A REFINED METHOD FOR LIVE-LOAD DISTRIBUTION PREDICTION OF BRIDGES AND COMPARATIVE STUDY

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ABSTRACT

A refined analysis method is proposed for predicting the distribution of vehicle lived loads on bridge girders. An effective and efficient iteration scheme is used to solve the nonlinear equations. Two representative bridge systems are investigated. The obtained results from the proposed method are compared to experimental data and those obtained from other analysis methods. The prediction method for live-load distribution implemented in the current bridge design code is carefully examined. The paper concludes with a number of actual bridge examples and recommendations.

INTRODUCTION

Bridge structures are difficult to be analyzed due to their complex geometries and the vehicle live loads involved. In practice, such systems are usually analyzed and designed by a simple design method that is largely based upon the past experience. It is also seen that, in cases that research evidence is not sufficient, empirical or semi-empirical methods are often used instead. Such methods are found to lead to overly conservative designs in many incidences. They even result in unsafe results in some cases. The proposed new bridge specification [1] realizes the concerns described above, and recommends the use of a refined analysis method when deemed necessary. Furthermore, under the proposed code [1] design forces may be reduced by fifteen percent if a refined method such as finite element method is used.

As identified in Reference [2], there are several bridge issues that require further studies by employing refined analysis methods. This paper focuses on the prediction of the lateral distribution of vehicle live loads on bridge girders, which is being an important issue to the economical design of bridges. The method for determining the live-load distribution factor for "I-shape" girder bridges implemented in the present bridge design code [3] is still based on a simple and empirical formula [i.e. eqn (12) to be discussed later in the paper]. To verify the accuracy and adequacy of that simple formula, a finite-element based refined approach is proposed and discussed in details in the paper.

PROBLEM DESCRIPTION

Two simple-span bridge systems, "BRIDGE 1" and "BRIDGE 2" as shown in Fig. 1, were considered in the pilot study. The former has the span length (L) of 68.5 feet and center-to-center girder spacing (S) of 8 feet. The latter has L of 78 feet and S of 7 feet. Both bridge deck systems are made of I-shape prestressed concrete (PC) girders whose cross-sections are shown in Fig. 2 (AASHTO TYPE III). The analyses were performed with the aid of the general-purpose finite element program MSC/NASTRAN [4]. Transverse diaphragms were provided at the midspan of each girder for both bridge systems, as shown in Fig. 2(b).

EQUATIONS OF MOTIONS

The physical problem on hand is being studied statically. As a good practice, linear analysis always precedes nonlinear analysis. The finite element system equilibrium equations for a linear static analysis can be expressed by the vectorial equation as follows

$$[K]\ \{U\} = \{R\} \qquad (1)$$

where [K] is the structural stiffness matrix, {U} the displacement vector, and {R} is the externally applied load vector.

While for nonlinear static analysis the equilibrium equations to be solved are:

$$\{R\}_{t+\Delta t} - \{F\}_{t+\Delta t} = \{0\} \tag{2}$$

where $\{R\}_{t+\Delta t}$ is the vector of externally applied nodal loads at load step $t+\Delta t$, and $\{F\}_{t+\Delta t}$ is the force vector equivalent (in the virtual work sense) to the element stresses at load step $t+\Delta t$.

To solve the incremental equilibrium equations [eqn (2)], an effective and efficient quasi-Newton iteration scheme similar to Reference [5] was used. The algorithm of the iteration scheme can be described by eqns (3) and (4) as follows:

$$[K^*]_{t+\Delta t}{}^{(i-1)}\ \{\Delta U\}^{(i)} = \{R\}_{t+\Delta t} - \{F\}_{t+\Delta t}{}^{(i-1)} \tag{3}$$

$$[U]_{t+\Delta t}{}^{(i)} = \{U\}_{t+\Delta t}{}^{(i-1)} + \beta^{(i)}\ \{\Delta U\}^{(i)} \tag{4}$$

where $[K^*]_{t+\Delta t}{}^{(i-1)}$ is an updated stiffness matrix, $\{F\}_{t+\Delta t}{}^{(i-1)}$ the consistent nodal force vector corresponding to the element stresses due to the displacement vector $[U]_{t+\Delta t}{}^{(i-1)}$, $\{\Delta U\}^{(i)}$ the incremental displacement vector in iteration i, and $\beta^{(i)}$ is an acceleration factor obtained from a line search in the direction $\{\Delta U\}^{(i)}$ such that

$$\frac{\{\Delta U\}^{(i)}\quad [\{R\}_{t+\Delta t} - \{F\}_{t+\Delta t}{}^{(i)}]}{\{\Delta U\}^{(i)}\quad [\{R\}_{t+\Delta t} - \{F\}_{t+\Delta t}{}^{(i)}]} \leq \text{convergence limit} \tag{5}$$

It is noted that $[K^*]_{t+\Delta t}{}^{(i-1)}$ is not explicitly formed, but instead the inverse of the stiffness matrix is updated using vector products to provide a secant approximation to the stiffness matrix in successive iterations. The update of matrix is performed only at the solution steps specified by the user.

FINITE ELEMENT MODELING

The analyses were accomplished using the general-purpose finite element program MSC/NASTRAN [4]. The bridge deck structural system was modeled using both "shell" and "beam" (stiffeners) elements, as shown respectively in Figs 3 and 4. A standard quadrilateral (four-node) shell element of constant thickness coupling bending with membrane action was incorporated in modeling the horizontal slab. Stiffeners were described using a standard isoparametric (two-node) beam element. The composite action of the beam and slab was effected by connecting the centers of the slab and beam elements with rigid links/elements, as shown in Fig. 2. This produced correct constraint relations for displacements of the slab and beam.

Because the slab was modeled separately from the beam, it was possible to use different material properties for each structural element. This was desirable since, typically, the concrete strength for the cast-in-place topping is lower than the one in the precast concrete tees. To better represent the structural behavior of the deck slab, it was modeled as an orthotropic plate. To do this, an orthotropy factor, D_y, based on the ratio of center-to-center spacing, S, to clear span was introduced (see Fig. 5). Its value is

$$D_y = (\text{girder spacing/clear slab span})^2 \qquad (6)$$

In MSC/NASTRAN program, this can be done easily by multiplying the material properties in the transverse direction by D_y.

