NONLINEAR ANALYSIS USING A MODAL BASED REDUCTION TECHNIQUE

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ABSTRACT

This paper presents a solution to nonlinear formulated problems using eigenfunctions computed by a linear free vibration solution. The system of equations is extremely reduced. The solution is unique in its formulation as the governing equations represent the problem continuously and do not require an iterational or incremental solution. Energy consideration is used and the Ritz method is applied to render the governing equations. An integrated system was built in which the current analysis functioned as a MSC/NASTRAN dummy module integrated with MSC/NASTRAN SOL 3 and SOL 24 to render the mode shapes and geometrical and material properties respectively. Several numerical examples are presented and compared to solutions from the literature.

INTRODUCTION

The solution of nonlinear static and dynamic problems has been intensively investigated over the last decade. The main effort lies in trying to create general purpose codes and making them as efficient and computationally cheap as possible. The direct approach [1,2,3] was based on the formulation of a certain problem using the nonlinear strains/displacements relations and then developing the displacements into series of products of functions and coefficients and then applying a Ritz method or any weighting residual technique such as the Galerkin method. The set of unknowns was more efficiently chosen by replacing the inplane displacements by Airy stress functions [1,3]. Basically the above mentioned works are followed by today's methods but the key change lies in the choice of functions. In [1,2] transcendental functions were chosen such that geometrical boundary conditions are satisfied. Stein [2] used the Kanterovitch like method in which the coefficients are functions as well. Sheinman and Frostig [3] chose natural modes of a degenerated problem for each component of the displacement vector. A different approach usually used in the finite element method was the updated Lagrangian [4] in which an iterational procedure was applied on a linear formulation. Another type of linearization [5] used an adapted Newton method. The use of the natural modes of the problem first started in the solution of problems in dynamics [6,7,8] then in static problems such as large deflection and postbuckling [9,10,11,12]. Most works used a finite element formulation in which the governing equation is discretized, a tangent stiffness matrix is generated and the solution is rendered by iterations. In [6] a 1-D piping problem was discretized into lumped masses and springs. Incrementation was used in time [8] and in space [12].

This paper offers a new approach in which the kinematics and the strains/displacements relations are nonlinear. The governing equations are continuous and discretization is done only in the representation of the mode shapes which are rendered by a dynamical solution using MSC/NASTRAN[14]. The solution utilizes an appropriate selection of the truncated eigenfunction series.

KINEMATICS

The formulation is based on energy consideration. The method presented in this paper is demonstrated on plate elements, while other types of elements are treated in the same manner respectively. The convention of geometry as well as stress resultants and couples is presented in Fig. 1.

A beginning is made with the choice of the order of the theory in which the displacements are described. Classical plate theory was chosen here as follows,

$$U_1 = u_1 - x_3 u_{3,1} \tag{1a}$$

$$U_2 = u_2 - x_3 u_{3,2} \tag{1b}$$

$$U_3 = u_3 \tag{1c}$$

in which U_1, U_2, U_3 are the displacements in a local Cartesian coordinate system oriented to 1, 2 and 3 directions respectively. u_1, u_2, u_3 are the mid plane displacements in the relevant local directions and x_3 is the 3rd coordinate.

The strain displacement relation expression is taken in the nonlinear form as follows,

$$\varepsilon_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i} + U_{r,i}U_{r,j}) \tag{2}$$

Substitute (1) into (2) to obtain,

$$\varepsilon_{11} = u_{1,1} - x_3 u_{3,11} + \frac{1}{2}u_{3,1}^2 \tag{3a}$$

$$\varepsilon_{22} = u_{2,2} - x_3 u_{3,22} + \frac{1}{2}u_{3,2}^2 \tag{3b}$$

$$\varepsilon_{12} = u_{1,2} + u_{2,1} - 2x_3 u_{3,12} + u_{3,1}u_{3,2} \tag{3c}$$

