NONLINEAR GAP-TYPE SOLUTIONS USING A LINEAR F.E.A. CODE

by

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Abstract

The most common form of structural Finite Element Analysis (FEA) is the linear static solution, in which the behavior of each element can be characterized as a linear equation. Linear static FEA cannot be used for problems with nonlinear gap-type elements, as their load vs. deflection behavior cannot be expressed with a single linear equation. Examples of gap-type elements include a cable (an axial element which can transfer tension between its ends, but not compression) and a bearing contact (two interfering surfaces that can compress against each other, but do not adhere when separated). For a gap element, the load vs. deflection equation depends on the sense and magnitude of deflection each loading condition imposes on the element.

Many common FEA codes do not support gap elements; for those that do, adding a gap element complicates the solution by requiring extensive changes to the linear model, and by increasing the CPU time required (often several times over). As the gap behavior can vary from one loading condition to the next, a separate solution for each condition must be obtained.

The Enforced Strain Method uses an approach in which a compensating enforced strain is used to give linear elements gap-like load vs. deflection behavior. The technique can be used with linear FEA codes that do not support gap elements, or can be used as an alternate solution for gap-capable codes. Benefits of the method are reduced CPU requirements, the ability to run multiple loading cases, and no need for superelements.

The Enforced Strain Method is a more efficient gap solution, particularly when a given model has a relatively small proportion of gaps, and when multiple loading conditions are required. An example problem is presented in which the required CPU time was reduced by 43% as compared to the fastest MSC/NASTRAN gap solution. Though presented as a program external to MSC/NASTRAN, the method could be implemented through DMAP alters to the standard linear static solution. Run as a DMAP, CPU time savings for the example problem would have increased from 43 to 66% as compared to MSC/NASTRAN's nonlinear gap Solution 66.

Introduction

Linear static analysis represents the simplest form of structural Finite Element Analysis (FEA) in that each element's behavior is represented by a single linear equation; benefits include the ability to solve many loading conditions simultaneously for relatively little CPU time. Gap-type nonlinear elements are more complex in that their behavior depends on the loading condition; nonlinear solutions require more CPU time, and that each condition be solved seperately. The Enforced Strain Method is a technique that uses linear static analysis to solve many nonlinear load conditions more efficiently than nonlinear analysis.

A linear finite element is one whose stiffness (the slope of the element's load vs. deflection curve) is constant regardless of the magnitude and algebraic sense of its deflection. An example is a non-buckling column, whose load vs. deflection curve is a single continuous line (see Figure 1A). Linear problems can be written as a series of simultaneous equations, which can then be solved directly using matrix methods. FEA codes using the force/ isplacement method solve the basic equation

$$\{d\} = [k]^{-1} \{P\} \qquad \text{(eq. 1)}$$

where $\{d\}$ = vector of grid point deflections,

$[k]^{-1}$ = inverse of the grid point stiffness matrix $[k]$,

$\{P\}$ = vector of forces and moments acting on each grid point due to the external loads for the given loading condition;

It should be pointed out that inverting the stiffness matrix, $[k]$, is computationally the most "expensive" part of the problem; once inverted, additional loading conditions can be analyzed for little added expense using the same inverted stiffness matrix $[k]^{-1}$ and a new load vector $\{P\}$.

Gaps are elements whose load vs. deflection curve is not a single continuous line; frequently, their behavior can be idealized using two or more linear segments. A cable has nonzero stiffness as its ends move apart, but no stiffness (carries no compressive load) if its two ends move closer together than its initial length. A contact gap represents interfering surfaces (a pin in a socket, pressing against one side of the hole) and has only compressive stiffness. A mechanical stop (a pin in a slot) has zero stiffness until a minimum deflection takes place. The load vs. deflection curves for these examples are shown in Figures 1B - 1D.

MSC/NASTRAN's Solution 66 solves gap-type problems by iteratively changing the stiffness matrix [k] as the Finite Element model (FEM) deflects under load. Changing the stiffness of even one element requires reinversion of the modified stiffness matrix. With several iterations required per loading condition, gap-type solutions can become prohibitively expensive; additional loading conditions compound the expense.

