

Determining Tube Stress From CBEND Element Forces and Moments

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Abstract

MSC/NASTRAN is used extensively in the design of external tubing for turbo-fan aircraft engines at Pratt & Whitney. It accurately calculates the stress of tubes under pressure, thermal, and case displacements and also natural frequencies. Many of the external tubes are small diameter (under 3/4 inch) and are part of a complex tube system. The most effective element type for these tubes is a “beam” element such as the CBEND. A complex small diameter tube system modeled with CBEND elements is very efficient compared with the same system modeled with CQUAD4 plate elements. However, while the MSC/NASTRAN CBEND element uses the *ASME Code* equations to account for the ovalization of the tube in the bends, the stress output is not complete. The in-plane and out-of-plane bending moments are not combined while the torque stress and the hoop stress are ignored. Therefore, the correct principal stresses are not determined. Pratt & Whitney developed a CBEND post-processor which uses the *ASME Code* equations to determine the complete stress field from the MSC/NASTRAN calculated forces and moments. This paper presents the *ASME Code* equations used by the CBEND post-processor and compares the results to equivalent plate models. Based upon these comparisons, the use of the MSC/NASTRAN CBEND element has been implemented in the design of small diameter tubes.

Introduction

MSC/NASTRAN is used extensively in the design of external tubing for turbo-fan aircraft engines at Pratt & Whitney. It accurately calculates the stress of tubes under pressure, thermal, and case displacements and also natural frequencies. Pratt & Whitney has a tube finite element pre-processor which will convert a tube design and manufacturing file into a MSC/NASTRAN input file. Currently, all tube models consist of the CQUAD4 plate element. However, many of the external tubes are small diameter (under 3/4 inch). The most effective element type for these tubes is a "beam." Beam models are much smaller than the equivalent plate model and therefore run faster and create smaller files. For example, a plate model of a 10" length of 0.5" diameter tube requires 900+ elements corresponding to 5600+ degrees of freedom assuming a plate element every 15 degrees and an aspect ratio of 2.0. The corresponding beam model would require 38 elements corresponding to 234 degrees of freedom. For complex tube systems, the use of plates would be prohibitive. To illustrate this, the 9 tube system of example 4 contains 1495 grids. The input file is .5 Megabytes and the OUTPUT2 file is .15 Megabytes. In comparison, the equivalent plate model contains 23,094 grids. Its input file is 5.4 Megabytes and OUTPUT2 file is 9.5 Megabytes. In addition, the beam model rms bandwidth is only 14% of the plate model. Because the model is smaller, less file space is required, and the analysis runs much faster, beam models are an attractive alternative to plate models. Therefore, an effort was begun to determine how to implement "beam" models which will give nearly the same answers as the base line CQUAD4 models.

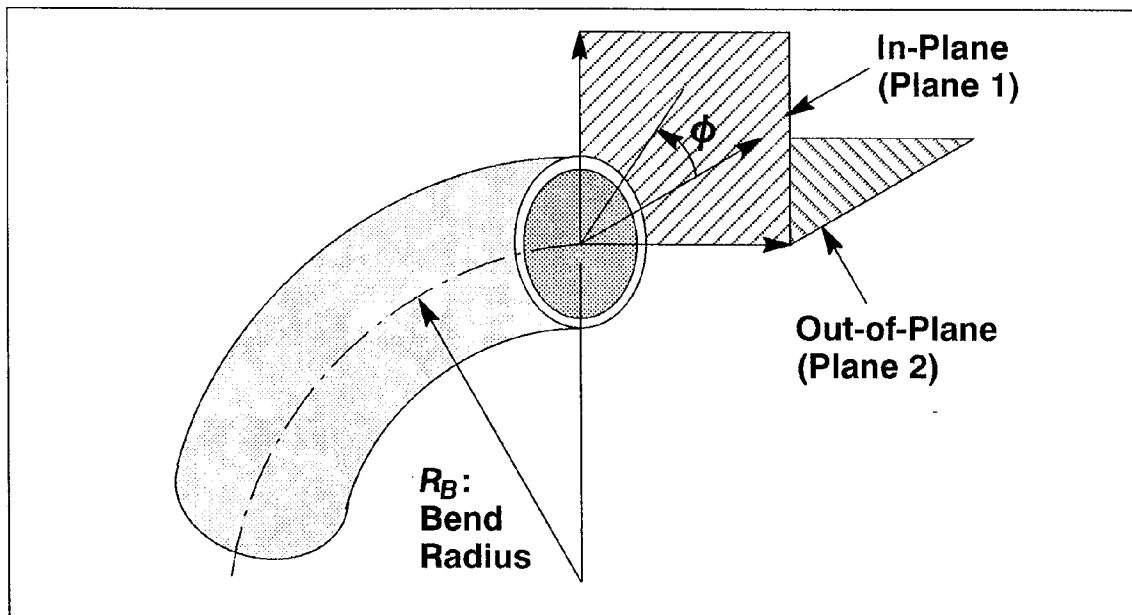


Figure 1: Tube Bend Description

Problem Definition

Straight tube sections behave according to simple beam theory. However, in the bends, radial stress parallel to the radius of curvature develop under bending which leads to ovalization (Ref. 4). Figure 1 shows the in-plane and out-of-plane directions. In-plane bending is bending in the plane of curvature which increases or decreases the bend radius. Figure 2A shows the ovalization due to an in-plane bending moment which is decreasing the bend radius of curvature. Out-of-plane bending is bending normal to the plane of curvature. Figure 2B shows that an out-of-plane bending moment causes the major axis of the resulting ellipse to be inclined at approximately 45 degrees. This ovalization within the bend creates a transverse stress not present in a straight tube. Figure 3 illustrates this for an in-plane bending moment. Transverse stress is similarly caused by the out-of-plane moment. Therefore, a 2-dimensional stress field is present within the bend. Since the transverse stress is in the hoop direction, hoop will be used to designate this stress direction.