Support for the structure consisted of a roller at each end of the beams. This roller provided resistance to vertical (z-direction) movements only. The beams were, therefore, free to rotate at their ends. For structural and numerical stabilities, as shown in Fig. 1, no x-displacement was allowed at A and B, and hinge support was applied at C and D. The finite element mesh was proportioned so that the maximum aspect ratio of the quadrilateral elements always remained at about two to one or less.

Typical discretization of the bridge deck structure is shown in Fig. 1.

MATERIAL MODELS

For linear analyses, an orthotropic material model was used for the slab. Elastic material properties are summarized in Table 1, where E is the modulus of elasticity and μ is Poisson's ratio. For nonlinear analyses, the complete concrete model as shown in Fig. 6 was adopted [6]. The curve portion, AB, is described by

$$f_c = f_c' \left(\frac{2\ \epsilon_c}{0.002} - \frac{\epsilon_c}{0.002} \right) \qquad (7)$$

where f_c is the compressive stress, f_c' the maximum compressive stress, and ϵ_c is the compressive strain.

Elastic analysis uses the tangent stiffness, E_t, as shown in Fig. 6. E_t can be determined by

$$E_t \text{ (ksi)} = 57 \sqrt{f_c'} \tag{8}$$

where f_c' is in pound/inch^2.

In the present study, material softening (i.e. straight lines BC and CD in Fig. 6) was not considered since a strain value higher than 0.002 would be practically unacceptable. Maximum tensile stress for reinforced concrete materials was set to be 0.10 f_c'. For damping considerations, 3% of critical viscous damping was used for linear analyses, and 7% for nonlinear analyses.

TRUCK LOAD

The movable wheel loads of a test vehicle used in the refined static analysis are shown in Fig. 7, where the average wheel width of 6.2 feet was used for simplification. Since the truck can move to anywhere on the bridge, it is unlikely that the locations of the wheel loads coincide with the nodes of the finite element model. Current version of MSC/NASTRAN program [4] requires that concentrated loads be specified at the nodal points. Therefore, manual calculation of the equivalent nodal loads was needed. Details for calculating such forces are provided in Appendix I.

The studied bridge systems belong to short-span type ($\leq$ 140 feet). For such type of bridges, wheel loads usually control moment and shear which are being of interest [3]. So, AASHTO lane loading [3] was not considered in the analyses.

COMPUTATION OF THE COMPOSITE GIRDER MOMENT

The finite element program MSC/NASTRAN [4] requires the input of the bare beam ("stiffener") properties: A (cross-sectional area), I (bending moment of inertia), J (torsional moment of inertia), E (Young's modulus) and G (shear modulus), Fig. 2, in addition to the slab ("shell") properties. The output then lists the axial force, P, and moment, M, that pertain to the beam element at the center of gravity. These allow the computation of the combined stress at the centerline of the bottom flange, σ_{comb}, as follows

$$\sigma_{comb} = \frac{P}{A} + \frac{M}{S_{b,nc}} \tag{9}$$

where $S_{b,nc}$ is the section modulus at the bottom of the beam with "noncomposite" action.

The moment carried by one composite section, M_{comp}, can be determined from beam theory

$$M_{comp} = \sigma_{comb} * S_{b,c} \tag{10}$$

where $S_{b,c}$ is the section modulus at the bottom of the beam with "composite" action. The composite section includes an effective flange, b, with due consideration of shear lag effects. In general, however, b= girder spacing (S) unless S exceeds 10 feet.

The following equation is adopted in the present bridge design code [3] for calculating the bending stress at the bottom of the beam, σ_b:

$$\sigma_b = \frac{DF * M}{S_{b,c}} \tag{11}$$

where M is the bending moment produced by one single lane of vehicle load on a simple beam, and DF is the distribution factor of live load.

In design, for bridges made of "I-shape" concrete girders, DF is determined by the following simple empirical formula [3]:

$$DF = \frac{S}{5.5} \tag{12}$$

where S is the center-to-center girder spacing in foot ($S \leq 14$ feet).

NUMERICAL EXAMPLES AND RESULTS

Two representative bridge systems, "BRIDGE 1" and "BRIDGE 2", as described above, were studied. The bridges were modeled three-dimensionally. Both linear and nonlinear analyses were performed. Wheels were assumed to act at the middle of the bridge as shown in Fig. 8. For verification and comparison purposes, another recognized general-purpose finite element program ADINA [7] was employed for these analyses. The finite element results and the computed static responses using the method described are summarized in Tables 2-5. To check the modeling for the bridge deck, studies including one and two slab elements between the girders were conducted and their results are shown in Fig. 9 with the field test result.

Computation time on IBM3090 mainframe machine was about 10 seconds for a typical three-dimensional (3-D) linear analysis, and 2½ minutes for a typical 3-D nonlinear analysis.

DISCUSSIONS

Finite element programs are notorious for generating stacks of printouts and a variety of results. It is essential that the designer conduct some checks by independent means to detect any gross error that may be introduced in the analysis through incorrect input data. To achieve that objective, two types of quality control checks were used:

1. Checking the general adequacy of the finite element prediction by comparing to field test and other analysis results. As shown in Fig. 9, the correlation is excellent, especially when the two-slab element model is used (i.e. "Analytic Results 1"). Parametric studies show that two-slab element model is required if the girder spacing is greater than 8 feet. As shown in Tables 2-5, MSC/NASTRAN's results were fairly close to ADINA's.
2. Checking the total elastic results statically. To show it, the 68.5-foot bridge is assumed as a simple beam subjected to the wheel loads described in Figs. 7 and 8. Using ordinary beam theory, this beam will produce an elastic moment of 10,330 kip-in at the midspan, which is quite close to what was computed by using the finite element programs (± 2%, see Table 2 and 4).

The bending stress at the bottom of the middle girder (i.e. girder 3) under the same wheel loads (Fig. 8) would be 0.4868 ksi for "BRIDGE 1" and 0.5452 ksi for "BRIDGE 2" as calculated by eqn (11), which was 24% higher than the linear finite element results (Tables 2 and 4) or about 43% higher than the nonlinear results (Tables 3 and 5). Such difference would be even more evident should the 15% discount permitted by the proposed new bridge code [1] be considered in the refined analysis results. This indicates that the distribution factor of live load calculated by the AASHTO formula, eqn (12), is too conservative. Indeed, parametric studies showed that the distribution factors calculated by eqn (12) were higher by 20% to 30% (without considering discounts) for typical "I-shape" girder bridges than those derived from linear finite element analyses. The degree of conservatism was nearly doubled if compared to the nonlinear finite element results.