FEA codes that support superelements improve iterative performance by partitioning the stiffness matrix into several units; the analyst separates the stiffness matrix into a small and a large component where all grids associated with nonlinear elements are isolated into the smaller "residual" matrix. Only the residual matrix must then be reassembled and reinverted when the stiffness of gap elements change. Superelements speed nonlinear solutions at the price of adding complexity to modeling.

Enforced Strain Method

The Enforced Strain method differs from this approach in that the gap element's stiffness is not changed to obtain the desired load vs. deflection behavior; rather, a "compensating" enforced strain is added to the applied load vector {P} that allows, for instance, a cable to shorten without picking up the compressive load a linear element would have. Conceptually, this is as though the cable had been cooled to a temperature such that its thermal contraction was exactly equal to the contraction of the element for that loading condition. The technique can be broken down into 5 steps:

1) Model the gaps using equivalent linear elements (i.e, 1-D axial CRODs with the appropriate non-zero stiffness in place of CGAPs);

2) Obtain a linear solution of resulting gap ROD loads for all required loading conditions;

3) Determine the mathematical relation between an enforced strain in each gap ROD and the resulting axial load in all others;

4) For each load condition, solve for a set of enforced strains that "cancel out" gap ROD axial loads that are inconsistant with the desired nonlinear gap behavior;

5) Add the enforced strains to the linear model and obtain a new linear solution (gap ROD axial loads should now agree with the results from nonlinear gap solutions).

Adding enforced strains to the linear solution alters equation 1 to

$$\{d\} = [k]^{-1} (\{P\} + [K] \{e\}) \qquad \text{(eq. 2)}$$

where $\{d\}$ = vector of grid point deflections,

$[k]^{-1}$ = inverse of the matrix of grid point stiffnesses,

$\{P\}$ = vector of forces and moments for the given loading condition;

$\{e\}$ = vector of gap compensating strains,

$[K]$ = matrix of gap element stiffnesses.

An enforced strain on one gap ROD changes the axial load in all others; the relation between strain and load can be expressed as

$$\{p\} = [C]\ \{e\} \qquad \text{(eq. 3)}$$

where $\{p\}$ = vector of gap ROD axial loads,

$[C]$ = "coefficient of influence" matrix defining the effect of a strain in one gap on the load in the others,

$\{e\}$ = vector of gap ROD enforced strains.

The matrix [C] is related to the stiffness matrices [k] and [K], and allows one to calculate an "equal and opposite" axial load to cancel inappropriate behavior of the linear gap RODs. How it is obtained will be discussed later.

Having solved for the linear gap RODs' axial loads for the required loading conditions and determined the [C] matrix for the linear model, the compensating strains must now be calculated. For a given loading condition, all gap elements that have a proper stiffness (i.e., cables in tension) are removed from [C]. The remaining partitioned matrix is inverted and multiplied by the loads in the inconsistent gap elements, or,

$$\{e_p\} = [C_p]^{-1}\ \{p_p\} \qquad \text{(eq. 4)}$$

where $\{e_p\}$ = enforced strains on inconsistant gap RODs for a given loading condition,

$[C_p]^{-1}$ = inverse of the coefficient of influence matrix with all consistent gaps partitioned out,

$\{p_p\}$ = inconsistent gap loads from the linear solution;

The resulting vector $\{e_p\}$ is a set of enforced strains that, when combined with the condition's linear solution, results in zero load in the formerly inconsistent gap RODs, and alters the load in the remaining RODs. The compensated gap loads can be predicted using

$$\{p'\} = [C]\ \{e\} + \{p\} \qquad \text{(eq. 5)}$$

where $\{p'\}$ = gap loads with compensation,

$[C]$ = full (unpartitioned) coefficient of influence matrix,

$\{e\}$ = enforced strain vector that includes compensating strains $\{e_p\}$ for the inconsistent gap RODs, and zeroes for the consistant gap RODs,

$\{p\}$ = uncompensated gap loads from the linear solution;

It is necessary to check the compensated gap load vector, $\{p'\}$, for newly inconsistant gaps. If any are found, [C] is repartitioned and the process is repeated until a fully consistant solution is found. Each additional loading condition requires a new enforced strain solution.