Eq. (3) may be separated into the midplane strains,

$$\overset{\circ}{\varepsilon}_{11} = u_{1,1} + \frac{1}{2}u_{3,1}^2 \tag{4a}$$

$$\overset{\circ}{\varepsilon}_{22} = u_{2,2} + \frac{1}{2}u_{3,2}^2 \tag{4b}$$

$$\overset{\circ}{\varepsilon}_{12} = u_{1,2} + u_{2,1} + u_{3,1}u_{3,2} \tag{4c}$$

and curvatures,

$$\chi_{11} = -u_{3,11} \tag{5a}$$

$$\chi_{22} = -u_{3,22} \tag{5b}$$

$$\chi_{12} = -2u_{3,12} \tag{5c}$$

ENERGY CONSIDERATION

The expressions for the strains, eq. (4) and curvatures, eq. (5) are used to build the total energy functional Π as follows,

$$\Pi = V + Q + U \tag{6}$$

in which V is the strain energy. U is the potential energy of the initial inplane force resultants applied to the plane in the prebuckled state. The latter loads are combined of the applied inplane edge loads plus the loading induced by bending when transverse load is applied. Q is the potential energy of the external loads in all directions, distributed over the surface. Q is built up in conjunction with the virtual work done by the components of the external loading n_1, n_2, n_3, through an infinitesimally small virtual displacements δu_1, δu_2, δu_3 respectively.

The strain energy V is,

$$V = \frac{1}{2} \int_{-\frac{h}{2}}^{+\frac{h}{2}} \iint_A \sigma_{ij}\varepsilon_{ij} \, dx_1 dx_2 dx_3 \tag{7}$$

Using plane stress approach in (7) we obtain,

$$V = \frac{1}{2} \int_{-\frac{h}{2}}^{+\frac{h}{2}} \iint_A (\sigma_{11}\varepsilon_{11} + \sigma_{22}\varepsilon_{22} + \sigma_{12}\varepsilon_{12}) \, dx_1 dx_2 dx_3 \tag{8}$$

Integrating (8) over the thickness, all stresses turn into the resultants N_{ij} in accordance with the midplane strains, while those related to the curvature create stress couple terms M_{ij}. Therafter substitution of (4) gives,

$$\begin{aligned} V = \frac{1}{2}\iint_A [&\tfrac{1}{2}(N_{11}u_{3,1}^2 + N_{22}u_{3,2}^2 + 2N_{12}u_{3,1}u_{3,2}) + \\ &N_{11}u_{1,1} + N_{22}u_{2,2} + N_{12}(u_{1,2} + u_{2,1}) - \\ &M_{11}u_{3,11} - M_{22}u_{3,22} - 2M_{12}u_{3,12}] \, dx_1 dx_2 \end{aligned} \tag{9}$$

The potential energy U is formulated as follows,

$$U = \iint_A (\bar{N}_{11}\varepsilon_{11}^* + \bar{N}_{22}\varepsilon_{22}^* + \bar{N}_{12}\varepsilon_{12}^*) \, dx_1 dx_2 \tag{10}$$

in which $\bar{N}_{ij}$ are the prebuckled inplane force resultants and ε^{*}_{ij} are the nonlinear part of the nonlinear form of ε_{ij}.
Thus,

$$U = \frac{1}{2}\iint_A (\bar{N}_{11}u^2_{3,1} + \bar{N}_{22}u^2_{3,2} + 2\bar{N}_{12}u_{3,1}u_{3,2})\, dA \tag{11}$$

Finally the potential energy due to external loads,

$$Q = -\iint_A [n_1u_1 + n_2u_2 + n_3u_3 + m_1u_{3,2} + m_2u_{3,1} + m_{12}(u_{1,2} + u_{2,1})]\, dx_1dx_2 \tag{12}$$

Eqns. (9), (11) and (12) altogether form the total energy functional Π in terms of the displacements u_i, the stress resultants N_{ij} and the stress couples M_{ij} and the external loads m_i and n_i as given by,

$$\begin{aligned}\Pi=\frac{1}{2}\iint_A \Big\{ & \frac{1}{2}\Big[\Big(N_{11} + 2\bar{N}_{11}\Big)u^2_{3,1} + \Big(N_{22} + 2\bar{N}_{22}\Big)u^2_{3,2} + \\ & 2\Big(N_{12} + 2\bar{N}_{12}\Big)u_{3,1}u_{3,2}\Big] + \\ & N_{11}u_{1,1} + N_{22}u_{2,2} + N_{12}(u_{1,2} + u_{2,1}) - \\ & M_{11}u_{3,11} - M_{22}u_{3,22} - 2M_{12}u_{3,12} - \\ & 2\Big[n_1u_1 + n_2u_2 + n_3u_3 + m_1u_{3,2} + \\ & m_2u_{3,1} + m_{12}(u_{1,2} + u_{2,1})\Big] \Big\}\, dx_1dx_2\end{aligned} \tag{13}$$

