

**NONLINEAR SEISMIC ANALYSIS OF BRIDGES: PRACTICAL APPROACH
AND COMPARATIVE STUDY**

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ABSTRACT

A simplified numerical model with an efficient computational scheme is proposed for nonlinear seismic analysis of bridges. The results obtained from the simplified model are compared to those from the refined model and other methods. The proposed model is shown to be especially effective for obtaining maximum responses, and is practical and economical. Effects of bridge skews on responses are also carefully examined. The paper concludes with a number of bridge examples and design recommendations.

INTRODUCTION

Bridge structures had been predominantly analyzed and designed elastically. Beginning at the year of 1971, researchers started to study the nonlinear seismic response of bridges. During the past two decades, with the funding primarily from the Federal Highway Administration, California Department of Transportation and National Science Foundation, researches in this particular area have been steadily progressed. A brief review of the major research achievements during that time span is provided in the following.

Tseng and Penzien [1] studied the coupled inelastic flexural behavior of pier columns by using a three-dimensional elasto-plastic model, and the discontinuous behavior of expansion joints by a nonlinear mathematical model. Chen and Penzien [2] proposed an efficient three-dimensional time-history analysis method by incorporating four different types of elements, namely solid elements for the surrounding soils, isoparametric beam elements for bridge deck, pier columns and pier caps, frictional elements for the interfaces between the soils and abutments, and boundary elements for considering the foundation flexibilities at the bases of pier columns. In their model, the effects of separation, impact and slippage at soil-abutment interfaces were taken into consideration. Toki [3] contributed to the study of nonlinear responses of continuous bridges subjected to traveling seismic waves. Imbsen and Penzien [4] developed a nonlinear beam-column element for pier columns for evaluating energy-absorption characteristics of highway bridges, which includes kinematic hardening feature. In addition, they proposed a tension-compression bar element having bilinear force-displacement relationship for studying nonlinear behaviors of expansion joints. Greimann et al. [5] were credited to nonlinear analysis of bridges with integral abutments. Ghusn and Saiidi [6] incorporated a bilinear biaxial bending element (five-spring element) for pier columns into a nonlinear static analysis computer program. Chaisomphob et al. [7] studied dynamic nonlinear behaviors of steel towers of long-span suspension bridges, and Razaqpur and Nofal [8] investigated composite bridges in a similar way. Nazmy and Abdel-Ghaffar [9] and Abdel-Ghaffar and Nazmy [10] examined long-span cable-stayed bridges under seismic loadings with the considerations of different sources of nonlinearities arising from changes of bridge geometry, sag effects of cables and axial force-bending interaction in the bridge tower. In their work, both synchronous and nonsynchronous support motions due to seismic excitations, and effects of nondispersive traveling seismic waves were carefully taken into account. Ahmed [11] contributed to nonlinear inelastic seismic analysis of highway bridges.

Though showing significant research evidence and improvement, the analysis methods or models mentioned above are generally too sophisticated from the viewpoint of bridge practice. Full recognition and implementation of them in the current bridge specification [12] is still miles away. Indeed, further study on nonlinear seismic analysis of bridges leading to the development of

simplified methods is imminent. This paper discusses a simplified and practical approach for nonlinear seismic analysis of bridges, and its comparative studies in details.

PROBLEM DESCRIPTION

As shown in Fig. 1, a representative two-span continuous bridge system made of four prestressed concrete box girders was investigated as a pilot study. Dimensions for the bridge and its structural components are summarized in Table 1. Actual cross-section of the girders is shown in Fig. 2. As shown in Fig. 3, the superstructures (girders) are connected to the substructures (abutments and pier) with the use of elastomeric bearing pads. As a common bridge practice, expansion joints allowing movement in the bridge longitudinal direction only were imposed at the abutments, and pinned-type connections imposed at the pier. It is noted that the girders are not going to displace freely at the expansion joints due to the friction existing between them and elastomeric bearing pads.