One key step in the enforced strain method is determining the relation between enforced strains and gap ROD axial loads as defined by equation 3. The coefficient of influence matrix [C] can be obtained by matrix operations on the stiffness matrix [k]. This method (as presented) is external to MSC/NASTRAN and cannot readily access [k], so an alternate method is used: for each gap element, a subcase with a unit enforced strain on only that gap's ROD is solved using the linear static solution. The resulting loads then define a "column" of the [C] matrix; solving all the unit strain subcases supplies the remaining columns of the matrix [C].

While this technique also iterates and inverts a matrix, it is more efficient than the stiffness iteration technique in that it deals with the much smaller coefficient of influence matrix [C]. In a typical model, each grid represents six degrees of freedom (translations and rotations in three directions). A 1000 grid model would have a stiffness matrix of 6000 degrees of freedom. If the model had 100 gaps in it, the [C] matrix would have 100 degrees of freedom (one per gap). Assuming half the gaps are inconsistent for a given condition, the enforced strain method iterates on a 50 degree of freedom coefficient matrix, rather than a 6000 degree of freedom stiffness matrix.

Using superelements for the model cited above, the residual stiffness matrix would have 200 grids (two per gap), or 1200 degrees of freedom. Reducing the matrix size by a factor of 5 results in a much quicker solution (CPU times are approximately proportional to the square of the matrix size). It should be noted that this residual matrix is 24 times as large as the partitioned coefficient matrix $[C_p]$; even with superelements, inverting the residual matrix once would require about 600 times as much CPU time as inverting the matrix $[C_p]$.

Analysis

Figure 2 shows a folding joint, in which the lugs of an inner and outer beam are connected by a hollow pin, about which the two beams may rotate. The structure is nonlinear in that the pin transfers load into the beam lugs by bearing against the holes' inner edges; reversing the direction of applied load means load transfer occurs on the other side of the hole. A nonlinear solution is required to obtain stresses in the lug region.

Figure 3 shows a FEM of the inner beam's lug. The beam itself is considered ground. The lower end of the pin is a plane of symmetry. Load is introduced into the pin at its upper end, through 36 contact gaps representing the outer beam's lug (see Figure 1-C). 36 radial contact gaps connect the pin's circumference to the inner beam's lug inner diameter. The pin is modeled with bending plate elements and the lug is modeled using planar membrane elements.

The design load for the lug is 250,000 lb. That load can be applied in any direction perpendicular to the pin axis. 10 subcases are to be run, in which the applied load direction ranges from 0 to 180 degrees, or net lug axial tension (-X direction in Figure 3) to net compression (+X direction), in steps of 20 degrees about the pin axis. Lateral symmetry obviates the need to run cases in the 180 to 360 degree range.

MSC/NASTRAN's Version 66, Solution 66 was used for the nonlinear (stiffness iterating) solution. CGAP elements were used with a compression stiffness of 11.11E6 lb/in and a tension stiffness of 11.11 lb/in (the tension stiffness should ideally be zero, but MSC recommends that the CGAP's "open" stiffness be 1/1,000,000 of the "closed" value for improved convergence). To speed execution, superelements were used. All grids connecting to gaps (two rows on the pin, and the inner circumference row of the lug's hole) were put in superelement 0 (the "residual" superelement). The remaining pin grids were put in superelement 1, and non-gap lug grids were assigned to superelement 2. Default convergence criteria and the AUTO iteration scheme were used. Table 1 shows the CPU time required for each step of the 10 runs made. The initial database setup and solution of the first loading condition took 60 seconds;

the 9 additional loading case restarts took an average of 21 seconds. Total CPU usage was 251 seconds in 10 separate runs.

The Enforced Strain method used Version 66's Solution 101 (linear statics with restart). CGAPs were replaced with linear CRODs of the same 11.11E6 lb/in stiffness. Superelements were not needed and were not used. The first run solved for CROD loads for the 10 loading conditions. A second run solved 72 additional load cases in which a unit enforced strain was applied to the 72 gap RODs. An external program then calculated the enforced strains required to make gap loads consistant with their deflections in the 10 loading cases; this step took 5 CPU sec. The DEFORM cards were added to the database and rerun for the final nonlinear solution. Total CPU usage was 139 sec. of NASTRAN time in 3 separate runs, and 5 sec in the external compensation program (shown in Table 2).