In order to turn (13) into a whole displacements expression, we use the constitutive relations to render strains and curvatures out of the resultants and couples. Classical laminate theory is used,

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \mathring{\varepsilon} \\ \chi \end{Bmatrix} \tag{14}$$

In which A, B and D are the well known rigidity matrices. Then the strains - displacements relations (4) and (5) are used again, to obtain Π in terms of displacements rigidity coefficients and external loads.

GOVERNING EQUATIONS VIA RITZ METHOD

Since the functional Π is self adjoint, one may use the Ritz method in which first the displacements are developed into series of functions multiplied by coefficients. The self-adjointness allows one to choose functions which are of the admissible functions space which means that they have to satisfy the geometrical boundary conditions only (neither the differential equation nor the dynamic boundary conditions need to be satisfied) and be p times differentiable (the differential equation is 2p times differentiable). The above requirements are in addition to the regular requirements from trial functions which have to be linear independent orthogonal functions taken from a complete set. The Ritz method is the basis for modern numerical approximations such as the finite elements method and also equivalent to other widely used methods such as the Galerkin method. The major disadvantage of the Ritz method is in the choice of the trial functions which have to satisfy the above requirements and may be a very difficult task for a generally shaped structure. In this paper it is proposed that trial functions for the nonlinear formulation be chosen from the best inventory which is the set of the eigenfunctions rendered by a linear solution of the relevant structure. These functions also known as mode shapes, satisfy inherently the geometrical boundary conditions and have physical meaning rather than being a pure mathematical choice. The overall mode shapes may be separated into components in the three local directions 1, 2 and 3 in conjunction with the coordinate system. Since the components of each mode are computed with their true values within the mode (which means that they are related to each other numerically), there is a need for only one participation factor for the entire mode. This one factor is applied to all three components of a single mode within the developed series.

$$u_1=\sum_m \xi_m \Phi_m \tag{15a}$$

$$u_2=\sum_m \xi_m \Psi_m \tag{15b}$$

$$u_3=\sum_m \xi_m \Theta_m \tag{15c}$$

Where Φ_m, Ψ_m and Θ_m are the components of the m^{th} mode in the 1, 2 and 3 local directions respectively, and ξ_m is the participation factor of the m^{th} mode.

The Ritz method is formulated,

$$\frac{\partial \Pi}{\partial \xi_i}=0 \qquad i = 1..m \tag{16}$$

Substituting (14) into (13) in conjunction with (4) and (5), then replacing the displacements by (15) and performing the differentiation as stated in (16) yields m cubic equations with the unknowns ξ as follows,

$$\iint_A \Big\{ (\Theta_{i,1}\xi_i)(\Theta_{j,1}\xi_j)(\Theta_{k,1}\xi_k)\Big[A_{11}\Theta_{m,1} + A_{13}\Theta_{m,2}\Big]/2$$

$$+ (\Theta_{i,2}\xi_i)(\Theta_{j,2}\xi_j)(\Theta_{k,2}\xi_k)\Big[A_{22}\Theta_{m,2} + A_{23}\Theta_{m,1}\Big]/2$$

$$+ (\Theta_{i,1}\xi_i)(\Theta_{j,1}\xi_j)(\Theta_{k,2}\xi_k)\Big[A_{12}\Theta_{m,2} + 3A_{13}\Theta_{m,1} + 2A_{33}\Theta_{m,2}\Big]/2$$

$$+ (\Theta_{i,2}\xi_i)(\Theta_{j,2}\xi_j)(\Theta_{k,1}\xi_k)\Big[A_{21}\Theta_{m,1} + 3A_{23}\Theta_{m,2} + 2A_{33}\Theta_{m,1}\Big]/2$$

$$+ (\Theta_{i,1}\xi_i)(\Theta_{j,1}\xi_j)\Big[A_{11}\Phi_{m,1} + A_{12}\Psi_{m,2} + A_{13}(\Phi_{m,2}+\Psi_{m,1}) - B_{11}\Theta_{m,11} - B_{12}\Theta_{m,22} - 2B_{13}\Theta_{m,12}\Big]/2$$