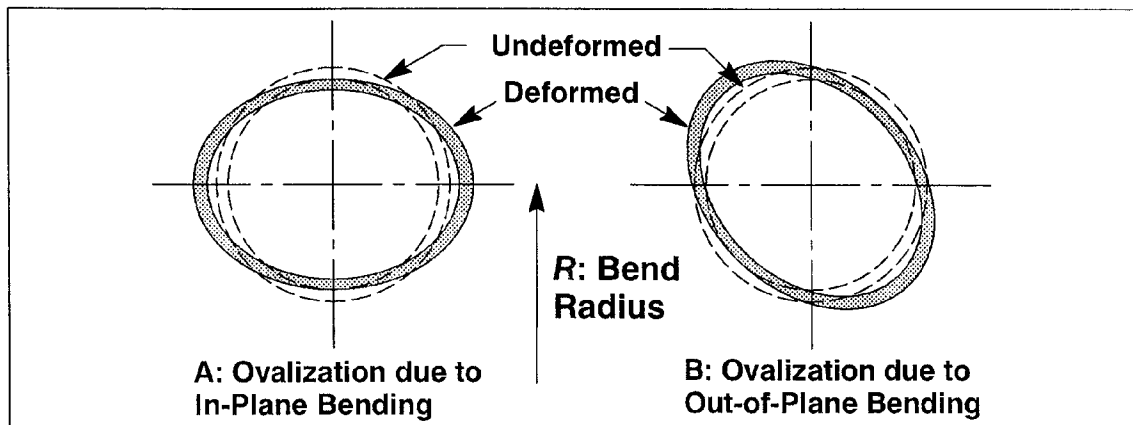


Figure 2: Ovalization of Tube Bends Due to Bending (from Ref. 6)

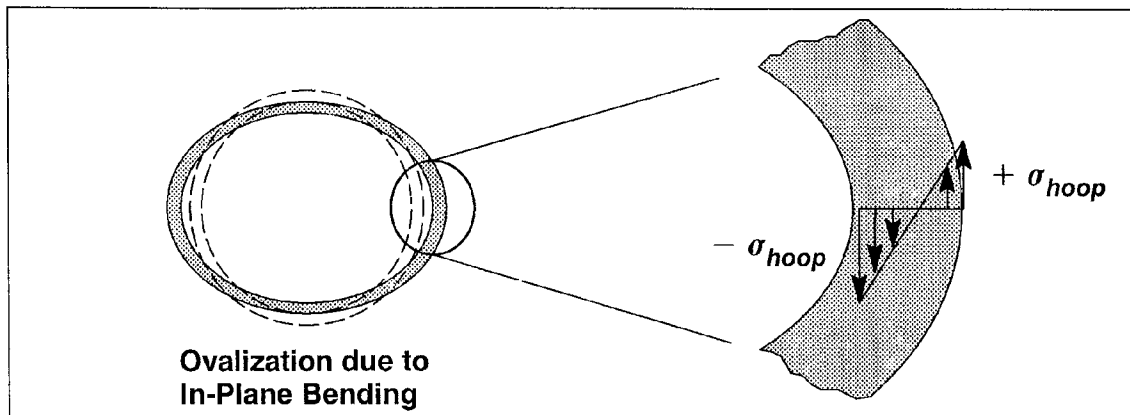


Figure 3: Example of Hoop Stress Distribution (In-Plane Bending) (from Ref. 6)

MSC/NASTRAN has many “beam” type elements. However, the bend element, CBEND, models tube ovalization. Since a straight tube on a turbo-fan is a rare exception, modeling the tube ovalization was of prime concern, and was the reason plate models were originally used. Therefore, a detailed investigation of the CBEND element was made. References (2) and (5) contain a detailed description of the CBEND element.

Accurate natural frequencies are obtained directly when using the CBEND element. However, MSC/NASTRAN does not compute many of the stress components. None of the hoop stress which develops in the bends is calculated. Also, any stress due to pressure or torque is not calculated. Finally, the combined effect of in-plane and out-of-plane bending is not considered. Therefore, a special post-processor was created to determine accurate stress from a CBEND model.

Analysis

Nomenclature

- D_o = tube outer diameter
- D_i = tube inner diameter
- t = tube wall thickness
- r = mean cross sectional radius = $(D_o - t) / 2$
- I = cross-sectional moment of inertia = $\pi (D_o^4 - D_i^4) / 64$
- J = cross-sectional polar moment of inertia = $\pi (D_o^4 - D_i^4) / 32$
- R_B = bend radius
- A = additional thickness from Section NB-3641.1 of reference (1).
- L = tube length
- P = internal fluid gage pressure
- F = MSC/NASTRAN average tensile force along the beam center-line
- M_1 = MSC/NASTRAN in-plane bending moment
- M_2 = MSC/NASTRAN out-of-plane bending moment
- T = MSC/NASTRAN torque moment about the geometric center
- E = Young's modulus (modulus of elasticity)
- ν = Poisson's Ratio

Tube Flexibility

The ovalization of the circular cross section within a bend reduces the strength and stiffness. Thus, the flexibility of the CBEND element is modified by k_p as found in references (1) and (2):

$$\begin{array}{ll} \text{Curved Tube} & k_p = \frac{1.65 r^2}{R_B t} \left[\frac{1}{1 + 6 \frac{Pr}{Et} \left(\frac{r}{t} \right)^{4/3} \left(\frac{R_B}{r} \right)^{1/3}} \right] \\ \text{Straight Tube} & k_p = 1.0 \end{array} \quad (1)$$

Axial Stress

The axial stress is caused by the tensile load, the blow-off load, and the in-plane and the out-of-plane bending moments:

$$\sigma_{axial} = \sigma_{tensile} + \sigma_{blow-off} + \sigma_{in-plane} + \sigma_{out-of-plane} \quad (2)$$

The tensile stress is determined by dividing the tensile force by the tube cross sectional area:

$$\sigma_{tensile} = \frac{F}{\frac{\pi}{4} (D_o^2 - D_i^2)} \quad (3)$$

The internal pressure creates a blow-off load except in cases where the tube is straight. For small diameter tubes with little or no pressure, the blow-off load is insignificant. The blow-off stress is calculated as follows:

$$\sigma_{blow-off} = \frac{P D_i^2}{(D_o^2 - D_i^2)} \approx \frac{P D_i}{4 (t - A)} \quad (4)$$

The ASME Code (Ref. 1) uses the approximate formula of equation (4) where the wall thickness, t , is reduced by an additional thickness, A . This additional thickness is not used in the beam post-processor and the exact formula in equation (4) is used.