Discussion

Tables 3 and 4 compare CGAP and CROD loads from the Solution 66 nonlinear and the Solution 101 linear compensated runs. The first 36 CGAPs are at the top of the pin (for load introduction), and the second 36 are between the pin and inner beam lug. Subcase 11 shows the bearing loads from the net tension case, while Subcase 15 is a combined lateral and axial condition in which the load is applied 80 degrees off centerline. Apart from a sign convention difference between the CGAP and CROD elements, the most pronounced discrepency between the two solutions is 12 lb out of an average 18,400, or 0.07%. This small difference is due first to the non-zero "open" CGAP stiffness required for Solution 66, and second, to a limit of 7 digits of accuracy on the CROD load output used to build the coefficient matrix [C] for the Enforced Strain solution.

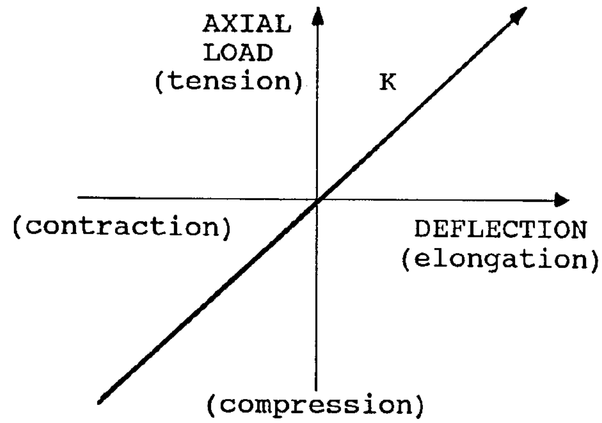
Table 2 shows that almost half of the CPU time (68 of 144 sec) used by the Enforced Strain method was in the Unit-Strain restart run, whose sole purpose was to obtain the coefficient matrix [C]. It has been noted that [C] could be formed directly from the stiffness matrix [k] using relatively simple matrix transformations. For this example, inverting [k] required 18 of the 51 CPU sec for the initial linear run, and constitutes a far more demanding task than generating [C]. Assuming it to take half as long as inverting [k], calculating [C] would take 9 sec instead of 68; coded as a DMAP alter to the linear solution, total Enforced Strain method CPU time would then be 85 CPU sec (versus 251 CPU sec for Solution 66), reducing CPU times by a factor of 3. Furthermore, a solution for all 10 cases would be obtained in a single run.

The Enforced Strain method has been used for several years at Bell and has simplified the analysis of large nonlinear FEMs with many loading conditions. The CPU time savings referenced are typical of the results obtained at Bell, and have been frequently exceeded.

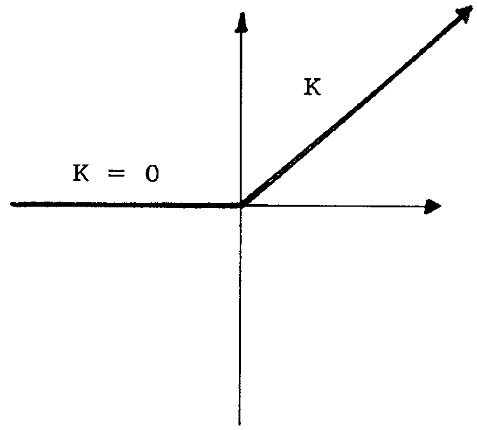
Conclusions

CPU time comparisons are important in that several FEM codes (MSC/NASTRAN, for example) charge royalties based on CPU usage; it is a useful measure of computational power required by a given job, and of its turnaround time. For the lug problem presented, the stiffness-iterating nonlinear GAP Solution 66 required the use of superelements, and that 10 separate runs be made (1 per loading condition). The Enforced Strain method as presented needed only three runs and 43% as much CPU time as Solution 66, and did not require superelements. Had the method been implemented within the FEA code (using DMAP alters, for instance), all 10 conditions could have been solved in a single run, and would have required only 33% of Solution 66's CPU time. Experience with larger models support the comparisons made here.