$$+ (\Theta_{i,2}\xi_i)(\Theta_{j,2}\xi_j)\Big[A_{21}\Phi_{m,1} + A_{22}\Psi_{m,2} + A_{23}(\Phi_{m,2}+\Psi_{m,1}) - B_{21}\Theta_{m,11} - B_{22}\Theta_{m,22} - 2B_{23}\Theta_{m,12}\Big]/2$$

$$+ (\Theta_{i,1}\xi_i)(\Phi_{j,1}\xi_j)\Big[A_{11}\Theta_{m,1} + A_{31}\Theta_{m,2}\Big]$$

$$+ (\Theta_{i,1}\xi_i)(\Psi_{j,2}\xi_j)\Big[A_{12}\Theta_{m,1} + A_{32}\Theta_{m,2}\Big]$$

$$+ (\Theta_{i,1}\xi_i)\Big[(\Phi_{j,2}+\Psi_{j,1})\xi_j\Big]\Big[A_{13}\Theta_{m,1} + A_{33}\Theta_{m,2}\Big]$$

$$- (\Theta_{i,1}\xi_i)(\Theta_{j,11}\xi_j)\Big[B_{11}\Theta_{m,1} + B_{31}\Theta_{m,2}\Big]$$

$$- (\Theta_{i,1}\xi_i)(\Theta_{j,22}\xi_j)\Big[B_{12}\Theta_{m,1} + B_{32}\Theta_{m,2}\Big]$$

$$- (\Theta_{i,1}\xi_i)(\Theta_{j,12}\xi_j)\Big[B_{13}\Theta_{m,1} + B_{33}\Theta_{m,2}\Big]*2$$

$$+ (\Theta_{i,2}\xi_i)(\Phi_{j,1}\xi_j)\Big[A_{21}\Theta_{m,2} + A_{31}\Theta_{m,1}\Big]$$

$$+ (\Theta_{i,2}\xi_i)(\Psi_{j,2}\xi_j)\Big[A_{22}\Theta_{m,2} + A_{32}\Theta_{m,1}\Big]$$

$$+ (\Theta_{i,2}\xi_i)\Big[(\Phi_{j,2}+\Psi_{j,1})\xi_j\Big]\Big[A_{23}\Theta_{m,2} + A_{33}\Theta_{m,1}\Big]$$

$$- (\Theta_{i,2}\xi_i)(\Theta_{j,11}\xi_j)\Big[B_{21}\Theta_{m,2} + B_{31}\Theta_{m,1}\Big]$$

$$- (\Theta_{i,2}\xi_i)(\Theta_{j,22}\xi_j)\Big[B_{22}\Theta_{m,2} + B_{32}\Theta_{m,1}\Big]$$

$$- (\Theta_{i,2}\xi_i)(\Theta_{j,12}\xi_j)\Big[B_{23}\Theta_{m,2} + B_{33}\Theta_{m,1}\Big]*2$$