As previously noted, both the in-plane and out-of-plane bending moments cause the tube to ovalize (Ref. 6) creating transverse or hoop strain. Thus, both axial and hoop strain contribute to the axial stress caused by these moments as follows:

$$\begin{aligned} \text{Outside surface} \quad \sigma_{in-plane \text{ or } out-of-plane} &= -\frac{M D_o}{2 I} [\sigma_{tm} + \nu \sigma_{nb}] \\ \text{Inside Surface} \quad \sigma_{in-plane \text{ or } out-of-plane} &= -\frac{M D_i}{2 I} [\sigma_{tm} - \nu \sigma_{nb}] \end{aligned} \quad (5)$$

where

$$\begin{aligned} M &= M_1 \text{ (in-plane moment) or } M_2 \text{ (out-of-plane moment)} \\ \sigma_{tm} &= \sin\phi + [(1.5X_2 - 18.75)\sin 3\phi + 11.25 \sin 5\phi] / X_4 \quad \text{(in-plane moment)} \\ \sigma_{nb} &= \lambda (9X_2 \cos 2\phi + 225 \cos 4\phi) / X_4 \quad \text{(in-plane moment)} \\ \sigma_{tm} &= \cos\phi + [(1.5X_2 - 18.75)\cos 3\phi + 11.25 \cos 5\phi] / X_4 \quad \text{(out-of-plane moment)} \\ \sigma_{nb} &= \lambda (9X_2 \sin 2\phi + 225 \sin 4\phi) / X_4 \quad \text{(out-of-plane moment)} \\ X_1 &= 5 + 6\lambda^2 + 24\psi \\ X_2 &= 17 + 600\lambda^2 + 480\psi \\ X_3 &= X_1 X_2 - 6.25 \\ X_4 &= (1 - \nu^2)(X_3 - 4.5X_2) \\ \phi &= \text{circumferential angle (See Figure 1)} \\ \lambda &= t R_B / (r^2 \sqrt{1 - \nu^2}) \quad \text{(Equations are valid for } \lambda \geq 0.2 \text{ only.)} \\ \psi &= P R_B^2 / E r t \end{aligned}$$

Equation (5) is from Ref. (1), Table NB-3685.1-2. The moment sign convention used by MSC/NASTRAN is opposite the convention of Ref. (1) thus necessitating the minus signs in equation (5).

Hoop Stress

The hoop stress is caused by the internal fluid pressure and the in-plane and out-of-plane bending moments:

$$\sigma_{hoop} = \sigma_{pressure} + \sigma_{in-plane} + \sigma_{out-of-plane} \quad (6)$$

The hoop stress caused by the internal fluid pressure is not uniform around the circumference in the bend. The following formula from Ref. 1, Table NB-3685.1-1, accounts for this:

$$\sigma_{pressure} = P \left[\frac{D_o - 0.8(t - A)}{2(t - A)} \right] \left[\frac{0.5(2R_B + r \sin \phi)}{R_B + r \sin \phi} \right] \quad (7)$$

Note that the wall thickness, t , is reduced by a calculated additional thickness, A , from Ref. (1), Section NB-3641.1. (This additional thickness is currently not used by the CBEND post-processor.) The first bracket term is approximately r/t . Therefore, equation (7) is the well known Pr/t hoop stress modified by the second bracket term which is dependent on the bend radius and angle ϕ . As the bend radius, R_B , gets larger (the tube is becoming straight), the second bracket term in equation (7) approaches 1 and the normal hoop stress distribution in a tube is present.

Similar to the axial stress, the hoop stress is also a combination of both the axial and hoop strain caused by the in-plane and out-of-plane bending moments as follows:

$$\begin{aligned} \text{Outside surface} \quad \sigma_{in-plane \text{ or } out-of-plane} &= - \frac{M D_o}{2 I} [\nu \sigma_{tm} + \sigma_{nb}] \\ \text{Inside Surface} \quad \sigma_{in-plane \text{ or } out-of-plane} &= - \frac{M D_i}{2 I} [\nu \sigma_{tm} - \sigma_{nb}] \end{aligned} \quad (8)$$

where the variables are the same as equation (5). Note that the only difference between equations (8) and (5) is that Poisson's ratio modifies the other stress index. As before, equation (8) is from Ref. (1), Table NB-3685.1-2. The moment sign convention used by MSC/NASTRAN is opposite the convention of Ref. (1) thus necessitating the minus signs in equation (8).

Radial Stress

The ASME Code, Ref. (1), assumes that the radial stress is zero. In contrast, the beam post processor defines the radial stress as follows:

$$\begin{aligned} \text{Outside surface} \quad \sigma_{radial} &= 0.0 \\ \text{Inside Surface} \quad \sigma_{radial} &= -P \end{aligned} \quad (9)$$