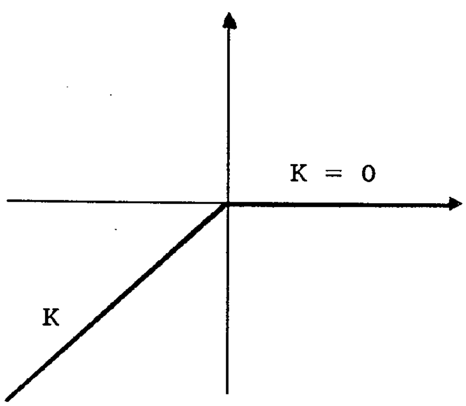
The Enforced Strain method is a viable alternative to the existing stiffness iterating nonlinear solutions, and can be used with linear FEA codes that support enforced strains and/or thermal expansion.



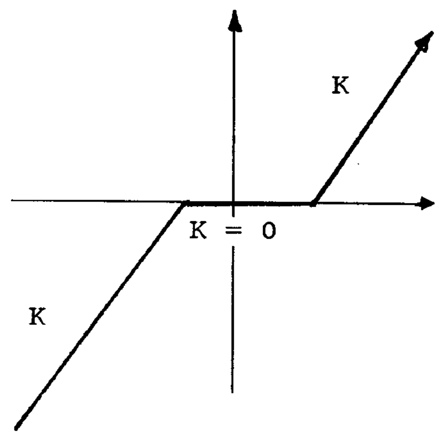
A: LINEAR ELEMENT
Non-Buckling Column, Tension and Compression Have Equal Stiffness.



B: CABLE
Tension-Only Element With Zero Stiffness in Compression



C: CONTACT GAP
Bearing Pressure Transfer Between Interfering Surfaces (Compression Only)



D: MECHANICAL STOP
Zero Stiffness Until A Minimum Deflection

FIGURE 1.

Load vs. deflection curves for a linear element (A) and three gap-type elements (B through D). The horizontal axis represents the element's change in length under load (extension to the right, contraction to the left), and the vertical axis is the element's resulting load (tension above the origin, compression below). Slope of the curve is the element's stiffness, K.

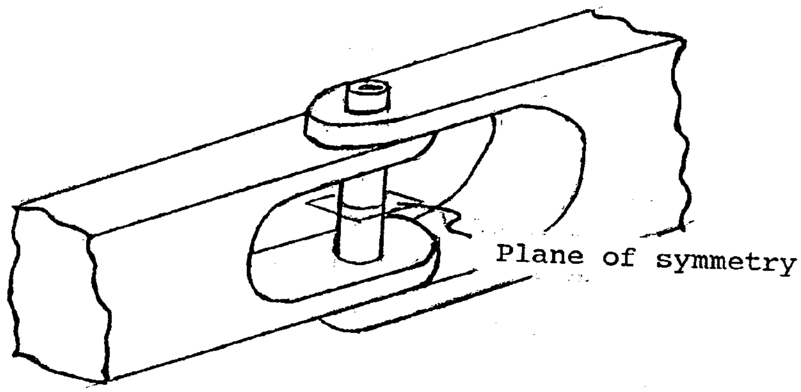


FIGURE 2.

Folding joint, with load transferred from the beam on the right to the beam on the left through a hollow steel pin. The bearing of the pin on the lug holes is nonlinear in that the gaps between them only transmit a compressive bearing stress (see figure 1C).

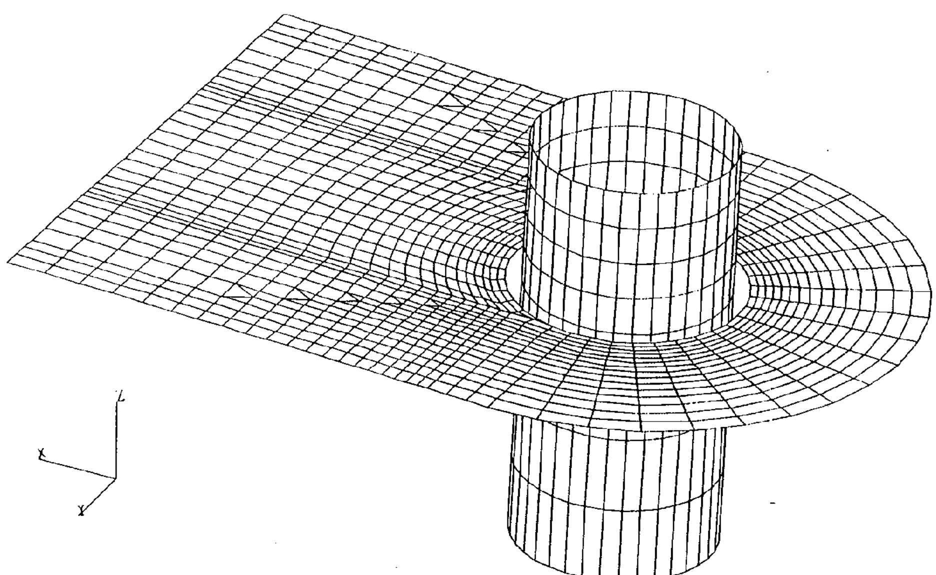


FIGURE 3.