CONCLUSIONS AND RECOMMENDATIONS

The AASHTO formula, eqn (12), is generally too conservative for predicting the distribution factor of vehicle live loads for "I-shape" girder bridges. The finite element method using MSC/NASTRAN program [4] not only gives good results, but also provides more information that can not be possibly obtained from a simplified method. The refined method using finite elements produces overall economy in bridge design, but must be used with great care. MSC/NASTRAN program [4] can model a general bridge system easily and efficiently because of the varieties of elements and material models built-in. It is predictable that finite element methods will soon be widely accepted by practicing bridge engineers and state and federal departments of transportation. To gain recognition and make the computer program [4] more suitable for bridge applications, several modifications can be made to the program: (1) specification of unsymmetrical I-shape and box-shape cross sections being often used for bridge girders and automatic calculation of their sectional properties, (2) implementation of AASHTO [3] vehicle movable loads (both truck and lane loads) so that the tedious manual calculation of equivalent nodal loads can be avoided.

ACKNOWLEDGEMENT

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APPENDIX I. CALCULATION OF EQUIVALENT NODAL LOADS

Referring to Fig. A1, the equivalent nodal forces are calculated according to the following equations:

$$P_1 = P * \frac{A_1}{\Sigma A} \tag{A1}$$

$$P_2 = P * \frac{A_2}{\Sigma A} \tag{A2}$$

$$P_3 = P * \frac{A_3}{\Sigma A} \tag{A3}$$

$$P_4 = P * \frac{A_4}{\Sigma A} \tag{A4}$$

where P is the wheel load, A_1, A_2, A_3 and A_4 the tributary areas, and ΣA is the total element area (= $A_1 + A_2 + A_3 + A_4$).

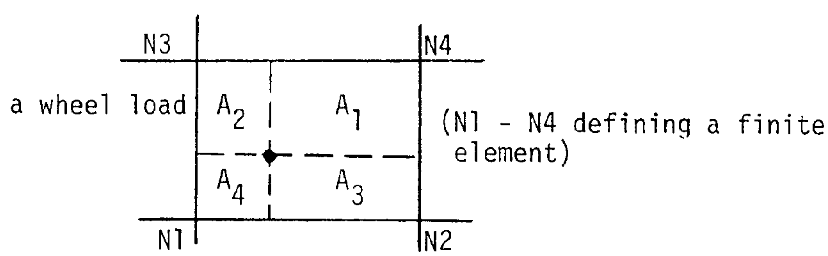

Figure A1. Distribution of wheel load to nodes.

Table 1. Summary of Elastic Material Properties ("BRIDGE 1" and "BRIDGE 2")

(units: kip, inch)

elastic property	slab	girder	diaphragm
E	3824	4488	3824
μ	0.20	0.20	0.15

Table 2. MSC/NASTRAN "Linear" Finite Element Results and Static Responses for "BRIDGE 1"
(Values in parentheses obtained from ADINA)

(units: kip, inch)

girder	P	M	σ_{comb} *	M_{comp} **	percentage of total moment
1	2.997 (3.043)	338.4 (343.5)	0.060 (0.0609)	615.1 (624.5)	5.9 (5.9)
2	58.59 (59.48)	922.3 (936.3)	0.2537 (0.2576)	2602.1 (2641.7)	25.0 (25.0)
3	97.31 (98.79)	1318 (1338)	0.3868 (0.3927)	3966.7 (4027.1)	38.1 (38.1)
4	58.59 (59.48)	922.3 (936.3)	0.2537 (0.2576)	2602.1 (2641.7)	25.0 (25.0)
5	2.997 (3.043)	338.4 (343.5)	0.060 (0.0609)	615.1 (624.5)	5.9 (5.9)
				Σ= 10401.1 (10559.5)	

* Calculated by eqn (9); ** Calculated by eqn (10).

Table 3. MSC/NASTRAN "Nonlinear" Finite Element Results and Static Responses for "BRIDGE 1"
(Values in parentheses obtained from ADINA)

(units: kip, inch)

girder	P	M	σ_{comb} *	M_{comp} **	percentage of total moment
1	2.66 (2.7)	287.6 (292.0)	0.0512 (0.0520)	525.3 (533.3)	5.8 (5.8)
2	52.1 (52.9)	783.7 (795.6)	0.2198 (0.2231)	2253.6 (2287.9)	25.0 (25.0)
3	86.6 (87.9)	1120.2 (1137.3)	0.3357 (0.3408)	3442.5 (3494.9)	38.2 (38.2)
4	52.1 (52.9)	783.7 (795.6)	0.2198 (0.2231)	2253.6 (2287.9)	25.0 (25.0)
5	2.66 (2.7)	287.6 (292.0)	0.0512 (0.0520)	525.3 (533.3)	5.8 (5.8)
				Σ= 9000.2 (9137.3)	

* Calculated by eqn (9); ** Calculated by eqn (10).

Table 4. MSC/NASTRAN "Linear" Finite Element Results and Static Responses for "BRIDGE 2"
(Values in parentheses obtained from ADINA)

(units: kip, inch)

girder	P	M	σ_{comb} *	M_{comp} **	percentage of total moment
1	3.357 (3.408)	378.9 (384.7)	0.0672 (0.0682)	688.9 (699.4)	5.9 (5.9)
2	65.62 (66.62)	1032.9 1048.7)	0.2842 (0.2885)	2914.3 (2958.7)	25.0 (25.0)
3	108.99 (110.64)	1476 (1499)	0.4332 (0.4398)	4442.7 (4510.4)	38.1 (38.1)
4	65.62 (66.62)	1032.9 (1048.7)	0.2842 (0.2885)	2914.3 (2958.7)	25.0 (25.0)
5	3.357 (3.408)	378.9 (384.7)	0.0672 (0.0682)	688.9 (699.4)	5.9 (5.9)
				Σ= 11649.2 (11826.6)	

* Calculated by eqn (9); ** Calculated by eqn (10).