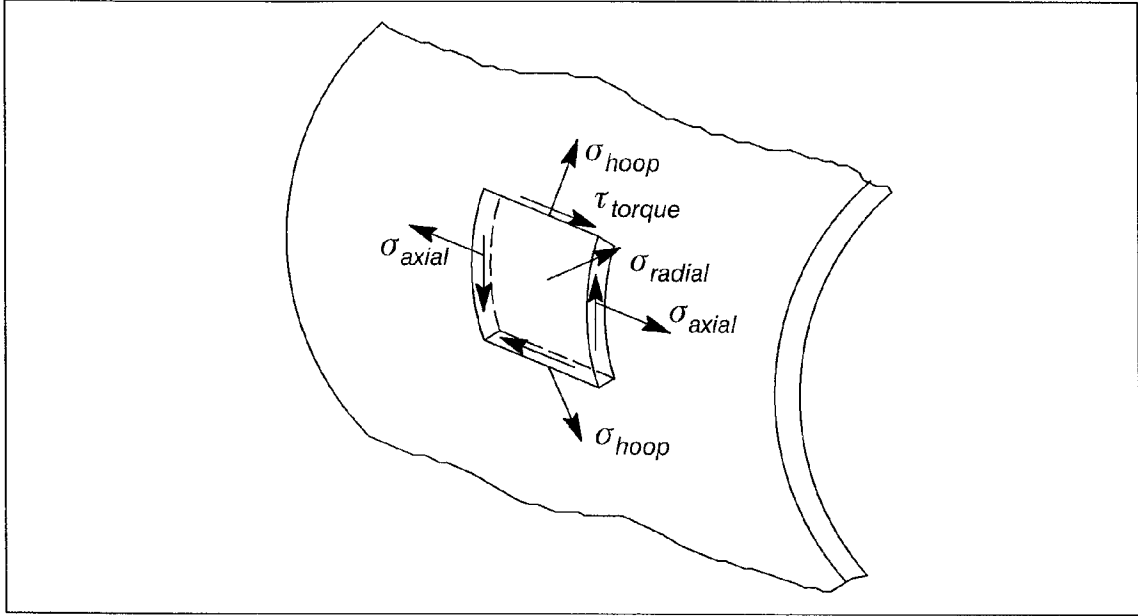


Figure 4: Combined Stress Field in the Tube

Torque Stress

The torque moment causes a shear stress in the axial-hoop stress plane as follows:

$$\begin{array}{ll}
 \text{Outside surface} & \tau_{torque} = \frac{T D_o}{2 J} \\
 \text{Inside Surface} & \tau_{torque} = \frac{T D_i}{2 J}
 \end{array} \quad (10)$$

Shear Stress

As usual in the case of long, slender beams which these tubes are, it is assumed that the bending stress is much larger than the shear stress caused by the shear force. The hoop stress is of the same magnitude as the bending stress. Therefore, the shear stress is ignored by the CBEND post-processor.

Combined Stress

Figure 4 shows the stress on an element of the tube. The axial, hoop, radial, and torque (shear) stresses are calculated from (2), (6), (9), and (10) respectively. The principal stresses are calculated as follows:

$$\sigma_1 = \left[\frac{\sigma_{axial} + \sigma_{hoop}}{2} \right] + \sqrt{\left(\frac{\sigma_{axial} - \sigma_{hoop}}{2} \right)^2 + \tau_{torque}^2} \quad (11)$$

$$\sigma_2 = \sigma_{radial} \quad (12)$$

$$\sigma_3 = \left[\frac{\sigma_{axial} + \sigma_{hoop}}{2} \right] + \sqrt{\left(\frac{\sigma_{axial} - \sigma_{hoop}}{2} \right)^2 + \tau_{torque}^2} \quad (13)$$

In addition, the Von Mises equivalent stress is determined from the principal stresses:

$$\sigma_{vm} = \sqrt{\frac{1}{2} (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad (14)$$

CBEND Post-Processor

The CBEND post-processor reads the CBEND element forces and moments from the OUTPUT2 file. The required geometry information such as the bend radius, dimensions, and pressure is read from the bulk data file. For tube systems, the title of each system tube (written as a comment) and its beginning and end elements are also read from the bulk data file. While the element geometry information can be written to the OUTPUT2 file, comment cards cannot. It would be helpful if either comment cards or the bulk data deck could be written to the OUTPUT2 file. Then the bulk data file would not be required.

At each grid, the forces and moments are averaged from the 2 connecting beam elements. It was observed that the torque could vary from end-to-end of each element especially if the element was in a bend. The results were torque plots with oscillations and spikes. However, using the element's average torque removed the spikes and dampened the oscillations. Therefore, the torque is averaged within the element and then averaged with the connecting element's average torque at each grid. Averaging the forces and moments before the stress calculation avoids having to determine two stress values at each grid and then averaging the stress. The axial, hoop, torque, minimum principal, maximum principle, and Von Mises stresses are determined at small angular increments of $\Delta\phi$ for both the inside and outside surface at each grid. Then, the maximum value of each stress type is retained for plotting. Plot files are created for both an in-house plot program (used in this paper) and PATRAN (Ref. 3).

Discussion

Example 1: 10 Inch Straight Tube, Ends Fixed

Table 1 shows the results from analyzing a 10 inch long straight tube in which the ends were fixed. To approximate a straight tube, a bend radius of 100,000 inches was given for the CBEND elements. The theoretical results for a beam were calculated from Roark (Ref. 4). The CBEND results more closely match the base line CQUAD4 results than the CBAR results. The CBAR results, though, almost exactly matches simple beam theory.

D_o	L/D_o	t	r/t	Mode	CQUAD4	% differ. CBEND	% differ. CBAR	% differ. beams
.375	26.67	.01	18.75	1	870.1	.77	2.55	2.85
				2	2348	.21	4.68	4.98
.375	26.67	.028	6.7	1	830.1	.92	2.66	2.78
				2	2244	.40	4.68	4.72
.75	13.33	.028	13.39	1	1649	-1.09	7.22	7.40
				2	4225	-2.53	15.34	15.46

Table 1: Frequency Comparison of CBEND, CBAR, and CQUAD4 Elements

Example 2: 20 Inch Tube with a 90 Degree Bend, Ends Fixed

Table 2 shows the results from analyzing a 20 inch tube with a 90 degree bend in the middle as shown in Figure 5. Again, the CBEND matched the base line CQUAD4 results more closely than the CBAR.

D_o	L/D_o	t	t/t	Mode	CQUAD4 Element	% differ. CBEND	% differ. CBAR
.75	24.95	.028	13.39	1	360.4	1.14	6.60
				2	1081	-2.59	13.04
				3	1166	-1.72	12.26
				4	1883	-0.48	16.04

Table 2: Frequency Comparison of CBEND, CBAR, and CQUAD4 Elements

The increased flexibility in the bend reduces the stress level under applied displacements and thermal loads. For example, Table 3 shows how the increased flexibility of the CBEND element lowers the reaction forces at end A in Figure 5. The increased flexibility is the main reason for choosing the CBEND element instead of the CBAR element.

