid points along the slave and master lines are called the slave and master nodes, respectively. Similarly, line segments joining adjacent slave and master nodes are called slave and master segments. In a finite element context, contact is determined between slave nodes and the master line. Slave nodes are constrained to slide on the master line after contact and must remain on the master line until a tensile interface force develops.

Slideline contact regions are specified by the user. For a contact region, a user needs to specify the normal to the slideline plane called the slideline plane vector, slave and master lines, and friction properties. Currently, only a Coulomb friction law with equal static and kinetic coefficient of friction is permitted. Slave and master lines are defined by providing a list of grid points in a certain topological order. Also, a user can specify contact between the master nodes and the slave line in addition to the the contact between the slave nodes and the master line without defining a new contact region. This is called symmetric penetration.

THEORY:

MSC/NASTRAN uses a penalty method to solve the contact problem. The residual force vector and the stiffness matrix are derived using a consistent linearization procedure based on the work done by Ju [1], Simo, Wriggers and Taylor [2], Stein, Wriggers and Van [3], and Allahabadi [4].

A three-node slideline element, shown in Figure 2, is automatically created whenever a contact is detected between a slave node and a master segment. The first node of the slideline element is the slave node, the second node is the first node of the master segment and the third node is the second node of the master segment. Each node has two degrees of freedom. Hence, there are six degrees of freedom for the element. The first degree of freedom for a node is displacement in the element tangential direction (t) and the second degree of freedom is the displacement in the element normal direction (t). The tangential direction (t) is the direction from master node 1 to mater node 2 and is given by

$$t = \frac{x_2 - x_1}{|x_2 - x_1|} = \frac{x_2 - x_1}{l} \tag{1}$$

where $x_1 = X_1 + u_1$, $x_2 = X_2 + u_2$ are the current position of the two master nodes (X_1, X_2) are the reference coordinates, and u_1 , u_2 are the current nodal displacements for the master nodes 1 and 2, respectively), and l is the current length of the master segment.

The element normal direction (n) is the direction perpendicular to the tangential direction in the slideline plane, and is given by

$$n = e_3 \times t \tag{2}$$

where e_3 is the unit vector normal to the slideline plane.

Surface coordinate (a) is the current projection of the slave node (s) on the master segment, and is given by

$$a = \frac{x_s - x_1}{|x_2 - x_1|} \cdot t = \frac{x_s - x_1}{l} \cdot t \tag{3}$$

where $x_s = X_s + u_s$ is the current position of the slave node.

The finite element formulation of the contact constraint is divided into three parts. The first part gives the formulation for frictionless contact, the second part gives the formulation for the friction sticking part and the third part gives the formulation for the friction slipping part. The friction sticking is the lastic part of the friction phenomenon.

Normal Contact:

MSC/NASTRAN determines the contact between a slave node and the master line by measuring how close the slave node is to the master segment in the normal direction. The normal gap g_n of the slave node s into the master segment is the projection of the vector from x_1 to x_s in the normal direction, and is given by

$$g_n = (x_s - x_1) \cdot n \tag{4}$$

Penetration is indicated by a negative value of the gap. The location of the penetration is given by the surface coordinate (Equation 3).

In the penalty method, the gap constraint (i.e., the slave node can not penetrate the master segment) is imposed by adding a penalty term to the energy potential, and the solution is obtained by minimizing the modified energy potential. Mathematically, the modification to the energy potential (π_n) due to the gap constraint for a slave node is

$$\pi_n = \frac{\varepsilon_n}{2} g_n^2 \tag{5}$$

where ε_n is the penalty value.

The solution to the constraint problem is obtained by minimizing the modified potential with $\varepsilon_n \to \infty$. The contact residual (i.e., the right hand side or load vector) and the contact stiffness (i.e., the left hand side) in the finite element formulation due to the contact constraint are obtained by the first and the second variations of π_n , respectively. A stiffness matrix obtained by the second variation of π_n is called a consistently linearized stiffness matrix. The residual force vector G_n and the contact stiffness matrix K_n for normal contact are:

$$G_n = -\varepsilon_n g_n N_s \tag{6}$$

$$K_n = \varepsilon_n \left[N_s N_s^T - \frac{g_n}{l} \left(T_s N^T + N T_s^T + \frac{g_n}{l} N N^T \right) \right]$$
 (7)

where
$$N_s^T = \{ 0, 1, 0, -(1-a), 0, -a \} \cdot n$$

 $N^T = \{ 0, 0, 0, -1, 0, 1 \} \cdot n$
 $T_s^T = \{ 1, 0, -(1-a), 0, -a, 0 \} \cdot t$

The vectors N_s and T_s represent displacements at element degrees of freedom corresponding to a unit displacement in the normal and the tangential direction, respectively at the slave node. The vector N represents displacement at the master segment degrees of freedom due to a unit displacement in normal direction at the master node 2.