Table 5. MSC/NASTRAN "Nonlinear" Finite Element Results and Static Responses for "BRIDGE 2"
(Values in parentheses obtained from ADINA)

(units: kip, inch)

girder	P	M	σ_{comb} *	M_{comp} **	percentage of total moment
1	2.98 (3.02)	322.1 (327.0)	0.0574 (0.0582)	588.3 (597.3)	5.8 (5.8)
2	58.4 (59.2)	877.7 (891.1)	0.2461 (0.2499)	2524.0 (2562.4)	25.0 (25.0)
3	97.0 (98.4)	1254.7 (1273.8)	0.3760 (0.3817)	3855.6 (3914.3)	38.2 (38.2)
4	58.4 (59.2)	877.7 (891.1)	0.2461 (0.2499)	2524.0 (2562.4)	25.0 (25.0)
5	2.98 (3.02)	322.1 (327.0)	0.0574 (0.0582)	588.3 (597.3)	5.8 (5.8)
				Σ= 10080.3 (10233.8)	

* Calculated by eqn (9); ** Calculated by eqn (10).

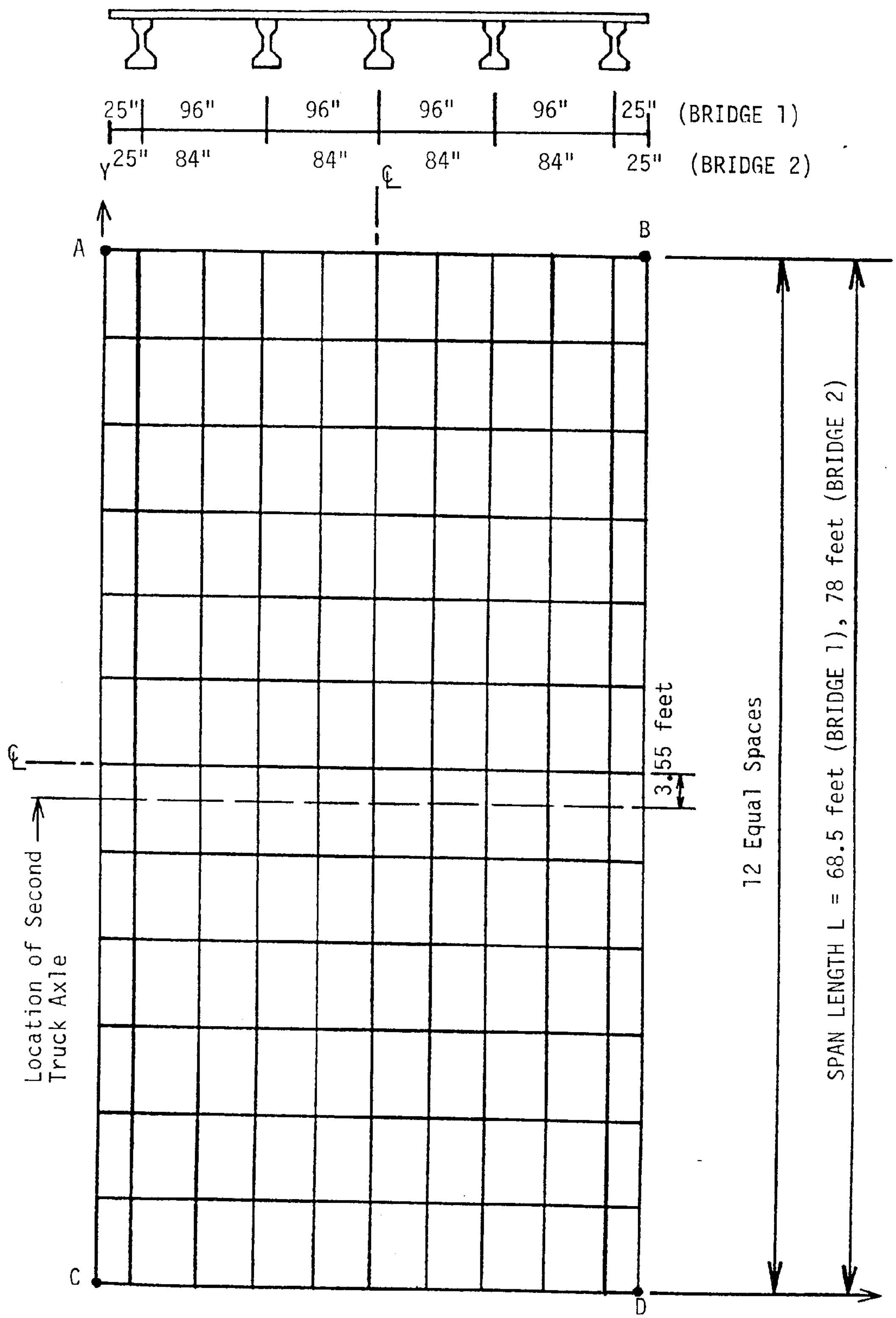

Figure 1. Investigated bridge systems and finite element discretization.