$$
\begin{aligned}
&+ (\Theta_{i,1}\xi_i)(\Theta_{j,2}\xi_j)\Big[A_{13}\Phi_{m,1} + A_{23}\Psi_{m,2} + A_{33}(\Phi_{m,2}+\Psi_{m,1}) \\
&\qquad\qquad - B_{31}\Theta_{m,11} - B_{23}\Theta_{m,22} - 2B_{33}\Theta_{m,12}\Big] \\
&+ \xi_i\Big(\Phi_{i,1}\Big[A_{11}\Phi_{m,1} + A_{12}\Psi_{m,2} + A_{13}(\Phi_{m,2}+\Psi_{m,1}) \\
&\qquad - B_{11}\Theta_{m,11} - B_{12}\Theta_{m,22} - 2B_{13}\Theta_{m,12}\Big] \\
&\quad + \Psi_{i,2}\Big[A_{21}\Phi_{m,1} + A_{22}\Psi_{m,2} + A_{23}(\Phi_{m,2}+\Psi_{m,1}) \\
&\qquad - B_{21}\Theta_{m,11} - B_{22}\Theta_{m,22} - 2B_{23}\Theta_{m,12}\Big] \\
&\quad + (\Phi_{i,2}+\Psi_{i,1})\Big[A_{31}\Phi_{m,1} + A_{32}\Psi_{m,2} + A_{33}(\Phi_{m,2}+\Psi_{m,1}) \\
&\qquad - B_{31}\Theta_{m,11} - B_{32}\Theta_{m,22} - 2B_{33}\Theta_{m,12}\Big] \\
&\quad - \Theta_{i,11}\Big[B_{11}\Phi_{m,1} + B_{12}\Psi_{m,2} + B_{13}(\Phi_{m,2}+\Psi_{m,1}) \\
&\qquad - D_{11}\Theta_{m,11} - D_{12}\Theta_{m,22} - 2D_{13}\Theta_{m,12}\Big] \\
&\quad - \Theta_{i,22}\Big[B_{21}\Phi_{m,1} + B_{22}\Psi_{m,2} + B_{23}(\Phi_{m,2}+\Psi_{m,1}) \\
&\qquad - D_{21}\Theta_{m,11} - D_{22}\Theta_{m,22} - 2D_{23}\Theta_{m,12}\Big] \\
&\quad - 2\Theta_{i,12}\Big[B_{31}\Phi_{m,1} + B_{32}\Psi_{m,2} + B_{33}(\Phi_{m,2}+\Psi_{m,1}) \\
&\qquad - D_{31}\Theta_{m,11} - D_{32}\Theta_{m,22} - 2D_{33}\Theta_{m,12}\Big] \\
&+ \bar{N}_{11}(\Theta_{i,1}\Theta_{m,1}) + \bar{N}_{12}(\Theta_{i,1}\Theta_{m,2}+\Theta_{i,2}\Theta_{m,1}) + \bar{N}_{22}(\Theta_{i,2}\Theta_{m,2})\Big)/4 \Big\} \, dA \\
&= \iint_A \Big\{ m_1\Theta_{m,2} + m_2\Theta_{m,1} + m_{12}(\Phi_{m,2}+\Psi_{m,1}) + n_1\Phi_m + n_2\Psi_m + n_3\Theta_m \Big\} \, dA
\end{aligned}
\tag{17}
$$

The indexes in eq. (17) act in a tensorial sense following Einstein summation rule.

SOLUTION PROCEDURE

Once all terms in eq. (17) are computed it may be written in a matricial form,

$$
[\mathbf{K}_L^1]\{\xi_i\} + [\mathbf{K}_P^1]\{\xi_i\} + [\mathbf{K}^2]\{\xi_i\xi_j\} + [\mathbf{K}^3]\{\xi_i\xi_j\xi_k\} = \{P\} \qquad i,\ j,\ k = 1..m \tag{18}
$$

where,

$[\mathbf{K}_L^1]$ - generalized stiffness matrix same as in linear analysis, order: $[m \times m]$

$[K_p^1]$- generalized linear stiffness matrix consists of force terms
order: $[m \times m]$

$[K^2]$- generalized stiffness matrix related to quadratic terms,
order: $[m \times \frac{m(m+1)}{2}]$

$[K^3]$- generalized stiffness matrix related to cubic terms,
order: $[m \times \frac{m(m+1)(m+2)}{6}]$

{P}- generalized forces vector of order [m]

The set of the m nonlinear algebraic equations (18) was solved by the IMSL routine ZSPOW[15] for ξ and then enabled to render the displacements by substitution into eq. (15).

Eq. (18) may solve several types of problems. Dropping the cubic and quadratic series products and observing the remains it is realized that one may solve a linear stretching and bending problem. If in addition to the above one drops the RHS terms and ends up having the homogeneous equations for a linear buckling solution. On the other hand, considering all nonlinear terms one may compute a large deflection problem related to inplane and out of plane loads or one may ignore the out of plane loads and solve for a postbuckling problem. Materials of different orientation behavior such as isotropic or orthotropic materials may be handled by relevant treatment of the A, B and D matrices components.