The first term in the stiffness matrix is the only one that arises in the case of linear kinematics. The extra terms in the stiffness matrix are the result of nonlinear kinematics or consistently linearizing the residual force vector.

Friction Sticking:

Similar to the normal contact, the constraint potential π_i due to friction sticking for a slave node is given by

$$\pi_i = \frac{\varepsilon_n}{2} g_i^2 \tag{8}$$

where ε_t is the friction penalty value and g_t is the tangential slip. In the literature different definitions of tangential slip are given. Ju[1] defines tangential slip as

$$g_t = (a - a^0)l (9)$$

where a^0 is the previous surface coordinate. On the other hand, Stein et al. [3] define tangential slip as

$$g_t = (a - a^0)l^0 (10)$$

where l^0 is the previous length. Different mathematical definition for tangential slip results in different expressions for the residual force vector and the consistently linearized stiffness matrix. For MSC/NASTRAN instead of using a mathematical definition of slip, Allahabadi [4] derived the expressions for the residual force vector and the stiffness matrix from the physics of the problem. The expressions as used in MSC/NASTRAN are:

$$G_t = -\varepsilon_t g_t \left[T_s + \frac{g_n}{l} N \right] \tag{11}$$

$$K_{t} = \varepsilon_{t} \left[T_{s} T_{s}^{T} + \frac{g_{n}}{l} \left(T_{s} N^{T} + N T_{s}^{T} + \frac{g_{n}}{l} N N^{T} \right) + \frac{g_{t}}{l} \left(N_{s} N^{T} + N N_{s}^{T} \right) \right]$$

$$(12)$$

where
$$N^T = \{ 0, 0, 0, -1, 0, 1 \} \cdot n$$

 $T_s^T = \{ 1, 0, -(1-a), 0, -a, 0 \} \cdot t$
 $T^T = \{ 0, 0, -1, 0, 1, 0 \} \cdot t$

The difference in the expressions for residual force vector and stiffness matrix for friction sticking between Ju [1], Stein et al. [3], and Allahabadi [4] are in the higher order terms. It should be noted that the first four terms in the friction sticking stiffness matrix are the only ones that arise in the case of linear kinematics. The additional terms in the friction sticking stiffness matrix are due to nonlinear kinematics.

Friction Slipping:

Coulomb's law of friction is used for the finite element formulation of the friction slipping part. The coefficient of static and kinetic friction are assumed to be equal. As per Coulomb's law slipping occurs when the magnitude of tangential contact force for a slave node is equal to the normal force times the coefficient of friction. The friction force (F_i) is opposite to the slip direction and remains constant. That is for a slave node

$$F_t = -sgn(g_t)\mu\varepsilon_ng_n \tag{13}$$

where $sgn(g_i)$ gives the slip direction, μ is the coefficient of friction, ε_n is the contact (gap) penalty value, and g_n is the normal gap.

The expressions for the residual force vector and the friction slipping stiffness matrix as used in MSC/NASTRAN [4] are:

$$G_{t} = sgn(g_{t})\mu\varepsilon_{n}g_{n}\left[T_{s} + \frac{g_{n}}{l}N\right]$$
(14)

$$K_{t} = -sgn(g_{t})\mu\varepsilon_{n}\left[T_{s}N_{s}^{T} + \frac{g_{n}}{l}NN_{s}^{T} + \frac{g_{n}}{l}(N_{s}N^{T} + NN_{s}^{T})\right]$$
(15)

It must be noted that the first two terms of the stiffness matrix are the only ones that arise in linear kinematics. Even in linear kinematics, the matrix is unsymmetric. The other terms in the stiffness matrix are due to nonlinear kinematics. In MSC/NASTRAN, a symmetric form of stiffness matrix is used to avoid decomposition of an unsymmetric matrix. The use of the symmetric form should not cause any difference in the result as the correct expression for the residual force vector is used.