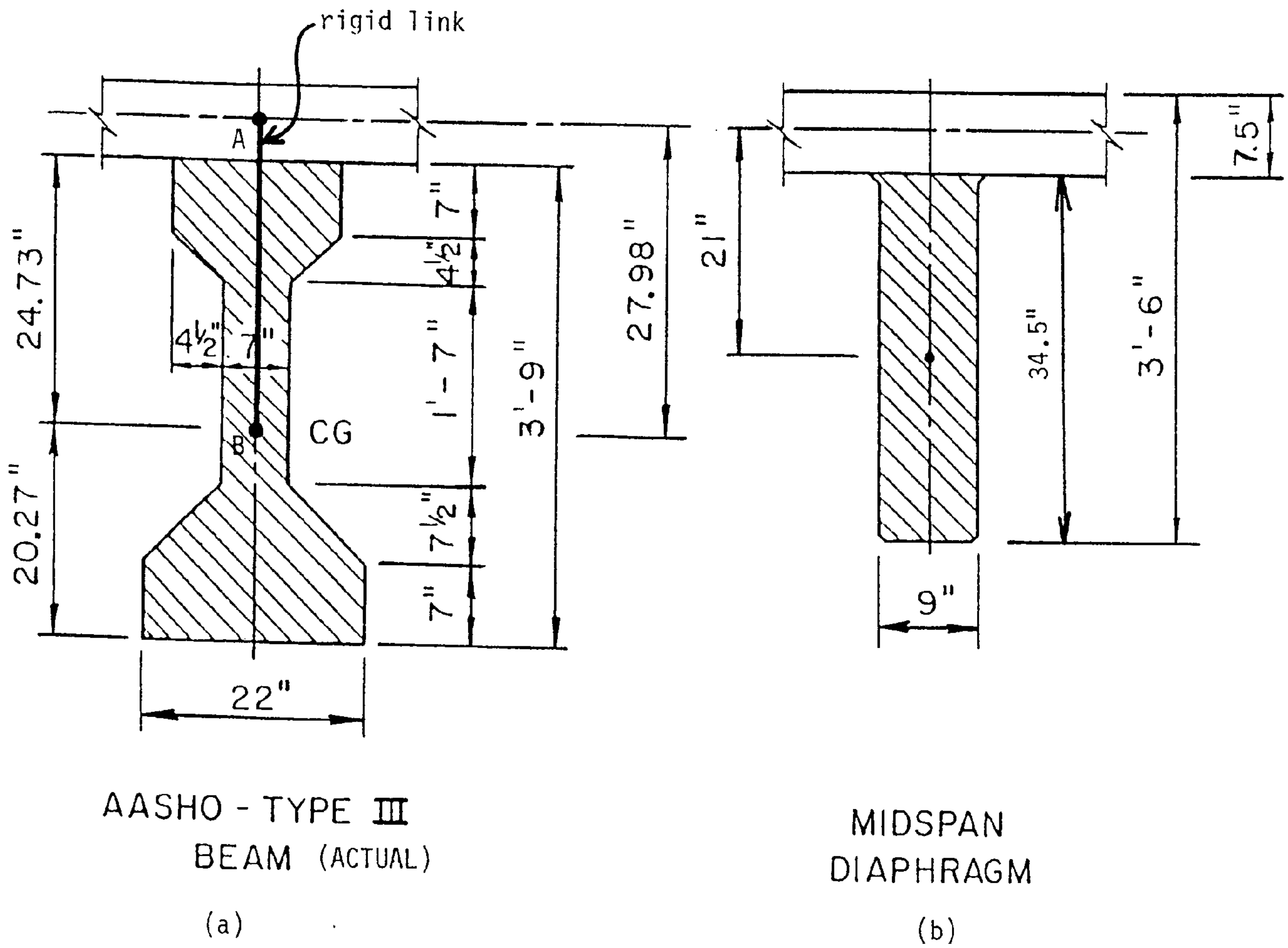

Figure 2. Cross-sections of bridge girders and diaphragms, and rigid link AB.

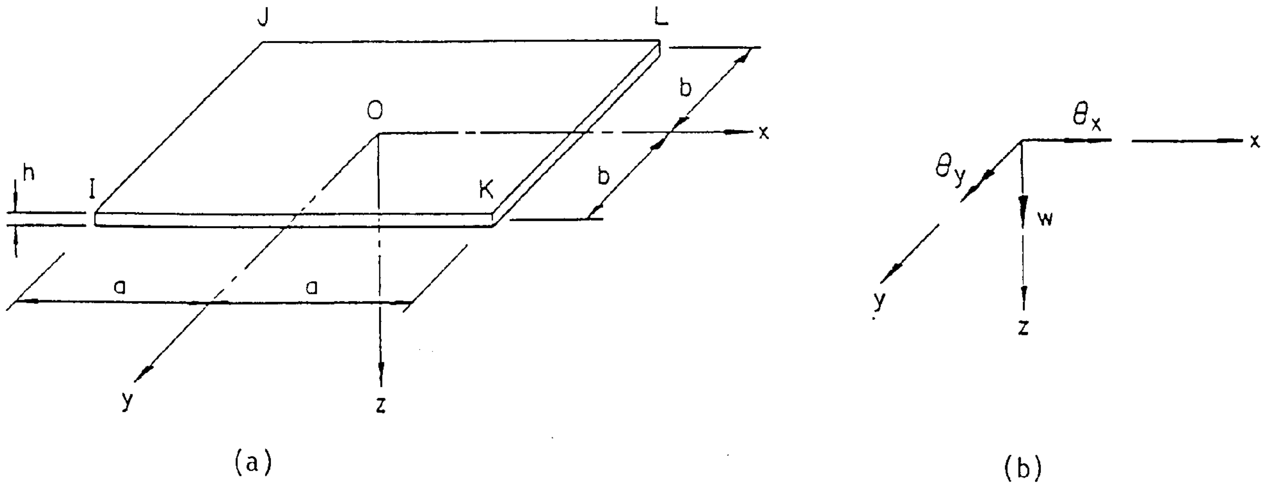

Figure 3. Quadrilateral plate element and basic displacement components.