Based on the above described analysis, a system was developed in which a computer code called ANLISA [16] (Advanced NonLinear Integrated Structural Analysis) was regarded as a MSC/NASTRAN "dummy module", combined with two MSC/NASTRAN solutions: SOL 24 and SOL 3. The solution procedure is as follows. A MSC/NASTRAN related finite element model is built. The user runs MSC/NASTRAN with additional cards which leads to the special system. At first, SOL 24 is executed in order to create the data blocks in conjunction with the model topology and material properties. SOL 24 which is a linear static analysis, provides the information about the linear stage such as prebuckling for a stability case. This information is necessary for the creation of the potential energy U shown in eq. (10). Next, SOL 3 is executed automatically, providing the components of the mode shapes which are then used in the dummy module ANLISA as Θ, Φ and Ψ. ANLISA gathers all the information into the process of building up the system of equations and solves for the unknowns which are the ξ_m. The integration and differentiation techniques used in the process of creating eq. (18) are the same as used by MSC/NASTRAN in order to achieve maximum compatibility with the source of the eigenfunctions. The resulting displacements are returned to MSC/NASTRAN and may be presented by the MSC/XL post-processor.

NUMERICAL EXAMPLES

Several numerical examples are presented. The types of the nonlinear problems covered are large deflection and postbuckling. The problems are applied on isotropic materials and composite materials chosen with respect to the literature for the sake of comparison. Two cases of large deflection of an isotropic square plate were examined. First case is for a plate simply supported all around and the second case is for a plate clamped all around. These cases were compared to graphs presented by Chia [1]. Next a postbuckling case of a laminated composite plate was computed and compared to results produced by a computer code developed in [3] and used by courtesy of the authors. Nonlinear analysis requires eigenfunctions which are highly located in the complete series. Therefore an advanced scheme[13] called "The fictitious masses method" was used to "reduce" the location of these eigenfunctions. This method incorporates fictitiously chosen large masses which artificially change the relations between the real stiffness and mass matrices. In result, what had to be computed as high mode shapes are "attracted down", performing as low mode shapes in correlation with the location and direction of the induced fictitious masses. This method helps in cases of local phenomena such as hole edges, stiffener caps etc. Large fictitious masses should then be put in these locations in relevant directions and the result of a free vibration analysis is that what usually appears as a very high mode in conjunction with the location of interest, appears now as one of the first few mode shapes of the structure. In the case of statics, no further action is required, but in the case of dynamics, this method results in unreal frequencies. Therefore it requires to filter out the unrealistic influence of the fictitious masses. A filtering technique for this case is presented by Karpel [13]. Any number of modes shown in the examples implies on these wisely chosen selection of eigenfunctions and not on contiguous modes from the original set.

Figure 2 shows the analysis carried out for a square simply supported laminated plate made of graphite epoxy AS4/3502 in which E_{11}=2.1e7 psi, E_{22}=2e6 psi, G_{12}=0.56e6 psi, ν_{12}=0.24 and the layer sequencing was $(\pm 45)_s$ with ply thickness of 0.005 in.

The plate was loaded axially by a distributed load. In the analysis following Sheinman et. al.[3], a geometrical imperfection was used as a trigger to pull the linearized system of equations out of equilibrium state. The usage of a geometrical imperfection leads to numerical problems. Too small geometrical imperfection provides erroneous results and therefore there is a lower bound on its size. In the current work though, a direct integration was used to solve the nonlinear system and a transverse load was chosen as the imperfection. This was proven very efficient since any small value met the requirement and it may be seen in figure 2 that the imperfection is hardly noticed. In fact the analysis of [3] yields the current analysis as its asymptotic solution.

In order to meet the curves presented by Chia [1], the same normalization scheme was used in the two cases of the large deflection analysis. The normalization terms may be seen at the axes of figures 3 and 4 for the simply supported and clamped cases respectively. The horizontal axis presents the normalized load and the vertical axis presents the normalized plate mid-point deflection. Four and eight mode shapes were taken for the analysis of the simply supported plate and the differences may be observed in figure 4 as they are compared to the curve presented by Chia [1]. For the clamped plate four mode shapes were used and the results as well as the comparison with Chia [1] is shown in figure 3.

CONCLUDING REMARKS

A new method was presented for nonlinear analysis of structures and compiled into an integrated computer system. Solutions of typical nonlinear problems were presented as well as comparisons with the literature. Quoting D. Tao and E. Ramm [11]: "The key question still remains how well the basis vectors capture the deformation modes of the structure". This problem is clearly recognized by the authors but since the current formulation provides a continuous set of nonlinear equations there is no need for 'new modes' to be used along different stages of the analysis. Nevertheless, the set of eigenfunctions has to be chosen in a way that it would be representative throughout the entire range of responses. Therefore only a selection of eigenfunctions, not necessarily contiguous, are chosen out of the complete set.