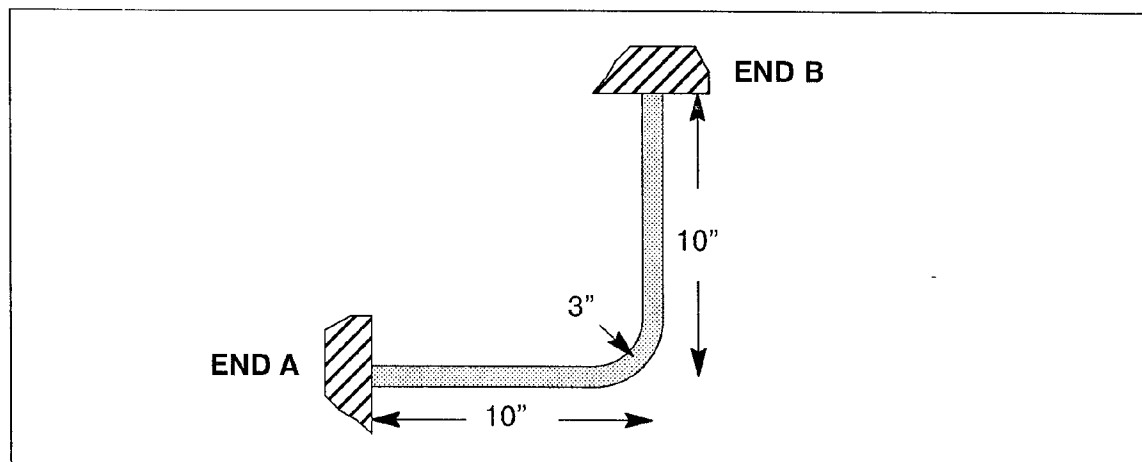


Figure 5: Beam / Plate Comparison Model

Reaction Force/Moment	CBAR Element	CBEND Element w/o Flexibility
Fx	1.13	1.10
Fy	1.43	1.39
Fz	1.51	1.45
Mx	1.21	1.17
My	1.33	1.22
Mz	1.05	1.04

Table 3: Reaction Forces of CBAR Element and CBEND Element Without Flexibility Normalized to CBEND Element with Flexibility

Example 3: 20 Inch Tube with a 90 Degree Bend, Ends Fixed

The tube model shown in Figure 5 was loaded with pressure, thermal, and displacement loads. Both a plate model with CQUAD4 elements and a beam model with CBEND elements were run. A comparison of the axial stress in the middle of the bend is shown in Figure 6. The CBEND post-processor was used to calculate the axial stress based on the MSC/NASTRAN calculated forces and moments. Grid point stresses were obtained from the plate model. In general, the 2 analyses agree. Also included in Figure 6 are the 4 axial stresses printed by MSC/NASTRAN from the CBEND analysis. It is obvious that by ignoring the blow-off load and the combined in-plane and out-of-plane moment effect, the MSC/NASTRAN stress values are not indicative of the actual stress. In this example, the maximum MSC/NASTRAN printed stress is approximately 4000 psi, while the CBEND post-processor calculates a maximum axial stress of approximately 8700 psi. Thus, relying on the MSC/NASTRAN printed results would result in a 55% error.