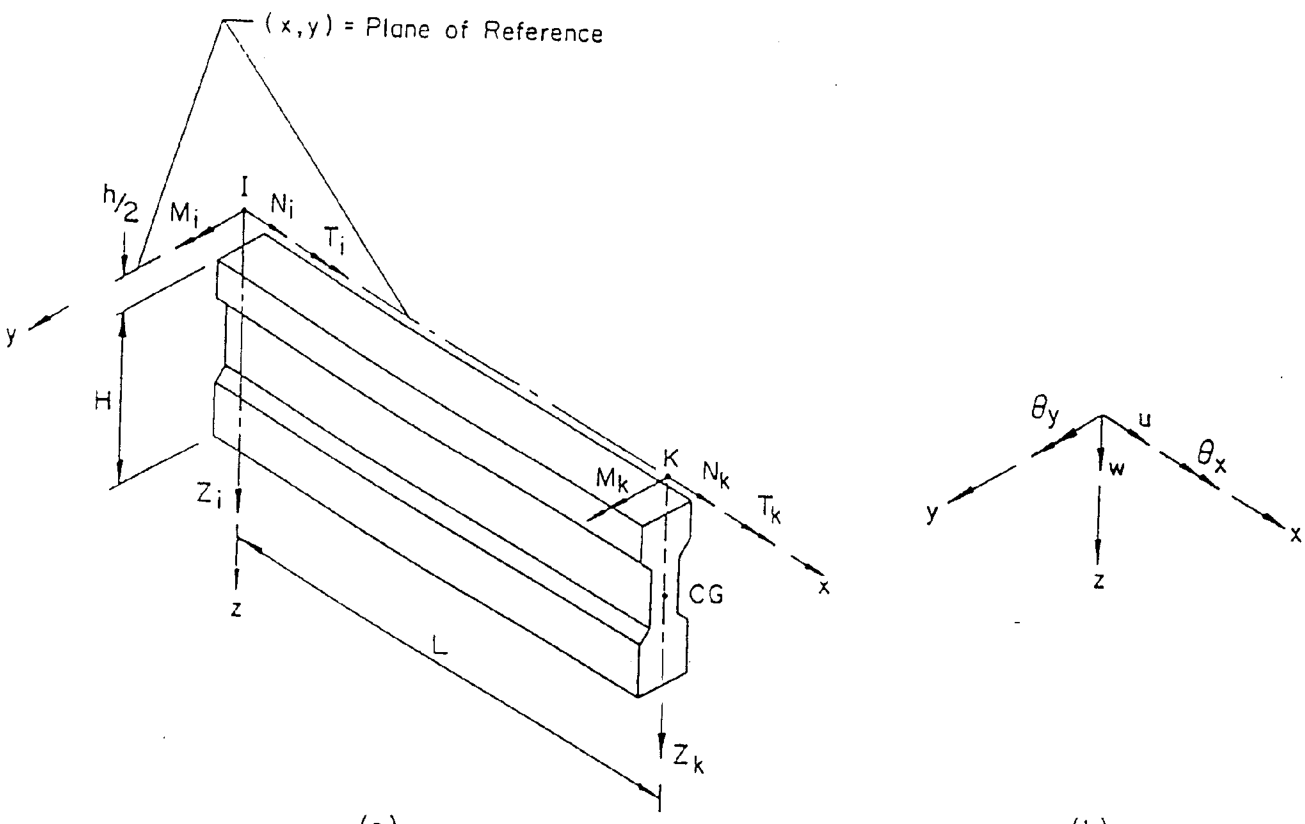

Figure 4. Eccentrically attached beam/stiffener element.

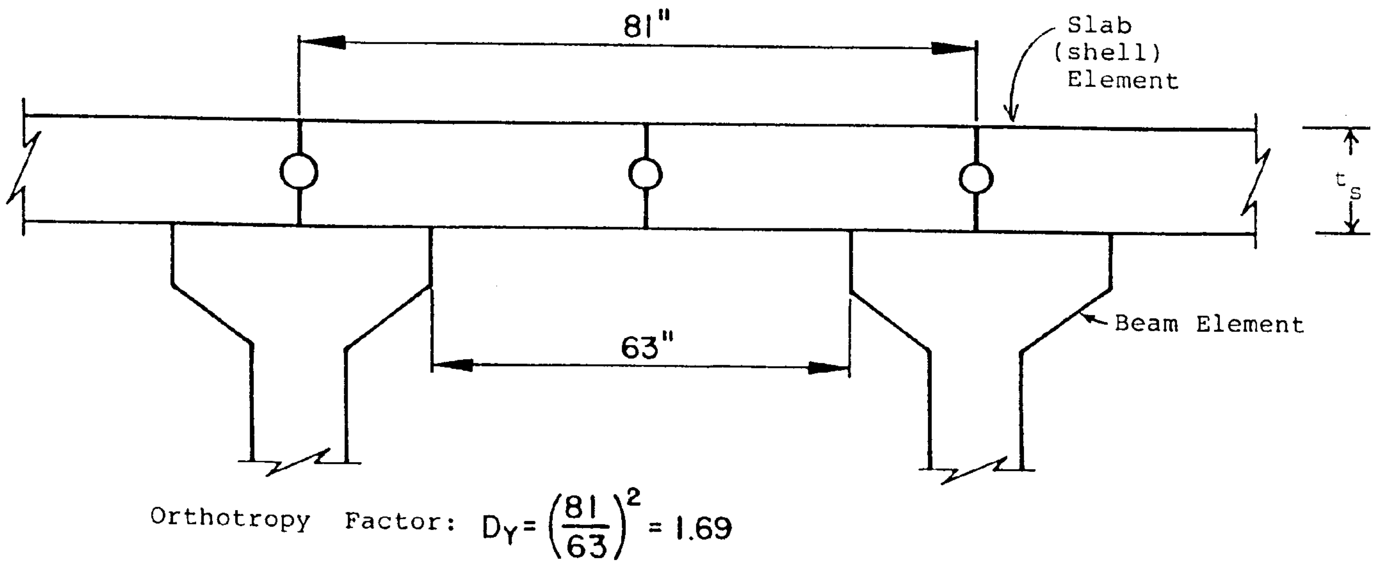

Figure 5. 2-plate mesh discretization and orthotropy factor (example).