USER INTERFACE:

The current user interface consists of five Bulk Data entries and one Case Control command. The five Bulk Data entries are: BLSEG, BCONP, BFRIC, BWIDTH and BOUTPUT. The Case Control command is BOUTPUT. The detail description of the Bulk Data entries and the Case Control command is given in Appendix A.

The Bulk Data entry BLSEG defines a slideline via a list of grid points in topological order. The identification of a slideline must be unique. The Bulk Data entry BFRIC defines the coefficient of friction. The Bulk Data entry BCONP is a top level entry that defines the contact region and its friction properties. It points to two BLSEG entries to define slave and master lines, a BFRIC entry to define the coefficient of friction for the contact region, a coordinate system entry to define the normal to the slideline plane, and also specifies symmetrical or unsymmetrical penetration.

In MSC/NASTRAN the contact and friction forces/stresses are associated with slave nodes. In order to compute the contact and friction stresses, an area is associated with each slave node. This area is based on the the contributory length and the width/thickness for 3–D/2–D components from the adjacent slave segments based on the initial geometry. Therefore, the user needs to specify widths associated with slave nodes. This is done by the use of BWIDTH Bulk Data entry.

The user can request results for any number of slave nodes for any number of slideline contact regions. BOUTPUT Bulk Data entry specifies a list of slave nodes for a particular slideline region. The identification of BOUTPUT entry is the same as the identification of the contact region (ID field in BCONP Bulk Data entry). The Case Control command BOUTPUT points to a set ID that selects the contact regions for which output is desired.

The output for a slave node consists of (a) the slideline contact region identification number, (b) the master segment to which it projects, (c) the parametric surface coordinate to identify the exact projection of slave node relative to the two master nodes, (d) contact and tangential forces and stresses in the element coordinate system, (e) a slip ratio to indicate whether the slave node is sticking, slipping or sliding, and (f) the status of the slave node (open, overhang, stick, slip or slide).

MSC/NASTRAN does not allow initial penetration between bodies. The initial gaps between the slave nodes and the master line are calculated based on the coordinates specified for the slave and the master nodes. To avoid having the user to specify the coordinates vary accurately, MSC/NASTRAN automatically adjusts the slave node coordinates to preclude penetration if the penetration is less than ten percent of the master segment length, and issues a warning message. However, if the initial penetration is more than ten percent, the analysis is terminated and a fatal message is issued.

It is the user's responsibility to make sure that the normals of the master segment point towards the slave line. This is easily accomplished by making sure that the cross product between the slideline plane direction and the master line direction points towards the slave line. If the normals do not point towards the slave line, the analysis may be terminated as the initial gaps will be taken as penetrations.

The slave and the master lines must lie in the slideline plane in the initial geometry. MSC/NASTRAN checks to make sure that this is indeed the case. Otherwise, analysis is terminated and a fatal message is issued. However, during the analysis relative motions outside the slideline plane are ignored, and therefore must be small compared to a typical master segment.

EXAMPLE PROBLEMS:

Three example problems taken form Allahabadi [5] are presented to demonstrate the 3–D slideline contact capability: Elastic punch sliding on an elastic foundation; Frictional Elastoplastic O–ring contact; and Contact of Two Beams.

Elastic Punch Sliding on an Elastic Foundation: An elastic punch is pushed into an elastic foundation made of same material, and then moved to the right till the punch is 90% overhanging. The Young Modulus and poisson ratio for both the punch and the elastic foundation are 1.0E+5 psi, and 0., respectively. The slave line consists of the three bottom nodes of the punch (See Figure 3.1). The master line consists of the top eleven nodes of the foundation. The coefficient of friction is 0.1. The analysis is performed in two subcases. In the first subcase the elastic punch is punched into the foundation with a total load of 4000 lbs. In the second subcase the elastic punch is moved to the right with a displacement increment of one inch until it overhangs by 90%. The deformed shapes at the end of subcase one and two are shown in Figures 3.2 and 3.3. Figure 3.2 is not drawn to scale to show small amount of penetration of punch into the foundation. This is acceptable as the penalty method does not satisfy the contact constraint exactly.