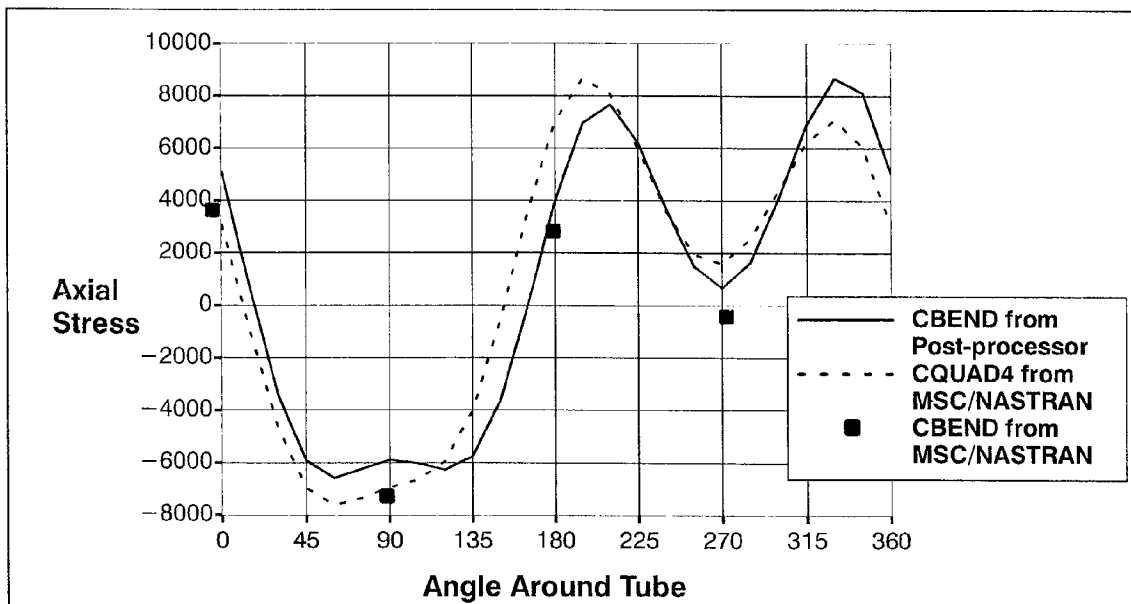


Figure 6: Axial Stress vs. Angular Position

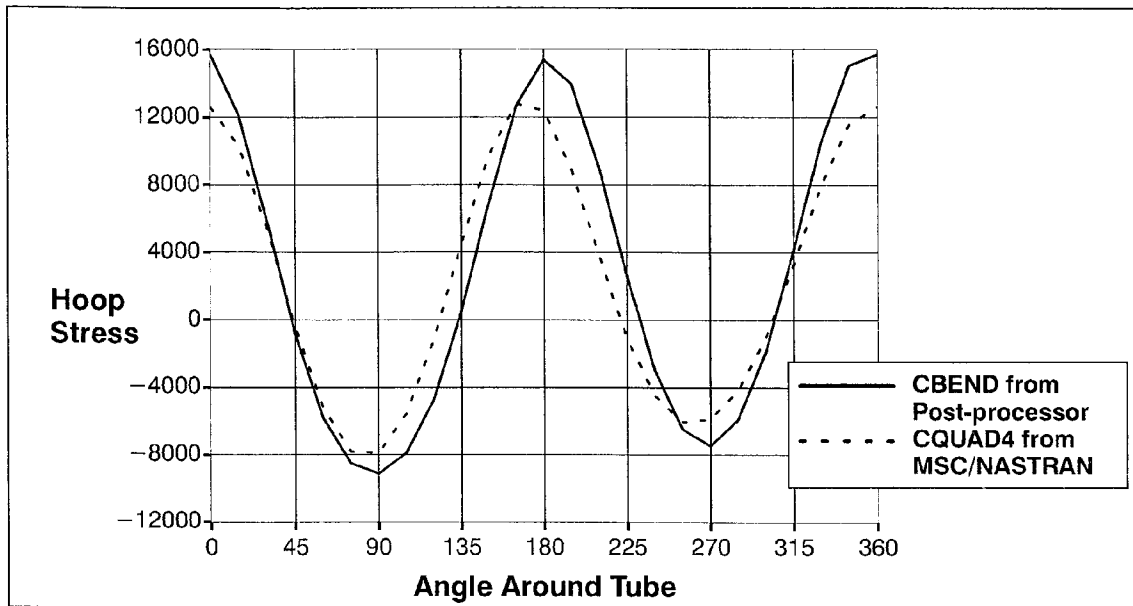


Figure 7: Hoop Stress vs. Angular Position

Figure 7 shows the hoop stress in the same bend location. Again, the agreement between the beam post-processor calculated stress and the plate stress is good. MSC/NASTRAN does not calculate or print the CBEND hoop stress.

The CBEND post-processor determines the maximum stress at each grid. Figure 8 compares the CBEND post-processor results to CQUAD4 results at each grid along the length of the tube. It is interesting to note that within the bend (the center hump in Figure 8) the maximum Von Mises stress occurs on the inner surface of the tube.

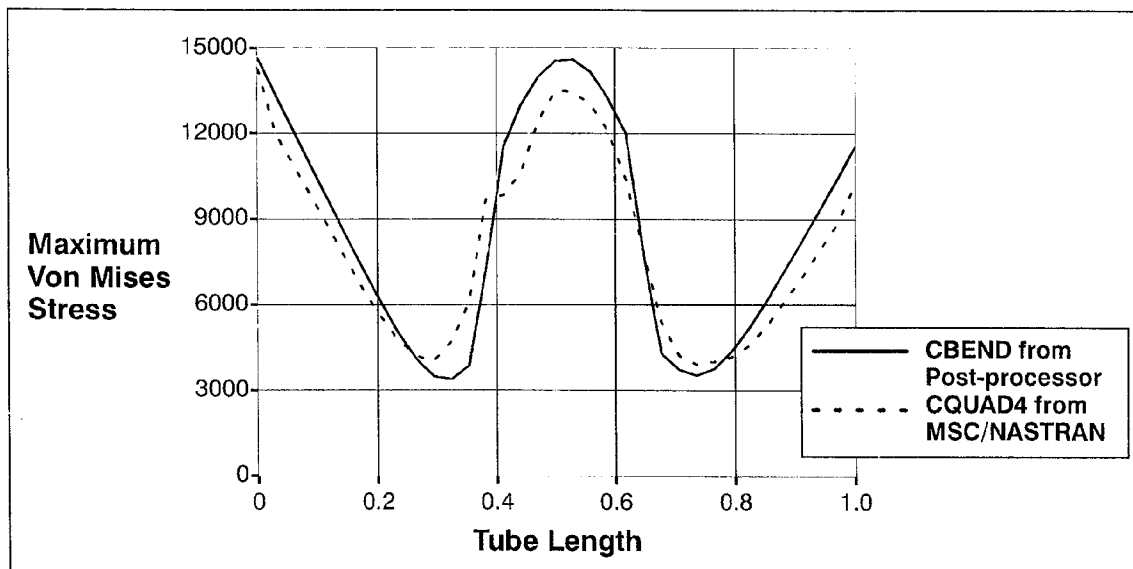


Figure 8: Maximum Von Mises Stress vs. Tube Length

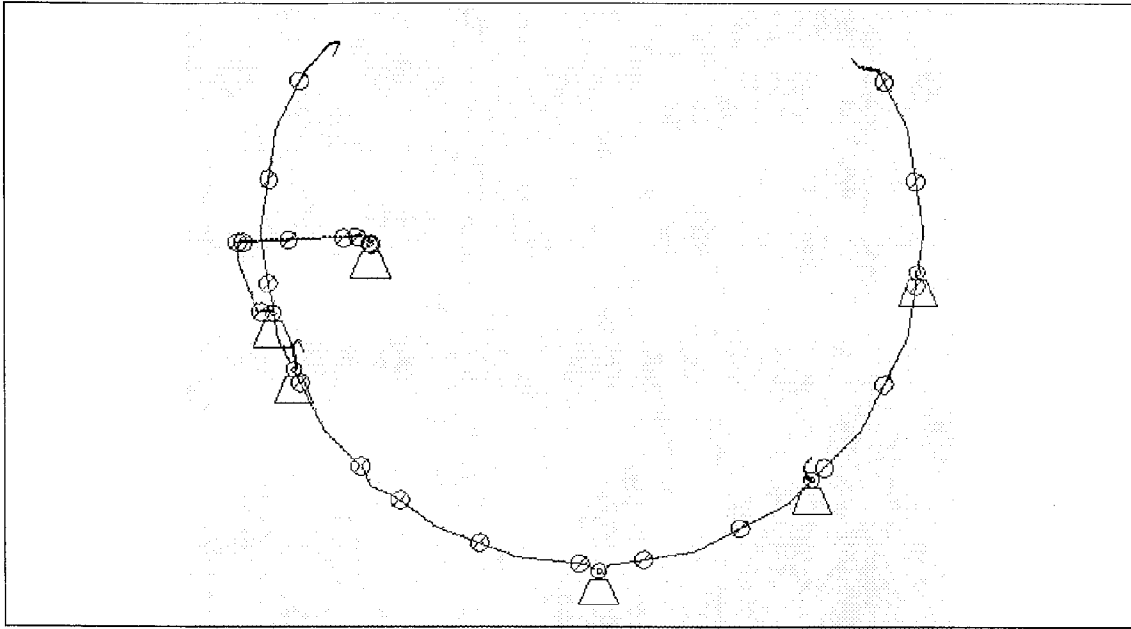


Figure 9: Tube System of Example 4

Example 4: Tube System

Generally a tube system rather than an individual tube is analyzed. A large tube system of 9 tubes is shown in Figure 9. This is a view from the front of the engine along its axis. The tube system wraps circumferentially around the case. The small circles represent brackets, while the small weights represent concentrated masses like fittings and valves.