The lug region of the inner beam of figure 2. The upper end of the hollow pin represents the lug of the outer beam, while the lower end is its symmetry plane. The beam itself is considered ground.

Initial Database, First Condition	60 sec.
Restart, Condition 2	20 sec.
Restart, Condition 3	22 sec.
Restart, Condition 4	21 sec.
Restart, Condition 5	21 sec.
Restart, Condition 6	21 sec.
Restart, Condition 7	22 sec.
Restart, Condition 8	21 sec.
Restart, Condition 9	23 sec.
Restart, Condition 10	20 sec.
Total	251 sec.

TABLE 1.

CPU requirements of the 10 Solution 66 Nonlinear runs. Restarts were used to save CPU time on processing the model and generating the initial stiffness matrix; superelements also were used, so that only nonlinear element GRIDs were in the iterated stiffness matrix.

Initial Database, 10 Conditions	51 sec.
Restart, 72 Unit-Strain Cases	68 sec.
External program, solve for Enforced Strains	5 sec. *
Restart, 10 Original Cases + Compensation	20 sec.
Total	144 sec.

TABLE 2.

CPU requirements for the Enforced Strain Solution using an external compensation code. Incorporating the technique into the FEM would significantly reduce the CPU time listed by eliminating the Unit Strain restart step.

* CPU time not in NASTRAN.

TABLE 3.

COMPARISON OF ENFORCED-STRAIN SOLUTION TO SOL. 66 GAP RESULTS
SUBCASE 1: 250,000 LB. AXIAL LOAD ON LUG (0 DEG., OUTBOARD C.L.)