Frictional Elastoplastic O-ring Contact: This problem demonstrate the 3-D slideline capability with elasto-plastic material. Consider an elastic plate in contact with an elasto-plastic metallic O-ring. Both the plate and the O-ring are modeled using QUAD4 elements, as shown in Figure 4.1. Advantage is taken of symmetry in the model. Young modulus and poisson ratio for the O-ring are 2.8E07 psi and 0.27, respectively. Young Modulus and poisson ratio for plate are 3.0E07 psi and 0.3 respectively. Isotropic hardening is assumed for the O-ring. The yield stress for the O-ring is 46417.0 psi. A contact region is defined between the bottom nodes of the plate and the outer nodes of the O-ring. The coefficient of friction is 0.3. The top of the plate is pushed down by 0.067 inched in ten load steps. Figure 4.2 shows the deformed shape. In the undeformed configuration only one plate grid point is in contact with the O-ring. In the deformed configuration four plate grid points are in contact with O-ring and the plate has moved down by 0.067 inches. Figure 4.3 shows the normal and tangential contact stress versus the contact radius for the O-ring. Figure 4.4 shows the plastic regions for the O-ring. The shaded portion in the plate indicates that the stresses in that portion are higher than 46417.0 psi, the yield stress value for the O-ring.

Contact of Two Beams: This problem demonstrates the 3–D slideline capability for geometric nonlinear problems. A beam with large deformations comes in contact with another beam (See Figure 5.1). Large deformations are specified by moving the tip of the top beam downward by 56 inches in 14 increments. The slave line consists of only one slave node which is at the tip of the top beam. The master line consists of five rightmost grid points of the lower beam. No friction is assumed between the two beams. The deformed shapes at various stages are shown in Figure 4.2.

Figure 4.3 shows the variation of contact force, beam reactions and the equivalent total load versus the tip deflection of the top beam.

SUMMARY:

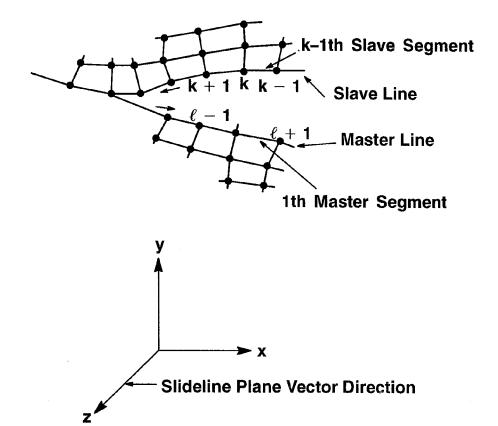
Three dimensional slideline contact capability will be available as a standard feature in MSC/NASTRAN SOL 106, Version 68 to perform Quasi-static nonlinear contact problems with inelastic deformations and nonlinear contact kinematics. Penalty method is used to enforce both the contact and the friction constraints. Coulomb's law with equal static and kinetic coefficient is used for friction. Penalty values are automatically calculated by the program. Users interested in using the capability in Version 67.5 need a special DMAP alter called **contact.v675** which is available in the DMAP alter library for Version 67.5. This capability can be used in combination with any other nonlinear static capability.

ACKNOWLEDGEMENT:

The author acknowledges the dedication and help of Dr. Bhoomaiah Alishetti of The MacNeal–Schwendler Corporation for coding the 3–D slideline contact capability in MSC/NASTRAN.

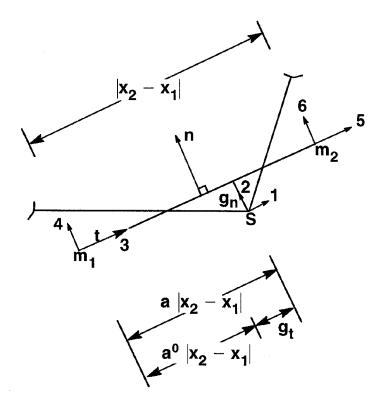
REFERENCES:

- 1. Ju, J. W., "On Finite Element Solution of Frictional Nonlinear Contact Problems: Perturbed and Augmented Lagrange Formulations," Department of Civil Engineering and Operations Research, Princeton University, Princeton, N.J., 1989.
- 2. Simo, J. C., Wriggers, P. and Taylor, R. L.,"A Perturbed Lagrangian Formulation for the Finite Element Solution of Contact problems," Report No. UCB/SESM-84/14. Department of Civil Engineering, University of California, 1984.
- 3. Stein, E., Wriggers, P. and Van, T. vu,"Models of Friction, Finite–Element–Implementation and Application to Large Deformation Impact–Contact Problems," University of Hannover, Hannover, Germany.
- 4. Allahabadi, R., "Software Requirement Specifications for 3–D Slideline Contact Project in MSC/NASTRAN," MEMO RAA–002A, June 20, 1991.
- 5. Allahabadi, R., "Development Test Plan and Report for Slideline Contact in MSC/NASTRAN V68," MEMO RAA–006, October 28, 1992.



- X-Y plane is the slide line plane. Unit normal in the Z-direction is the slide line plane vector.
- Arrows show positive direction for ordering nodes. Counter-clockwise from master line to slave line.
- Slave and master segment normals must face each other.

Figure 1: A Typical Finite Element Slideline Contact Region



- S, m_1 , m_2 = slave, master node 1 and master node 2, respectively.
- a, a^0 = current and previous surface coordinate.
- g_n = penetration of slave node into the master segment
- g_t = sliding of the slave node on the master segment
- n = normal direction for the master segment.

Figure 2: Three Node Slideline Element

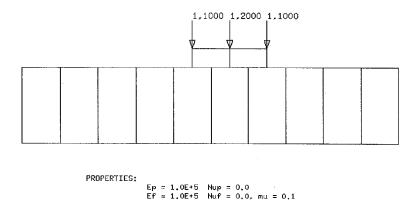


Figure 3.1: Elastic Punch Sliding On An Elastic Foundation (Undeformed)

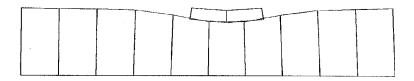


Figure 3.2: Elastic Punch Sliding On An Elastic Foundation (Subcase 1)

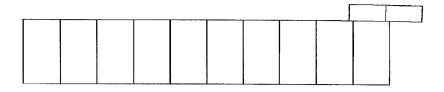


Figure 3.3: Elastic Punch Sliding On An Elastic Foundation (Subcase 2)

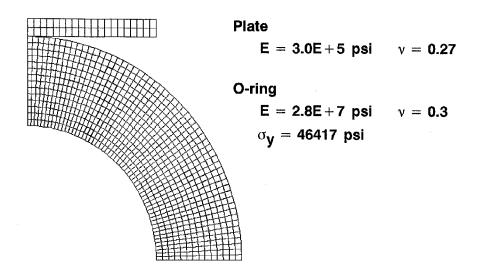


Figure 4.1: Frictional Elastoplastic O-ring Contact (Undeformed)

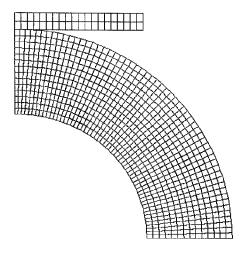


Figure 4.2: Frictional Elastoplastic O-ring Contact (Deformed)

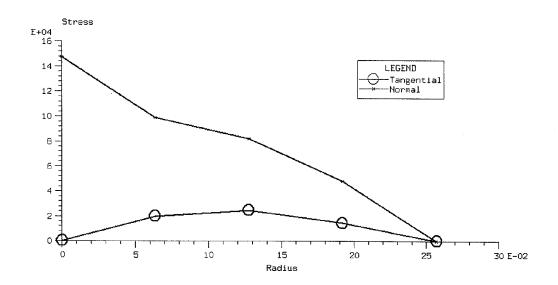


Figure 4.3: Normal and Tangential Contact Stress Versus Contact Radius for O-ring

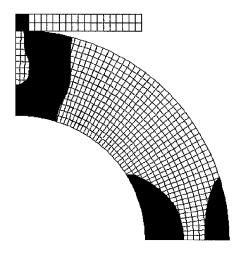


Figure 4.4: Frictional Elastoplastic O-ring Contact – Plastic Regions

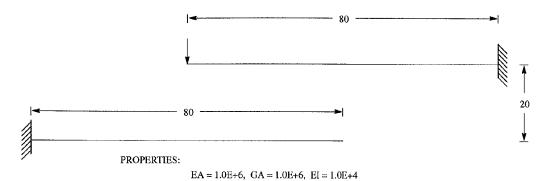


Figure 5.1: Contact of Two Beams (Undeformed)