GOVERNING EQUATION

Time-history mode superposition analysis was employed in this study. This method is effective for obtaining earthquake responses where only a few mode shapes are usually required. The governing finite element equilibrium equations for the solution of the response at time $t+\Delta t$ are

$$\begin{aligned} [M] \{\ddot{U}\}_{t+\Delta t}^{(i)} + [C] \{\dot{U}\}_{t+\Delta t}^{(i)} + [K]_{\tau} \{\Delta U\}^{(i)} \\ = \{R\}_{t+\Delta t} - \{F\}_{t+\Delta t}^{(i-1)} \quad (i=1, 2, \dots) \end{aligned} \quad (1)$$

where $[M]$ is the total mass matrix, $[C]$ the total damping matrix, $[K]_{\tau}$ the total stiffness matrix corresponding to the configuration at some previous time τ , $\{\ddot{U}\}$, $\{\dot{U}\}$ and $\{\Delta U\}$ respectively the acceleration, velocity and displacement vectors, $\{R\}$ the loading vector, and $\{F\}$ is the force vector.

The displacements at time $t+\Delta t$, $\{U\}_{t+\Delta t}$, can be determined from the following equation

$$\{U\}_{t+\Delta t} = \sum_{i=r}^s \{\phi\}_i x_{i,t+\Delta t} \quad (2)$$

where $\{\phi\}_i$ is the eigen-vector for i th mode, and $x_{i,t+\Delta t}$ is the i th generalized modal displacement at time $t+\Delta t$.

Through the use of the following transformation, uncoupling of eqn (1) is possible.

$$[K]_{\tau} \{\phi\}_i = \omega_i^2 [M] \{\phi\}_i \quad (i=r, \dots, s) \quad (3)$$

where ω_i and $\{\phi\}_i$ are the free vibration frequency and mode shape vector of the system at time τ .

Hence, the decoupled form of eqn (1) is

$$\begin{aligned} & \{\ddot{x}\}_{t+\Delta t}^{(i)} + [\Omega_1] \{\dot{x}\}_{t+\Delta t}^{(i)} + [\Omega_2] \{\Delta x\}^{(k)} \\ & = [\Phi]^T (\{R\}_{t+\Delta t} - \{F\}_{t+\Delta t}^{(i-1)}) \quad (i=1, 2, \dots) \end{aligned} \quad (4)$$

where $[\Omega_1]$ is a diagonal matrix consisting of $2 \xi_r \omega_r, \dots, 2 \xi_s \omega_s$ (ξ = damping ratio), $[\Omega_2]$ a diagonal matrix consisting of $\omega_r^2, \dots, \omega_s^2$, and $[\Phi]$ is a matrix consisting of $\{\phi\}_r, \dots, \{\phi\}_s$.

For comparisons, the above equations were solved by two well recognized general-purpose finite element codes: MSC/NASTRAN [13] and ADINA [14]. To extract the maximum modal responses, the complete quadratic combination method (CQC) proposed by Wilson et al. [15] was adopted, and the two load combinations specified in the current bridge design code [12], namely 1.0 L + 0.3 T and 0.3 L + 1.0 T (L: longitudinal; T: transverse), were considered.

NONLINEAR MODEL AND ELASTIC PROPERTIES FOR CONCRETE MATERIAL

The complete stress-strain relation for reinforced concrete materials was based on Kent and Park [16], as shown in Fig. 4. The curve portion, AB, is described by

$$f_c = f_c' \left[\frac{2 \epsilon_c}{0.002} - \left(\frac{\epsilon_c}{0.002} \right)^2 \right] \quad (5)$$

where f_c is the compressive stress, f_c' the maximum compressive stress, and ϵ_c is the compressive strain.

Elastic analysis simply utilizes the tangent stiffness, E_t , as shown in Fig. 4. E_t can be determined by

$$E_t \text{ (ksi)} = 57 \sqrt{f_c'} \quad (6)$$

where f_c' is in the unit of pound per square inch.

In the present study, material softening (i.e. the straight lines BC and CD in Fig. 4) was not considered since a strain value higher than 0.002 is practically unacceptable. The elastic properties (E : Young's modulus, and μ : Poisson's ratio) are summarized in Table 2. Maximum tensile stress for reinforced concrete materials was set to be 0.10 f_c' . For damping

considerations, 5% of critical viscous damping was used for linear analyses, and 8% for nonlinear analyses.