The Von Mises stress versus the grid points for all 9 tubes in the system is plotted in Figure 10. The labeled points are compared to an equivalent CQUAD4 plate model in Table 4. These points were selected because they are local maximum values in bends.

Location	Ratio	Location	Ratio	Location	Ratio	Location	Ratio
a	1.23	f	1.25	k	1.15	p	1.32
b	1.15	g	1.23	l	1.16	q	1.10
c	1.17	h	1.34	m	1.33	r	1.31
d	1.12	i	1.18	n	1.19	s	1.10
e	1.16	j	1.20	o	1.27	t	1.12
Comparison Summary							
High	1.34	Low	1.10	Average	1.21	Std. Dev.	.08

Table 4: Beam / Plate Stress Ratio for Tube System

A summary plot like Figure 10 can quickly show the analyst which tube or tubes contain the high stress. In this example, tube 5 has the highest stress. To further investigate, Figure 11 shows the Von Mises stress for tube 5 with the corresponding results for the plate analysis at the bend locations.

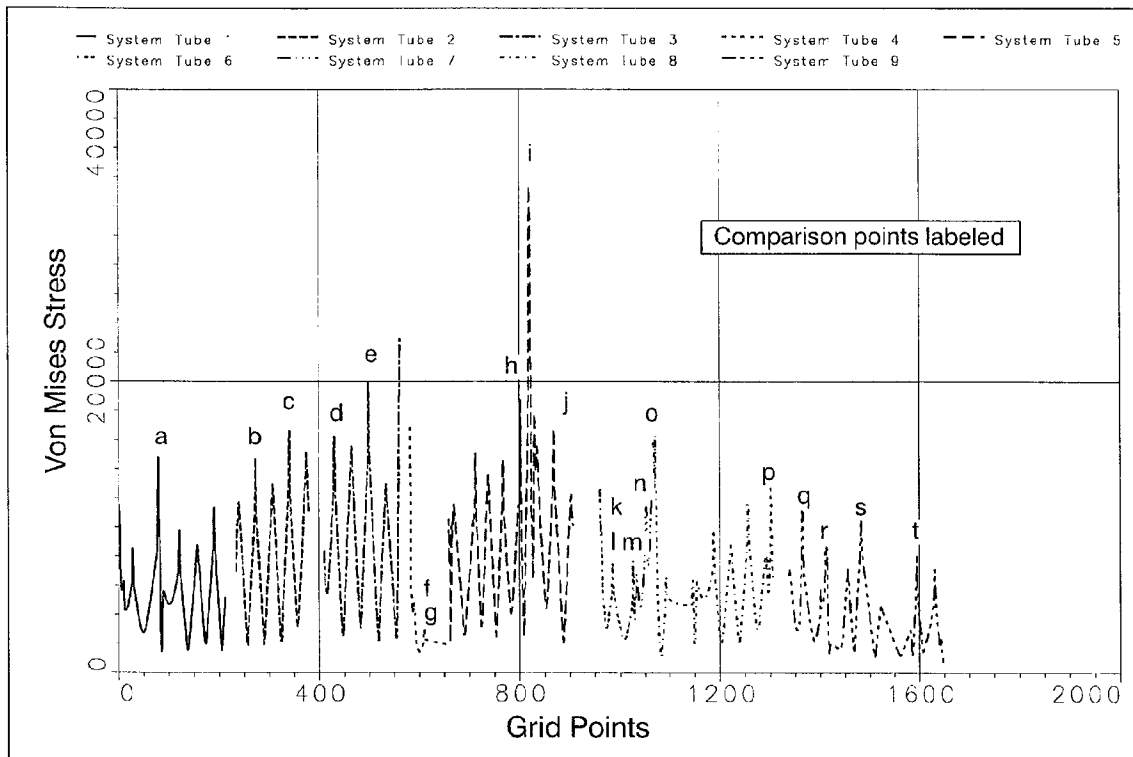


Figure 10: Von Mises Stress for Tube System

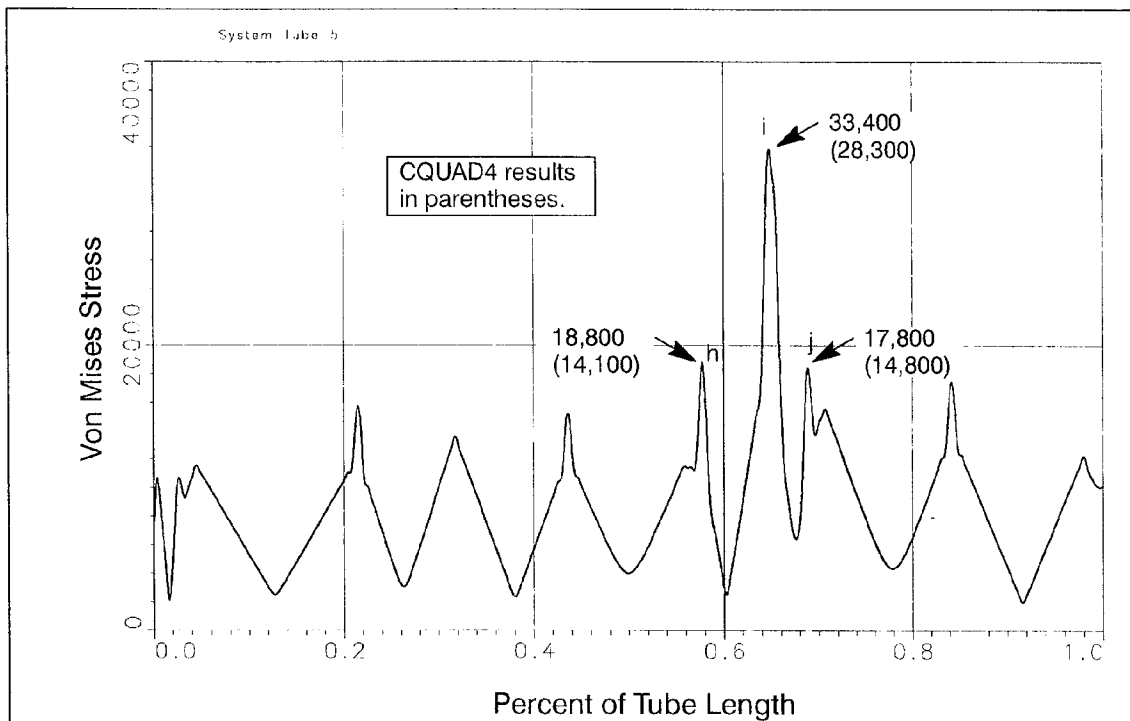


Figure 11: Von Mises Stress for System Tube 5

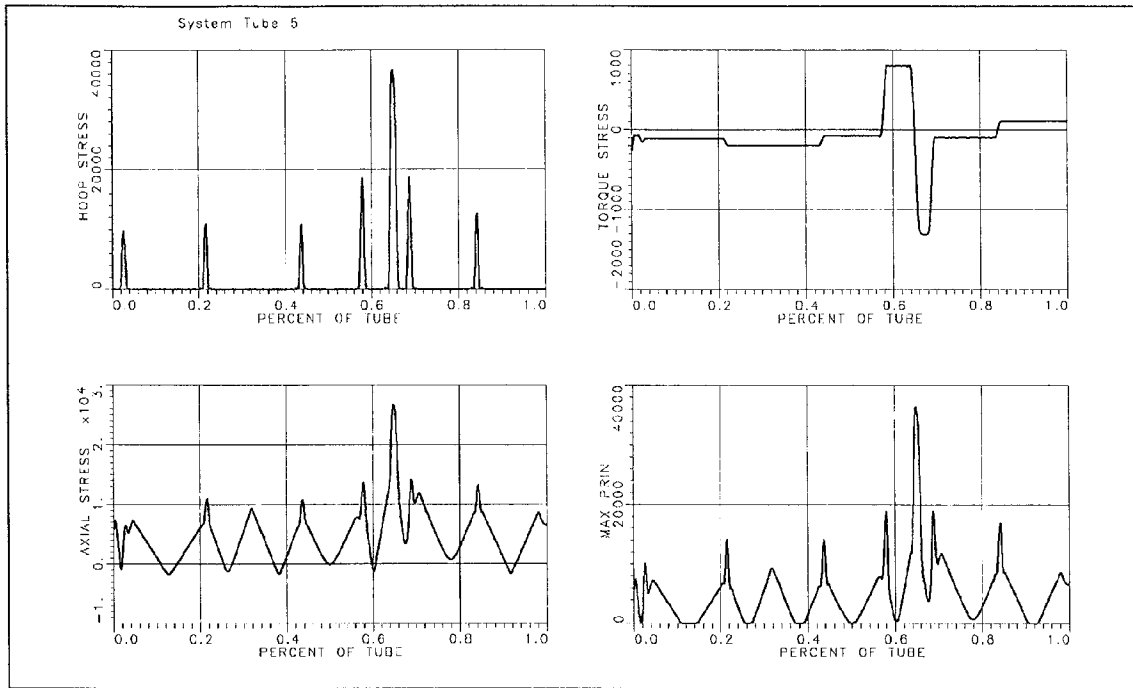


Figure 12: System Tube 5 Summary Stress Plots

Finally, Figure 12 plots the maximum axial, hoop, torque, and principal stresses for system tube 5. Note how the hoop stress is essentially zero between the bends while exceeding the axial stress in the bends. The maximum principal stress plot usually has the same shape as the maximum axial stress with an added amount at the bends due to the hoop stress.

Conclusions

Based upon the results presented in this paper, the use of CBEND elements to model round tubes under .75 inch diameter was implemented. The frequency comparison shows that a CBEND model closely matches the equivalent base line CQUAD4 plate model. The percent difference is less than the difference between experimental results (not presented) and analytical results using CQUAD4 elements. The calculated stress from the CBEND post-processor averages approximately 21% higher. This was deemed acceptable, especially since CBEND elements provide a conservative margin of error.