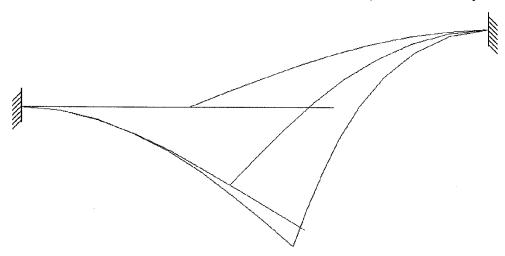


Figure 5.2: Contact of Two Beams (Deformed Shapes)

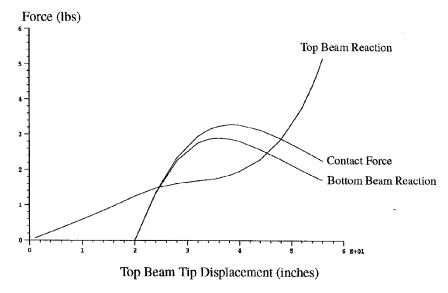


Figure 5.3: Contact of Two Beams (Forces – Deflection Plots)

APPENDIX A

Bulk Data Entry: BCONP — Contact Parameters

Defines the parameters for contact between two bodies.

Format:

1	2	3	4	5	6	7	8	9	10
BCONP	ID	SLAVE	MASTER	·	SFAC	FRICID	PTYPE	CID	
			· · · · · · · · · · · · · · · · · · ·		·		·		

Example:

	·						
BCONP	95	10	15	1.0	33	1	

Field	Contents
ID	Contact region identification number (Integer > 0). See Remark 1.
SLAVE	Slave region identification number (Integer > 0). See Remark 2.
MASTER	Master region identification number (Integer > 0). See Remark 3.
SFAC	Stiffness scaling factor (Real > 0 . or blank). Default $= 1.0$. This factor is used to scale the penalty values automatically calculated by the program. See Remark 4.
FRICID	Contact friction identification number (Integer > 0 or blank). See Remark 5.
PTYPE	Penetration type (1 ≤ integer ≤ 2 or blank). See Remark 6. 1: unsymmetrical (slave penetration only) (default) 2: symmetrical
CID	Coordinate system ID to define the slideline plane vector and the slideline plane of contact (Integer > 0 or blank). Default = Basic Coordinate System. See Remark 7.

Remarks:

- 1. ID field must be unique with respect to all other BCONP identification numbers.
- 2. The referenced SLAVE is the identification number in the BLSEG Bulk Data entry. This is the slave line. The width of each slave segment must also be defined to get proper contact stresses. See BWIDTH Bulk Data entry for the details of specifying widths.
- The referenced MASTER is the identification number in the BLSEG Bulk data entry. This is the master line.
 For symmetrical penetration, the width of each master segment must also be defined. See BWIDTH Bulk data entry for the details of specifying widths.
- 4. The SFAC field may be used to scale the penalty value automatically calculated by the program. The program calculates the penalty value as a function of the diagonal stiffness matrix coefficients that are in the contact region.

- 5. The referenced FRICID is the identification number of the BFRIC Bulk Data entry. The BFRIC defines the frictional properties for the contact region.
- 6. In an unsymmetrical contact algorithm only slave nodes are checked for penetration into master segments. This may result in master nodes penetrating the slave line. However, the error involve depends only on the mesh discretization. In symmetric penetration both slave and master nodes are checked for penetration. Thus, no distinction is made between slave and master. Symmetric penetration may be up to thirty percent more expensive than the unsymmetric penetration.
- 7. Unit vector in the the Z-axis of the coordinate system defines the slideline plane vector. Slideline plane vector is normal to the slideline plane. Relative motions outside the slideline plane are ignored, therefore must be small compared to a typical master segment. For a master segment the direction from master node 1 to master node 2 gives the tangential direction (t). The normal direction for a master segment is obtained by cross product of the slideline plane vector with the unit tangent vector (i.e., n = z x t). The definition of the coordinate system should be such that the normal penetration direction must point towards the slave region. For symmetric penetration the normals of master segments and slave segments must face each other. This is generally accomplished by traversing from master line to slave line in a counter—clockwise or clockwise fashion depending on whether the slideline plane vector forms right hand or left hand coordinate system with the slideline plane.