ELEM ID	LINEAR LOAD (LBS)	COMPENSATION (MICRO IN/IN)	NONLINEAR (LBS)	GAP EXTENSION (MICRO IN/IN)	GAP LOAD (LBS)
101	210.7	0.0	-15070.9	1355.8	15062.8
102	-2210.7	0.0	-15861.2	1427.2	15855.6
103	-4581.1	0.0	-17346.3	1561.0	17342.5
104	-6821.9	0.0	-19077.0	1716.8	19073.9
105	-8863.2	0.0	-20729.3	1865.5	20726.2
106	-10638.2	0.0	-22086.3	1987.7	22083.0
107	-12084.3	0.0	-23077.6	2077.0	23075.1
108	-13154.2	0.0	-23740.0	2136.6	23737.6
109	-13811.0	0.0	-24116.6	2170.6	24115.0
110	-14032.6	0.0	-24238.7	2181.5	24236.2
111	-13811.4	0.0	-24117.0	2170.6	24115.4
112	-13153.6	0.0	-23739.4	2136.5	23736.9
113	-12084.6	0.0	-23078.1	2077.0	23075.6
114	-10638.2	0.0	-22086.3	1987.7	22083.0
115	-8863.1	0.0	-20729.3	1865.5	20726.2
116	-6821.1	0.0	-19075.4	1716.7	19072.4
117	-4582.6	0.0	-17349.3	1561.2	17345.2
118	-2209.7	0.0	-15859.4	1427.0	15854.0
119	210.7	0.0	-15070.9	1355.8	15062.8
120	2608.5	0.0	-9404.9	847.7	9418.0
121	4917.9	2209.0	0.0	-2202.2	0.0
122	7062.1	10090.5	-0.0	-10077.5	-0.1
123	8988.2	22585.6	0.0	-22568.6	-0.2
124	10639.8	37760.2	0.0	-37743.3	-0.4
125	11972.9	53148.2	-0.0	-53135.3	-0.6
126	12950.2	66272.1	0.0	-66265.1	-0.7
127	13547.9	75062.2	0.0	-75060.1	-0.8
128	13748.2	78150.9	0.0	-78151.6	-0.9
129	13547.5	75062.4	0.0	-75060.6	-0.8
130	12950.7	66272.6	0.0	-66265.2	-0.7
131	11972.5	53148.5	0.0	-53135.3	-0.6
132	10639.8	37760.6	0.0	-37743.4	-0.4
133	8988.2	22586.0	0.0	-22568.7	-0.2
134	7062.9	10090.8	-0.0	-10077.5	-0.1
135	4916.3	2208.4	0.0	-2201.5	0.0
136	2609.5	0.0	-9404.4	847.7	9417.6
201	194.2	0.0	-14127.0	1271.6	14127.6
202	2604.1	0.0	-2504.5	224.0	2488.8
203	5081.8	5275.9	0.0	-5284.4	0.0
204	7345.9	15700.8	0.0	-15713.5	-0.2
205	9474.6	29730.3	0.0	-29746.7	-0.3
206	11369.8	45306.3	0.0	-45322.2	-0.5
207	12853.5	60204.7	0.0	-60212.6	-0.7
208	13966.1	72432.5	0.0	-72430.0	-0.8
209	14661.4	80430.3	0.0	-80422.0	-0.9
210	14892.6	83208.0	0.0	-83198.3	-0.9
211	14661.1	80430.3	0.0	-80421.9	-0.9
212	13966.3	72432.5	0.0	-72430.0	-0.8
213	12853.6	60204.7	0.0	-60212.5	-0.7
214	11369.7	45306.1	0.0	-45322.0	-0.5
215	9474.6	29730.2	0.0	-29746.5	-0.3
216	7345.6	15700.7	0.0	-15713.4	-0.2
217	5082.5	5276.5	0.0	-5285.0	0.0
218	2603.8	0.0	-2504.5	224.0	2488.8
219	194.1	0.0	-14127.0	1271.6	14127.6
220	-2159.9	0.0	-15919.4	1433.1	15922.2
221	-4419.8	0.0	-17342.0	1561.1	17344.0
222	-6499.3	0.0	-19043.9	1714.3	19045.9
223	-8352.2	0.0	-20648.0	1858.7	20649.7
224	-9934.7	0.0	-21953.5	1976.2	21955.2
225	-11208.0	0.0	-22872.2	2058.9	22873.9
226	-12140.8	0.0	-23439.1	2109.8	23440.2
227	-12709.0	0.0	-23738.9	2136.8	23740.3
228	-12900.2	0.0	-23831.5	2145.1	23832.2
229	-12709.3	0.0	-23739.1	2136.9	23740.5
230	-12140.5	0.0	-23438.9	2109.8	23440.0
231	-11208.1	0.0	-22872.1	2058.8	22873.8
232	-9934.8	0.0	-21953.6	1976.2	21955.3
233	-8352.2	0.0	-20648.0	1858.7	20649.7
234	-6499.6	0.0	-19044.8	1714.4	19046.7
235	-4419.3	0.0	-17340.3	1561.0	17342.4
236	-2160.0	0.0	-15920.1	1433.2	15922.8

TABLE 4.

COMPARISON OF ENFORCED-STRAIN SOLUTION TO SOL. 66 GAP RESULTS
SUBCASE 6: 250,000 LB. AXIAL + TRANSVERSE LOAD (100 DEG., OUTBOARD C.L.)