THE SIMPLIFIED AND PRACTICAL NONLINEAR ANALYSIS MODEL

For seismic design of bridges, the main interest is to find out the "maximum" earthquake responses (displacements and forces) at the connections/joints and pier columns. This goal can be achieved by a simplified and practically-acceptable model. As shown in Fig. 5, the girders were modeled as a "superbeam" representing the actual superstructure system. However, the pier system was modeled as a frame in order to incorporate the relative stiffnesses between the bridge girders and pier columns. General beam elements were used in the linear analyses, while isoparametric solid elements were employed in the nonlinear analyses since the beam elements in the current version of the programs (both MSC/NASTRAN and ADINA) can not be used with a nonlinear material model. Overall/Representative modeling (rather than detailed one) for the bridge deck, barriers and diaphragms was needed in the simplified model. All structural weights were accounted for in the model for determining the seismic response. Uniform distribution over the superbeam for the weights was assumed. The abutment walls and footings were modeled as "mass elements" since they are much stiffer than the adjacent structural components. To ensure the structural integrity and the composite behavior between the bridge deck and girders, "rigid elements" were used to model the connection between the pier cap and pier columns, and also between pier columns and pier footings.

As shown in Fig. 5, the behavior of the elastomeric bearing pads at the abutments was represented by "spring elements" (K_{bz} being the compressive element, and K_{by} being the frictional element). The interaction effects between the bridge and surrounding soils were incorporated by the use of soil springs being also modeled as spring elements. Of the possible 36 vibration modes (stiffness matrix $[K]$ having the dimension of 6×6), only eight of them, namely 3 sliding modes (K_{xx} , K_{yy} and K_{zz} with K_{zz} being the vertical sliding), 2 rocking modes (K_{rx} and K_{ry}), 1 twisting mode (K_{rz}) and 2 coupling modes (K_{yrx} and K_{xry}) are significant and considered. The values of the soil springs depend on the frequency of dynamic loading, footing type, footing dimensions, geological condition, and soil properties. In this study, the state-of-art formulae included in reference [17] were used to determine them. The soil used was a sand type with Young's modulus of 10 ksi and Poisson's ratio of 0.32. The calculated spring constant values are summarized in Table 3.

NUMERICAL EXAMPLES AND RESULTS

As described above, the bridge system of Fig. 1 and Table 1 was investigated as a pilot study. Typical bridge cross-section is shown in Fig. 2. For examining skew effects on bridge response, five skew angles, namely 0° , 20° , 40° , 60° and 80° , were considered. For comparison purposes, both single-mode and multi-mode time-history analyses were performed linearly and nonlinearly. ATC-6 type earthquake [18] shown in Fig. 6 was chosen as the design earthquake. The effects of ground motion type on bridge response were investigated in a previous related study [19]. The refined method as described in reference [20] was also employed for comparison, which modeled the real bridge system in details including the soil medium and individual bridge girders and their connectivities. Since the refined model is quite large, especially for nonlinear analyses, only two skew cases (0° and 40°) were included in this study.

All computations were carried out by MSC/NASTRAN [13] and ADINA [14] programs installed on IBM3090 machine. Maximum earthquake responses obtained from the analyses are summarized in Tables 4 through 8, where D_{long} is the longitudinal displacement of the bridge, P_{col} , V_{col} and M_{col} are the axial compression, total shear and total bending moment of pier column, respectively (pier columns being circular), and $V_{L,abut}$ and $V_{T,abut}$ are the longitudinal and transverse shear of abutment with respect to the abutment centerline. Torsional moments were found to be negligible, and hence they are not included in the tables.

DISCUSSIONS OF THE RESULTS

Comparing Tables 5 and 7 with Table 8, the simplified analysis model produced fairly good approximations for maximum earthquake responses (only about 8% higher than those produced by the refined model). Comparing Table 4 with 6 and Table 5 with Table 7, the linear results were about 32% higher than the nonlinear ones. As seen in Tables 4-8, MSC/NASTRAN results compared fairly well with ADINA ones, and the skew of bridge affected the bridge response quite significantly. In general, D_L decreased with the increase of skew angle. P_{col} , V_{col} and M_{col} decreased with the increase of skew angle for skew between 0° and 40° , while they started to increase when the skew angle exceeds 40° . The multi-mode analysis (MMA) method being more accurate showed higher critical responses than the single-mode analysis (SMA) method. Random variations of the results between SMA and MMA were observed. On the average, MMA's results were 43.5% higher than SMA's.

The degree of variation of the results may vary from this bridge to another one, but the general trends mentioned above shall remain the same.