Bulk Data Entry: BLSEG — Boundary line segments

Description: Defines a curve which consists of a number of line segments via grid numbers that may come in contact with other body. A line segment is defined between every two consecutive grid points. Thus, number of line segments defined is equal to the number of grid points specified minus 1. A corresponding BWIDTH Bulk data entry may be required to define the width/thickness of each line segment. If the corresponding BWIDTH is not present, the width/thickness for each line segment is assumed unity.

Format:

1	2	3	. 4	5	6	7	8	9	10
BLSEG	ID	G1	G2	G3	G4	G5	G6	G7	
	G8	THRU	G9	BY	G10				
	G11	G12							

Examples:

BLSEG	1 5	5	THRU	21	BY	4		
	. 27	30	32	33				
	35	THRU	44					

Field	Contents
ID	Line segments identification number (Integer>0). See Remark 1.
Gi	Grid numbers on a curve in a continuous topological order so that the normal to the segment points towards other curve. See Remark 2.

Remarks:

- 1. ID field must be unique with respect to all other BLSEG entries. Each line segment has a width in 3-D sideline and a thickness in a 2-D slideline contact to calculate contact pressures. The width/thickness of each line segment is defined via BWIDTH Bulk data entry. The ID in BLSEG must be same as the ID specified in the BWIDTH. That is, there must be one to one correspondence between BLSEG and BWIDTH. BWIDTH bulk data entry may be omitted only if the width/thickness of each segment is unity.
- 2. The grid numbers may be automatically generated using the THRU and BY keywords. For first line, THRU and BY can only be specified in the fourth and the sixth fields, respectively. For continuation lines, THRU and BY can only be specified in the third and the fifth fields, respectively. For automatic generation of grid numbers the default value for increment is 1 if grid numbers are increasing or -1 if grid numbers are decreasing. That is the user need not specify BY and the increment value.

The normal to the segment is determined by the cross product of the slideline plane vector (i.e. the Z direction of the coordinate system defined in the 'CID' field of BCONP bulk data entry) and the tangential direction of the segment. The tangential direction is the direction from node 1 to node 2 of the line segment.

A curve may be closed or open. A closed curve is specified by having the last grid number same as the first grid number.

Bulk Data Entry: BFRIC — Contact Friction

Description: Defines frictional properties between two bodies in contact.

Format:

1	2	3	4	5	6	7	8	9	10
BFRIC	FID		FSTIF	MU1					
Example(s	i):								
BFRIC	33		1	0.3					

Field	Contents
FID	Friction identification number (Integer > 0). See Remark 1.
FSTIF	Frictional stiffness in stick (Real $>$ 0.0). Default $=$ automatically selected by the program. See Remarks 2 and 3.
MU1	Coefficient of static friction (Real > 0.0).

Remarks:

- 1. This identification number must be unique with respect to all other friction identification numbers. This is used in the FRICID field of BCONP Bulk data entry.
- 2. The value of frictional stiffness requires care. A method of choosing its value is to divide the expected frictional strength (MU1 x the expected normal force) by a reasonable value of the relative displacement which may be allowed before slip occurs. The relative value of displacement before slip occurs must be small compared to expected relative displacements during slip. A large stiffness value may cause poor convergence, while too small value may cause poor accuracy.
- 3. The stiffness matrix for frictional slip is unsymmetric. However, the program does not use the true unsymmetric matrix. Instead the program uses only the symmetric terms. This is to avoid using the unsymmetric solver to reduce CPU time.

(Note: No distinction is made between static and kinetic friction.)

Bulk Data Entry: BWIDTH — Boundary line segments width/thickness.

Description: Defines width/thickness for line segments in 3-D/2-D slideline contact defined in the corresponding BLSEG BULK Data entry. This entry may be omitted if the width of each segment defined in the BLSEG entry is unity. Number of widths to be specified is equal to the number of segments defined in the corresponding BLSEG entry. If there is no corresponding BLSEG entry, the width specified in the entry are not used by the program.