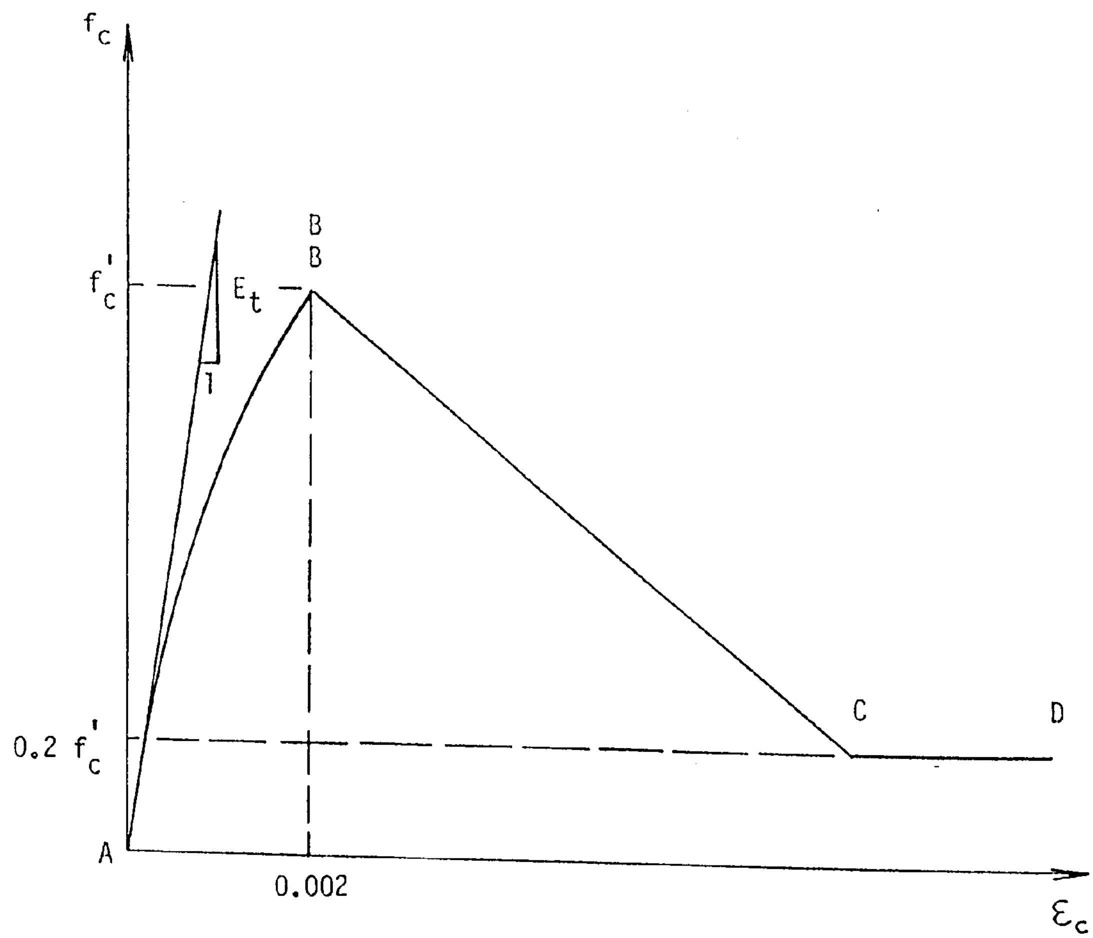

Figure 6. Nonlinear model for reinforced concrete materials.

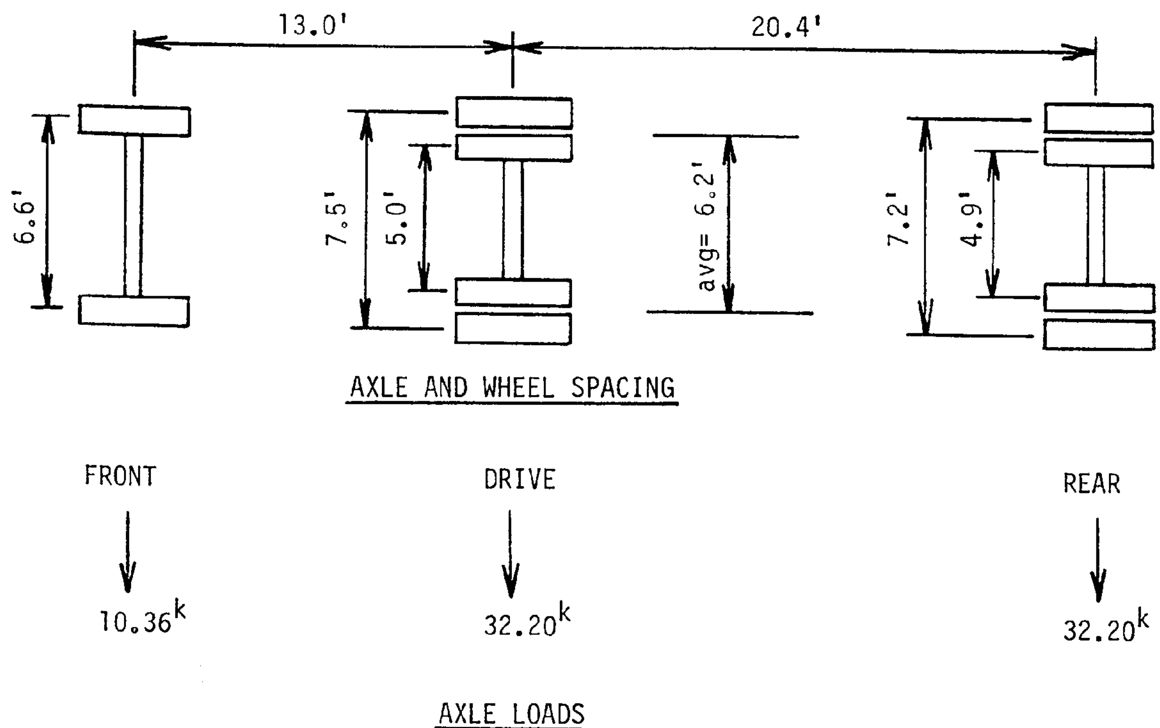

Figure 7. Wheel loads of test truck.