CONCLUSIONS AND RECOMMENDATIONS

The proposed simplified approach is computationally efficient and effective for nonlinear seismic analysis of bridges, and is a practical and economical model. Bridge systems can be easily handled by the proposed procedure since modeling effort is minimum. The computer program, MSC/NASTRAN, outplays the others in the following two major features for this type of studies: (a) the option of specifying "acceleration-time" history (ground accelerations) as the time-dependent load, which is necessary in order to obtain accurate seismic responses numerically, and (b) the availabilities of the "special elements" such as mass, spring and rigid elements which enable us to model the complex structural system in a more flexible and practical fashion. It is predictable that more and more bridge projects will require the use of refined methods. Therefore, it would be encouraging if these programs were made more suitable for bridge applications.

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Table 1. Summary of Bridge and Structural Dimensions

span length:	109 ft.	bridge deck:	218'x38'x9"
box girder:	218'x4'x5.5'	girder spacing:	10 ft.
pier cap:	38'x6'x4'		
pier column:	4' dia. x 14'	pier footing:	6'x6'x4'
abutment wall:	38'x4'x24.25'	abutment footing:	38'x6'x4'
end diaphragm:	18"x66"	intermediate diaphragm:	10"x39"

Table 2. Summary of Elastic Material Properties

	(units: kip, inch)						
	slab	girder	diaph.	abut.	pier cap	pier col.	ftg.
f_c' :	5200	8500	5200	3000	3000	3000	3000
E:	4110	5255	4110	3122	3122	3122	3122
μ :	0.20	0.20	0.20	0.15	0.15	0.15	0.15

Table 3. Summary of Calculated Spring Constants

	(units: kip, ft, rad)		
	Pier Footing	Abutment	Bearing Pad
K _{xx} :	10,468	86,948	-
K _{yy} :	26,199	62,378	-
K _{zz} :	22,354	70,489	-
K _{rx} :	386,903	10,843,832	-
K _{ry} :	183,704	1,406,722	-
K _{rz} :	568,855	21,931,398	-
K _{xry} :	13,957	115,930	-
K _{yrx} :	34,932	83,171	-
K _{by} :	-	-	14,000
K _{bz} :	-	-	210,000

Table 4. Maximum Earthquake Responses Obtained from "Linear" "Single-Mode" Analysis Using the "Practical" Model

	(units: ft, kip)				
	skew= 0°	skew= 20°	skew= 40°	skew= 60°	skew= 80°
D _L	0.74/0.75	0.67/0.68	0.51/0.52	0.33/0.33	0.19/0.20
P _{col}	283/288	226/230	179/182	274/278	382/388
V _{col}	257/261	218/221	213/216	274/279	362/368
M _{col}	3953/4013	3694/3750	3382/3434	3925/3986	5538/5623
V _{L,abut}	149/151	110/112	63/64	24/24	22/22
V _{T,abut}	0/0	37/38	79/81	106/108	122/124

Note: In each cell, the first value was per NASTRAN, while the second per ADINA, as separated by the slash.

Table 5. Maximum Earthquake Responses Obtained from "Linear"
"Multi-Mode" Analysis Using the "Practical" Model

	(units: ft, kip)				
	skew= 0 ⁰	skew= 20 ⁰	skew= 40 ⁰	skew= 60 ⁰	skew= 80 ⁰
D _L	0.78/0.80	0.75/0.76	0.73/0.74	0.64/0.65	0.60/0.62
P _{col}	328/334	323/328	419/425	450/458	524/532
V _{col}	287/291	282/286	373/379	406/412	479/487
M _{col}	4190/4254	4101/4164	5203/5282	5558/5643	6334/6431
V _{L,abut}	166/169	133/135	95/97	61/62	23/23
V _{T,abut}	0/0	48/49	93/94	108/110	129/131

Note: In each cell, the first value was per NASTRAN, while the second per ADINA, as separated by the slash.

Table 6. Maximum Earthquake Responses Obtained from "Nonlinear"
"Single-Mode" Analysis Using the "Practical" Model

	(units: ft, kip)				
	skew= 0 ⁰	skew= 20 ⁰	skew= 40 ⁰	skew= 60 ⁰	skew= 80 ⁰
D _L	0.50/0.51	0.45/0.46	0.35/0.35	0.22/0.22	0.12/0.13
P _{col}	192/195	154/156	121/123	185/188	260/263
V _{col}	174/177	147/149	144/146	187/190	244/248
M _{col}	2686/2727	2510/2548	2298/2334	2668/2709	3767/3824
V _{L,abut}	100/102	75/76	42/43	16/16	13/14
V _{T,abut}	0/0	25/26	54/55	71/73	83/84

Note: In each cell, the first value was per NASTRAN, while the second per ADINA, as separated by the slash.