Format:

1	2	3	4	5	6	7	8	9	10
BWIDTH	ID	W 1	W2	МЗ	W4	W5	W6	W7	
	W8	THRU	W9	BY	W10				
	W11	W12							

Examples:

BWIDTH	15	2.0	THRU	5.0	BY	1.0		
	2.0	2.0	2.0	2.0				
	35.	THRU	44.					

Field	Contents
ID	Width set identification number (Integer>0). See Remark 1.
Wi	Width values for the corresponding line segments defined in the BLSEG entry. See Remarks 2 and 3.

Remarks:

- 1. The ID field must be unique with respect to all other BWIDTH entries. It must be the same as the ID field in the corresponding BLSEG entry.
- 2. The widths may be automatically generated using the THRU and BY keywords. For first line, THRU and BY can only be specified in the fourth and the sixth fields, respectively. For continuation lines, THRU and BY can only be specified in the third and the fifth fields, respectively. For automatic generation of the width values the default value for increment is 1.0 if the width is increasing or -1.0 if the width is decreasing. That is the user need not specify BY and the increment value. If the number of width specified are less than the number of segments defined in the corresponding BLSEG entry, the width for the remaining segments is assumed to be equal to the last width specified.
- 3. If there is only one grid point in the corresponding BLSEG entry, there is no contributory area associated with the grid point. To compute correct contact stresses an area may be associated with the single grid point by specifying the area in field W1.

Bulk Data Entry: BOUTPUT — Output for Slideline contact.

Description: Defines slave nodes at which output is requested.

Format:

1	2	3	4	5	6	7	8	9	10
воитрит	ID	ALL							
	G1	G2	G3	G4	G5	G6	G7	G8	
	G8	THRU	G9	BY	G10				

Example:

BOUTPUT	15	ALL				
						l l

Field	Contents
ID	Boundary identification number for which output is desired (Integer > 0). See Remark 1.
GI	Slave node numbers for which output is desired. See Remark 2.

Remarks:

- 1. The ID is the identification number of a BCONP entry.
- 2. The grid numbers may be automatically generated using the THRU and BY keywords. For first line, THRU and BY can only be specified in the fourth and the sixth fields, respectively. For continuation lines, THRU and BY can only be specified in the third and the fifth fields, respectively. If output is desired for all the slave nodes, specify the word ALL in the third field of the first line or just include the contact region ID in the Case Control command BOUTPUT.

Note: ID is the same as the corresponding BCONP ID. This entry can selectively specify slave grid points for which OUTPUT is desired.

 $\textbf{Case Control Command:} \ \ BOUTPUT-Contact \ Output \ Requests$

Description: Selects slave nodes specified in the Bulk Data entry BOUTPUT for history output.

Format:

$$\label{eq:boutput} \text{BOUTPUT} \begin{bmatrix} \text{SORT1}, & \text{PRINT} \\ \text{SORT2}, & \text{PUNCH} \\ & \text{PLOT} \end{bmatrix} = \left\{ \begin{matrix} \text{ALL} \\ \text{n} \\ \text{None} \end{matrix} \right\}$$

Example:

BOUTPUT = ALL

BOUTPUT = 5

Field	Contents
SORT1	Output is presented as a tabular listing of slave nodes for each load or time depending on the solution sequence.
SORT2	Output is presented as a tabular listing of load or time for each slave node.
PRINT	The print file (FORTRAN I/O unit 6) is the output media.
PUNCH	The punch file is the output media.
PLOT	Generate slave node results history but do not print.
ALL	Histories of all the slave nodes listed in all the BOUTPUT bulk data entries are output. If no BOUTPUT bulk data entries are specified, histories of all the slave nodes in all the contact regions are output.
n	Set identification of previously appearing set command. Only contact regions whose identification numbers appear on the set command are selected for output. If there is a BOUTPUT bulk data entry for a contact region selected via the set command, histories for slave nodes listed in the bulk data entry are output. If there is no BOUTPUT bulk data entry for a contact region selected via the set command, histories for all the slave nodes in that contact region are output.
None	Result histories for no slave nodes are output.

Note: This command selects the contact region for which output is desired.