Once the proper set of eigenfunctions is selected, the system of equations is highly reduced compared to any other known method and thus proven to be very successful. In fact the generality of this method is obvious as it may be applied on any problem which can be presented through a model of linear finite element method code.

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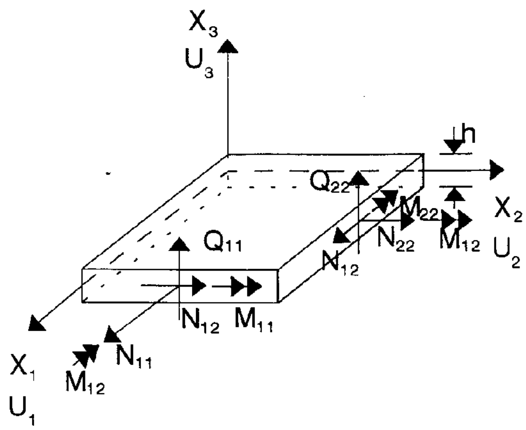

Fig. 1: Coordinate system of plate and
nomenclature for stress resultants and couples

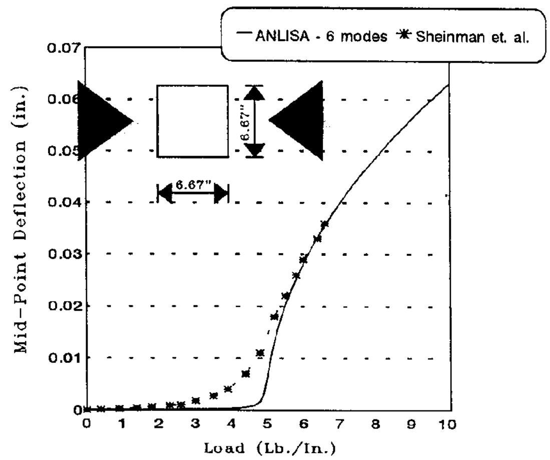

Fig. 2: Postbuckling behavior of square laminated plate
Gr/Ep (+45/-45)s , simply supported

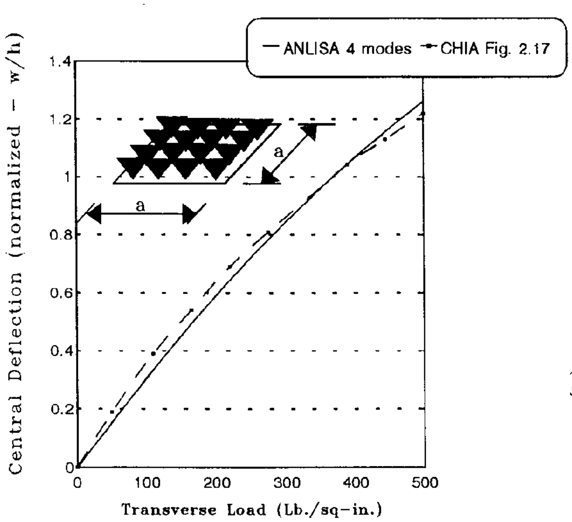

Fig. 3: Large deflection
square clamped isotropic plate

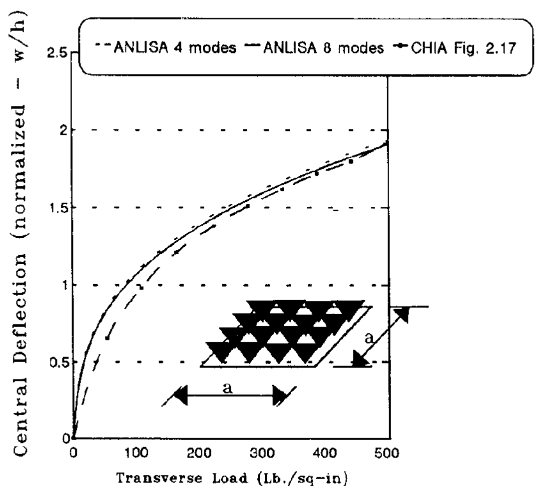

Fig. 4: Large deflection
square simply-supported isotropic plate