Table 7. Maximum Earthquake Responses Obtained from "Nonlinear"
"Multi-Mode" Analysis Using the "Practical" Model

	(units: ft, kip)				
	skew= 0 ⁰	skew= 20 ⁰	skew= 40 ⁰	skew= 60 ⁰	skew= 80 ⁰
D _L	0.52/0.54	0.51/0.52	0.50/0.51	0.43/0.44	0.40/0.42
P _{col}	225/228	221/225	282/286	304/310	355/360
V _{col}	197/200	193/196	253/258	276/280	326/331
M _{col}	2827/2870	2789/2832	3517/3571	3779/3837	4304/4370
V _{L,abut}	115/117	90/92	67/68	40/41	15/15
V _{T,abut}	0/0	32/33	63/65	72/74	88/89

Note: In each cell, the first value was per NASTRAN, while the second per ADINA, as separated by the slash.

Table 8. Maximum Earthquake Responses Obtained from "Multi-Mode"
Analysis Using the "Refined" model

	(units: ft, kip)			
	Linear Analysis		Nonlinear Analysis	
	skew= 0 ⁰	skew= 40 ⁰	skew= 0 ⁰	skew= 40 ⁰
D _L	0.73/0.74	0.67/0.68	0.49/0.50	0.453/0.46
P _{col}	301/306	383/390	206/209	261/264
V _{col}	265/268	345/350	181/183	233/237
M _{col}	3835/3893	4764/4835	2606/2646	3242/3290
V _{L,abut}	153/155	89/90	105/107	61/62
V _{T,abut}	0/0	84/85	0/0	58/60

Note: In each cell, the first value was per NASTRAN, while the second per ADINA, as separated by the slash.

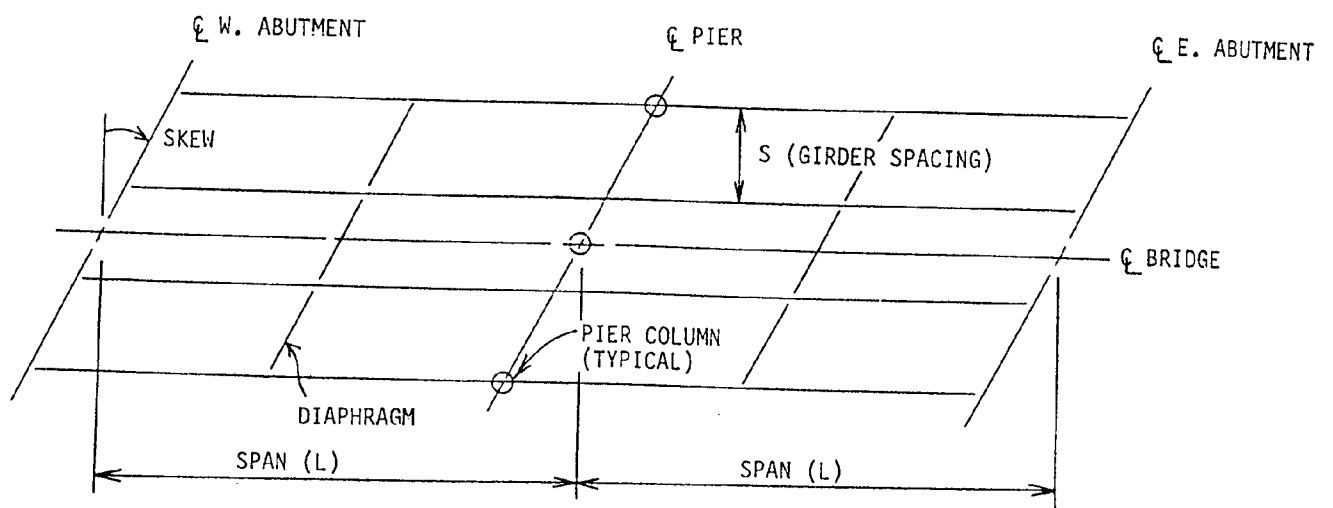


Figure 1. Bridge system for the pilot study (PLAN).