ELEM ID	LINEAR LOAD (LBS)	COMPENSATION (MICRO IN/IN)	NONLINEAR (LBS)	GAP EXTENSION (MICRO IN/IN)	GAP LOAD (LBS)
101	13599.8	74616.0	0.0	-74635.9	-0.8
102	13972.7	79109.5	-0.0	-79117.2	-0.9
103	13919.5	77292.6	0.0	-77286.9	-0.9
104	13421.4	69420.3	0.0	-69402.0	-0.8
105	12482.9	56709.8	-0.0	-56682.5	-0.6
106	11120.3	41183.8	0.0	-41153.7	-0.5
107	9371.1	25382.8	-0.0	-25366.5	-0.3
108	7291.6	11948.4	-0.0	-11930.7	-0.1
109	4949.4	3059.2	0.0	-3051.0	-0.0
110	2436.8	0.0	-8014.4	722.6	8028.5
111	-152.7	0.0	-15918.4	1432.4	15913.8
112	-2723.3	0.0	-16561.3	1490.4	16558.2
113	-5174.0	0.0	-17814.7	1603.4	17813.8
114	-7425.7	0.0	-19445.0	1750.2	19444.7
115	-9404.7	0.0	-21021.4	1892.1	21021.2
116	-11052.1	0.0	-22280.5	2005.5	22280.7
117	-12328.5	0.0	-23184.5	2086.8	23184.4
118	-13204.9	0.0	-23781.1	2140.6	23782.0
119	-13673.0	0.0	-24109.8	2170.1	24110.2
120	-13735.4	0.0	-24196.6	2178.0	24197.7
121	-13397.6	0.0	-24045.2	2164.3	24045.2
122	-12681.3	0.0	-23642.4	2128.1	23642.8
123	-11612.9	0.0	-22955.0	2066.2	22955.1
124	-10226.8	0.0	-21938.3	1974.6	21938.2
125	-8562.6	0.0	-20575.2	1851.9	20574.9
126	-6669.6	0.0	-18954.4	1706.0	18953.7
127	-4589.8	0.0	-17169.2	1545.4	17169.7
128	-2387.6	0.0	-15518.0	1396.8	15518.9
129	-114.8	0.0	-14717.2	1324.9	14719.6
130	2171.5	0.0	-12071.5	1086.1	12067.0
131	4404.9	1067.4	0.0	-1070.7	-0.0
132	6531.4	7900.3	-0.0	-7910.0	-0.1
133	8491.3	19680.0	0.0	-19698.3	-0.2
134	10228.1	34676.0	0.0	-34702.2	-0.4
135	11690.6	50478.4	-0.0	-50508.5	-0.6
136	12828.9	64539.3	0.0	-64567.3	-0.7
201	-13428.9	0.0	-23486.7	2113.8	23484.8
202	-14031.5	0.0	-23905.1	2151.6	23904.0
203	-14312.0	0.0	-24219.4	2179.9	24218.6
204	-13950.9	0.0	-24066.3	2166.2	24066.0
205	-13259.7	0.0	-23804.8	2142.7	23804.8
206	-11927.8	0.0	-22878.6	2059.3	22878.8
207	-10208.5	0.0	-21488.1	1934.2	21488.4
208	-8019.3	0.0	-19705.3	1773.6	19705.1
209	-5422.7	0.0	-17801.7	1602.3	17801.9
210	-2586.1	0.0	-15886.1	1429.8	15884.8
211	330.9	0.0	-10858.7	976.5	10849.2
212	3169.0	1388.9	0.0	-1392.7	-0.0
213	5744.0	8612.1	-0.0	-8617.7	-0.1
214	7979.3	20049.4	0.0	-20055.1	-0.2
215	9969.2	33970.9	0.0	-33976.0	-0.4
216	11400.1	48229.0	0.0	-48230.5	-0.5
217	12546.1	60832.1	0.0	-60827.2	-0.7
218	13127.6	70152.5	0.0	-70141.3	-0.8
219	13361.5	75118.9	0.0	-75104.2	-0.8
220	13333.0	75227.9	0.0	-75213.3	-0.8
221	12888.8	70494.3	0.0	-70481.7	-0.7
222	12129.3	61523.3	0.0	-61513.5	-0.5
223	11055.3	49415.4	0.0	-49407.8	-0.4
224	9701.8	35694.3	0.0	-35690.0	-0.2
225	8101.0	22162.8	0.0	-22163.2	-0.1
226	6293.8	10691.4	0.0	-10693.6	-0.0
227	4324.7	2868.8	0.0	-2869.3	-0.0
228	2240.1	0.0	-5936.5	534.2	5934.5
229	89.2	0.0	-12964.3	1166.9	12964.0
230	-2077.5	0.0	-14519.3	1306.8	14518.9
231	-4208.4	0.0	-16231.3	1460.9	16230.9
232	-6251.5	0.0	-18080.1	1627.3	18079.1
233	-8154.6	0.0	-19745.3	1777.1	19743.8
234	-9872.3	0.0	-21155.8	1904.0	21154.0
235	-11353.3	0.0	-22223.2	2000.1	22221.1
236	-12583.1	0.0	-23005.8	2070.6	23003.8