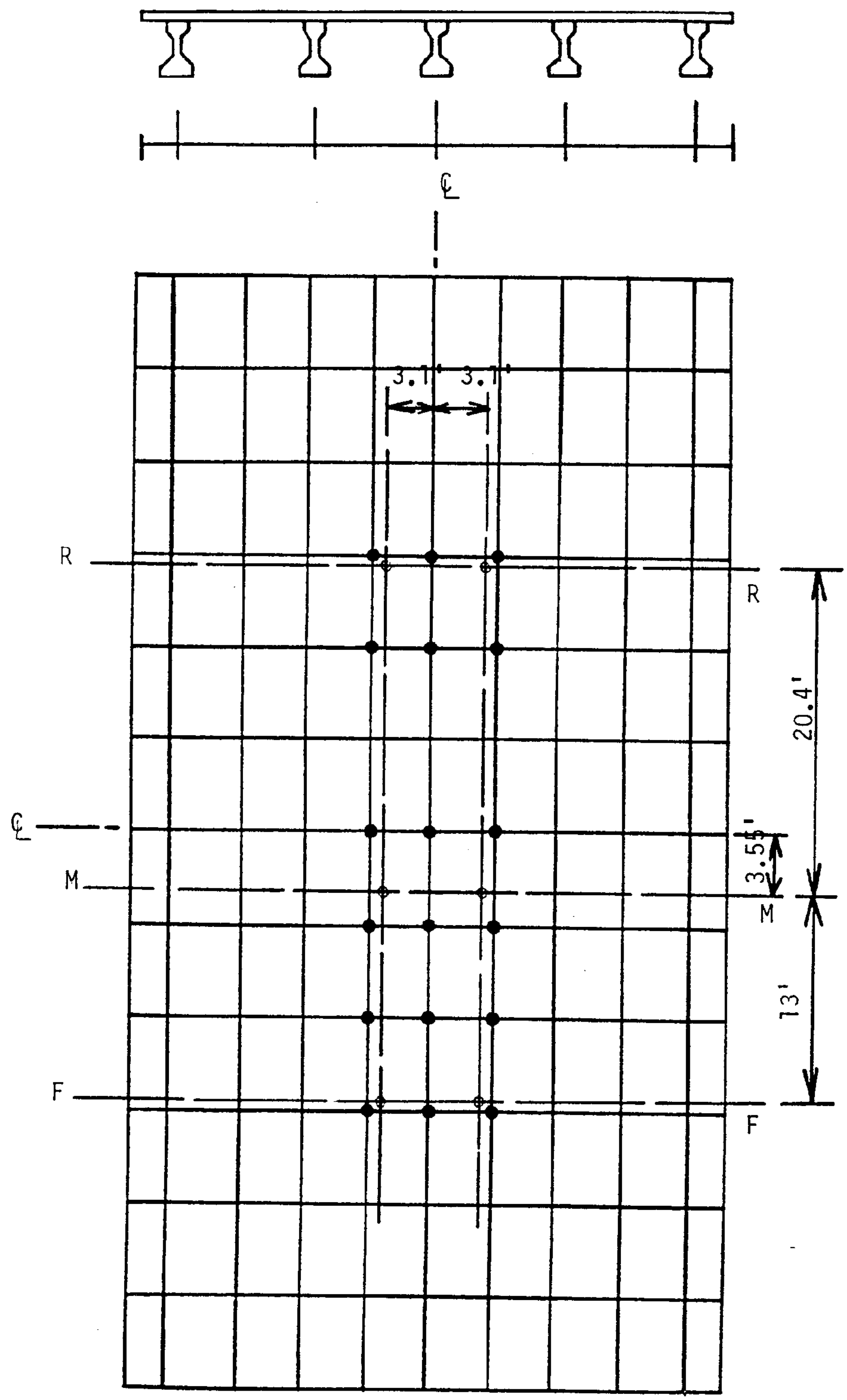

Figure 8. Wheel-load locations and loading nodal points (∘ : wheel-load location; • : loading nodal point).

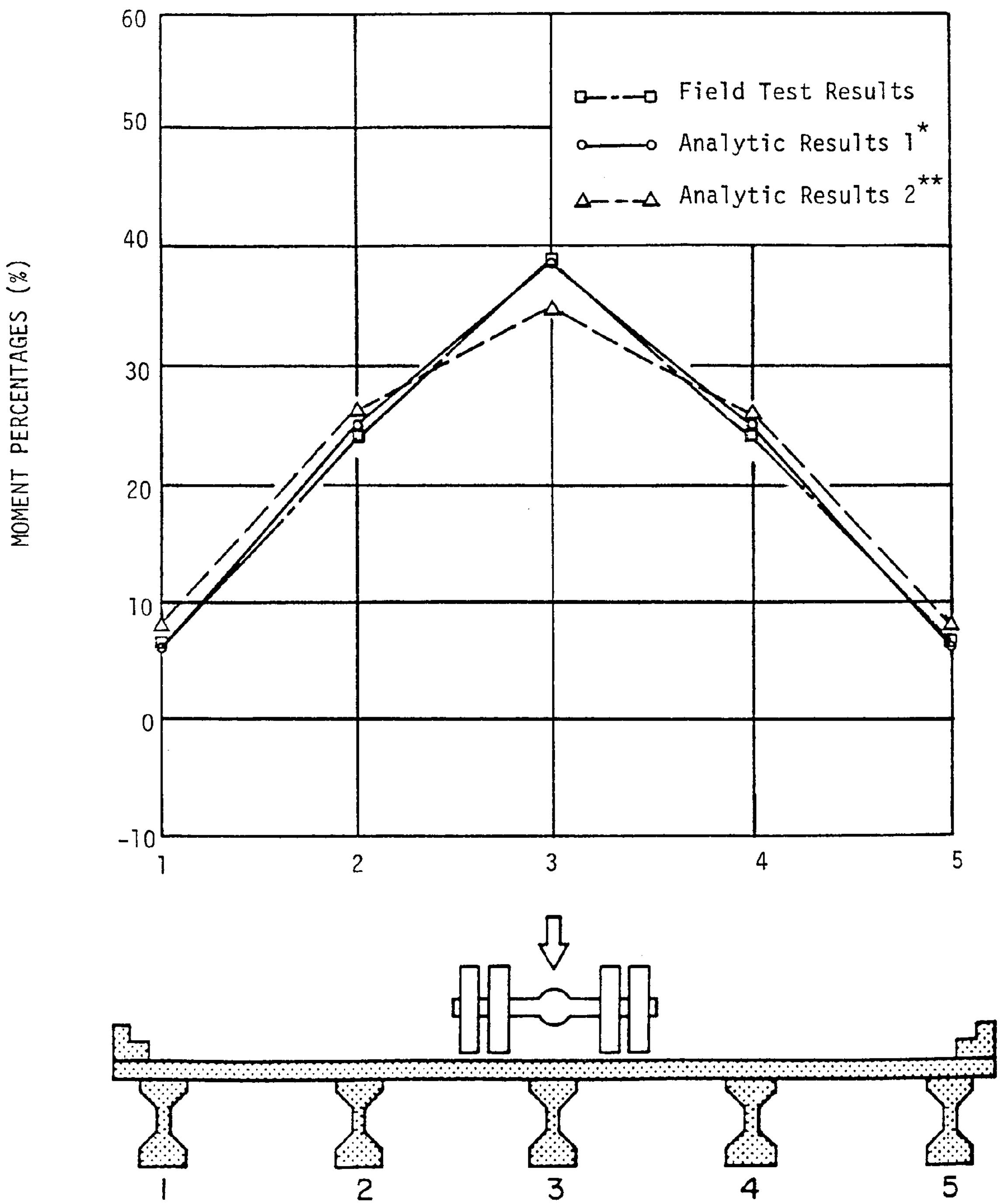

Figure 9. Correlation of moment percentages (*: 2-plate element model; **: 1-plate element model).