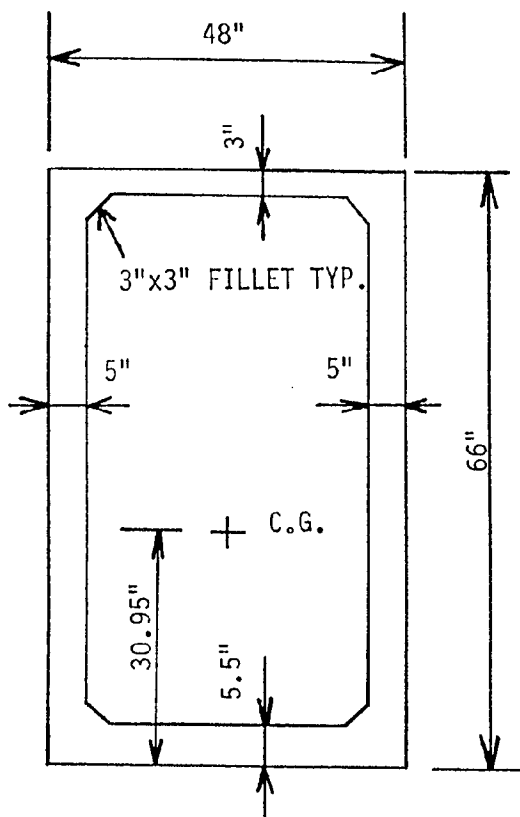


Figure 2. Typical cross-section of bridge girders.

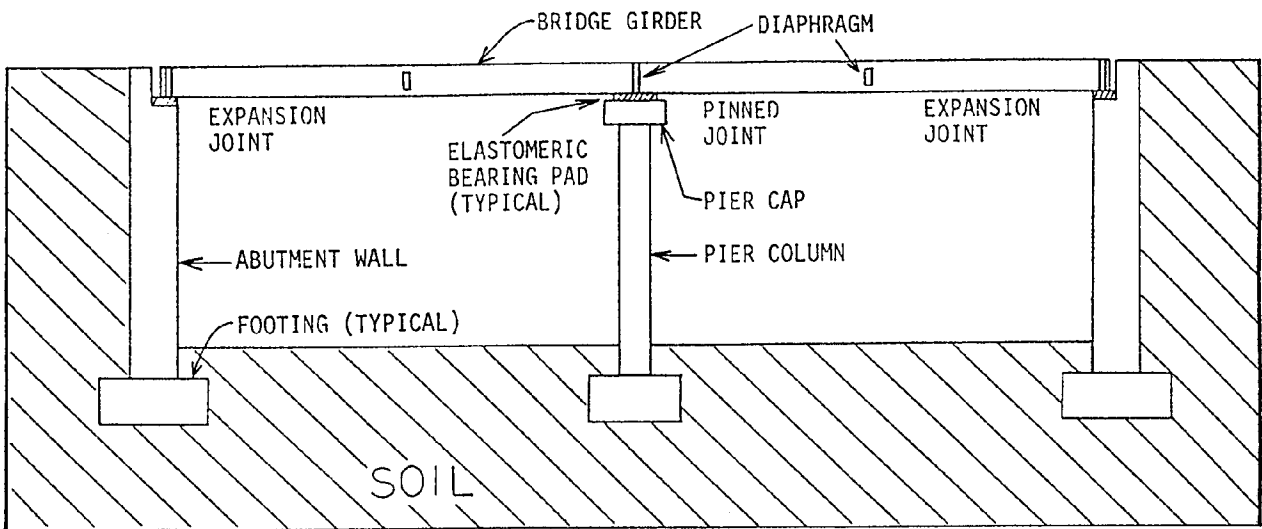


Figure 3. Bridge system for the pilot study (ELEVATION).