This paper has reviewed the theory used in MSC/NASTRAN CBEND elements and reviews the shortcomings of the stress output. Other users who may want to use the CBEND element can use the results presented here to guide them in their post analysis. MSC/NASTRAN could improve the utility of this element by: 1) Making the element default to a straight tube when the bend radius, R_B , is not defined or set to zero. 2) Providing more

complete stress output as defined in this paper. 3) Providing a way of writing the input and/or comments to the OUTPUT2 file.

Since MSC/NASTRAN has provided a tube bend element with modified flexibility, Pratt & Whitney will make use of it provide more rapid analysis of small diameter tubes. The use of these efficient “beam” models opens the door to optimization and forced response analyses.

Acknowledgements

Cullen Crain and Jean Nwachuku wrote the CBEND post-processor program. Scott Hadley provided the model for example 4.

References

1. *ASME Code*, Section III, NB-3683 – NB-3686, 1986.
2. *MSC/Nastran User's Manual, Version 67*, The MacNeal–Schwendler Corporation, Los Angeles, CA, August, 1991.
3. *PATRAN Plus User Manual*, PDA Engineering, Costa Mesa, CA, Release 2.4, January, 1990.
4. Roark, R. J. and Young, W. C., *Formulas for Stress and Strain, 5th Ed.*, McGraw-Hill, New York, 1965.
5. Schaeffer, H. G., *MSC/NASTRAN Static and Normal Modes Analysis*, Schaeffer Analysis, Inc., Mont Vernon, NH, 1979.
6. Utecht E. A., “Stress in Curved, Circular Thin-Wall Tubes,” *Trans. ASME, Journal of Applied Mechanics*, March, 1963, pp. 134–135.