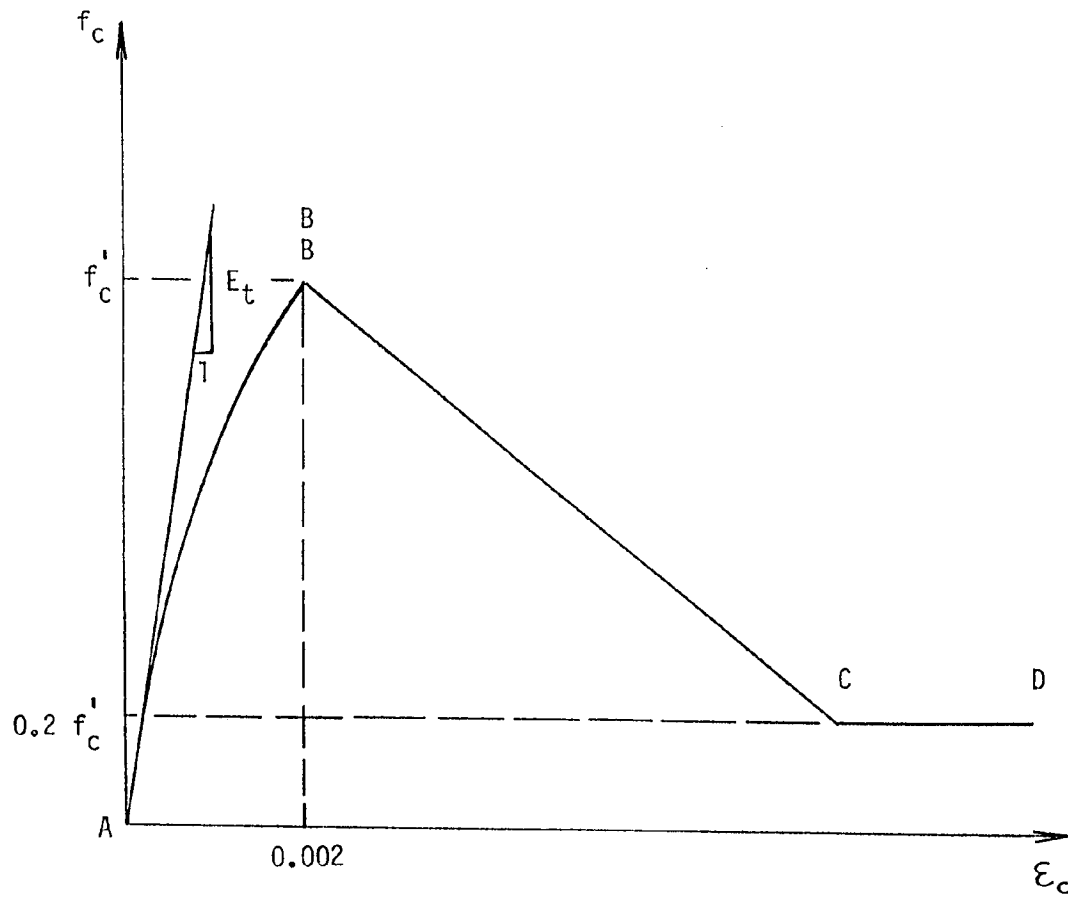
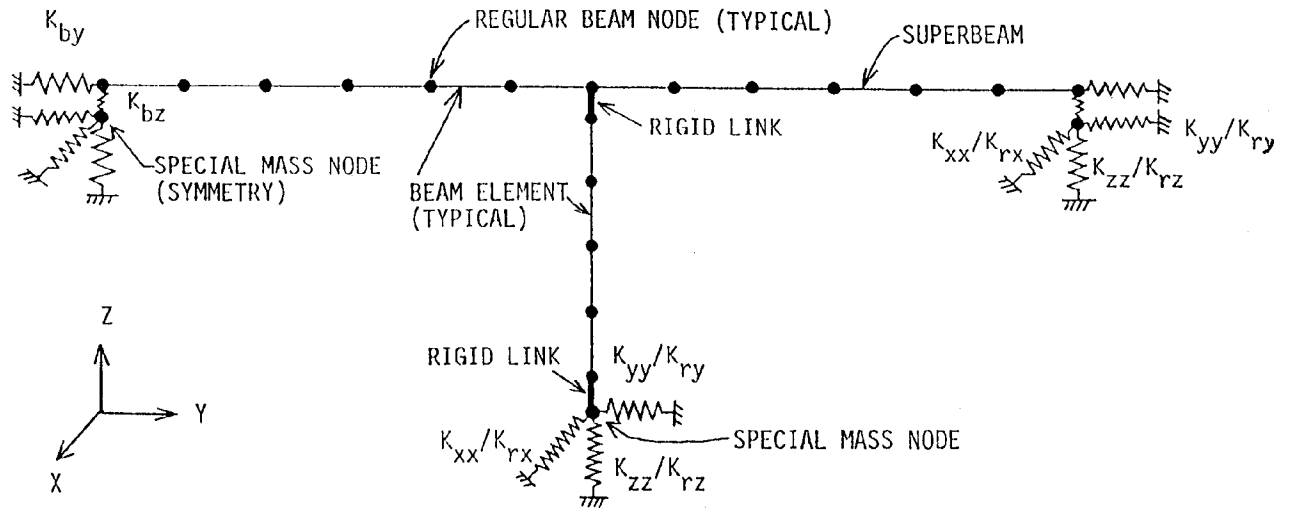
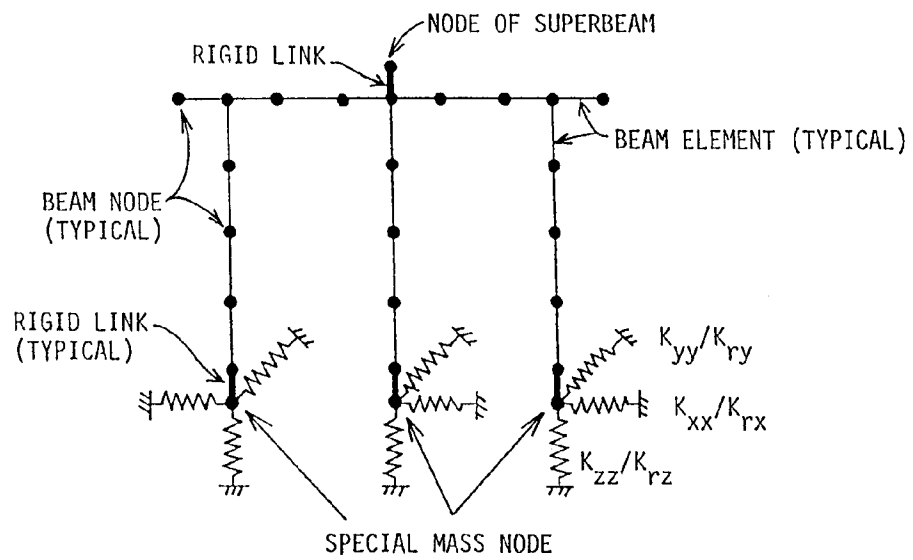


Figure 4. Nonlinear model for reinforced concrete materials.



(a) Bridge System



(b) Pier Frame

Figure 5. The simplified model.

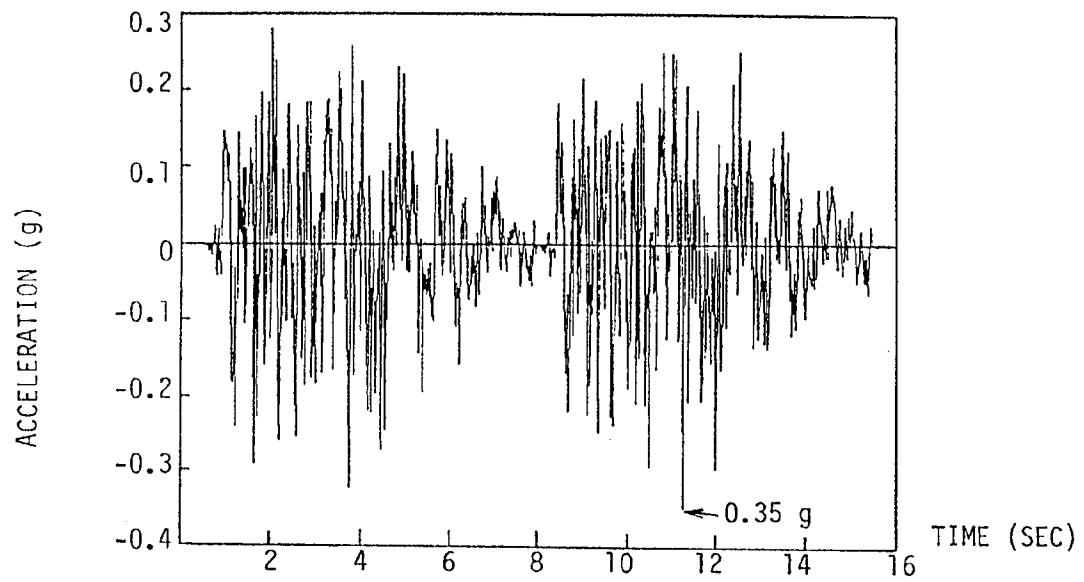


Figure 6. Ground acceleration-time history of ATC-6 earthquake.