

CROSS-ORTHOGONALITY CALCULATIONS FOR PRE-TEST PLANNING AND MODEL VERIFICATION

Ken Blakely and Ted Rose

The MacNeal-Schwendler Corporation
Los Angeles, CA

Abstract

Cross-orthogonality calculations can be used to compare the orthogonality of two different sets of mode shapes. Two types of calculations are described in this paper: MAC (Modal Assurance Criterion) and mass orthogonality. These calculations can be used for pre-test planning to guide proper selection of measurement instrumentation for a modal test, and can also be used to show the degree of correlation between modal test and MSC/NASTRAN mode shapes. This paper describes the implementation in Version 67.5 (via DMAP) and provides illustrative examples.

Introduction

Cross-orthogonality checks are useful in test-analysis correlation for modal testing. For pre-test planning, they are used to determine the best measurement locations by assessing the adequacy of a particular choice of A-set degrees of freedom (DOF) to accurately represent the system modes. This A-set represents the proposed set of measurement locations. Similarly, cross-orthogonality checks are used in post-model verification to assess the degree of correlation between test and MSC/NASTRAN mode shapes.

This paper describes the method and implementation in Version 67.5 and provides examples.

Pre-Test Planning Calculations

Proposed measurement locations for a modal test can be selected based on cross-orthogonality calculations. To do this, two MSC/NASTRAN runs are made with the same basic model, as follows:

1. Full (unreduced) model. This run computes modes of the full model and punches the G-set sized mode shapes (ϕ_g) in DMIG format for use in the second run.
2. Reduced model, with the selected A-set being the proposed measurement DOF. This run computes the A-set modes (ϕ_a) and mass matrix (M_{aa}), reads the punched modes (ϕ_g) from the first run, and makes the cross-orthogonality checks described below.

Two cross-orthogonality checks are made, comparing modes from the two runs. These checks are made to ensure the adequacy of the proposed A-set to accurately represent the modes computed with the full model. In order to perform the cross-orthogonality calculations, the G-set modes, ϕ_g , are partitioned into the A-set locations; call this ϕ_t . Figure 1 illustrates the relationship between ϕ_g , ϕ_a , and ϕ_t .

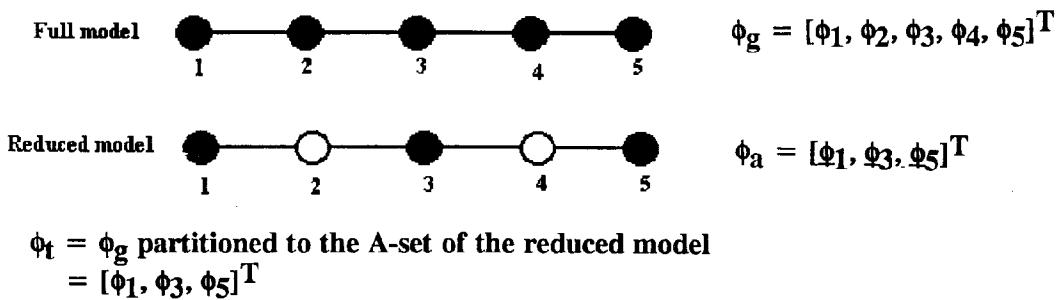


Figure 1: Relationship Between ϕ_g , ϕ_a , and ϕ_t

One type of cross-orthogonality check is the Modal Assurance Criterion, or MAC [Ref. 1]. MAC gives a measure of the degree of correlation between test and MSC/NASTRAN mode shapes, and is given by:

$$MAC_{ij} = ((\phi_t^T)_i (\phi_a)_j)^2 / (\phi_t^T \phi_t)_i (\phi_a^T \phi_a)_j \quad [1]$$

MAC values range from 0 (no correlation between shapes) to 1 (full correlation). In practice, neither extreme is reached, and values greater than 0.9 or so indicate good correlation while values less than 0.1 or so indicate no correlation.

Another type of cross-orthogonality check is mass orthogonality, given by

$$ORTHOA_{ij} = (\phi_t^T)_i M_{aa} (\phi_a)_j \quad [2]$$

Ideally, ORTHOA should be a diagonal matrix, since the modes should be orthogonal with respect to the mass matrix. As with MAC, good values are greater than 0.9 or so and acceptable "zeroes" are less than 0.1 or so.

In computing MAC and ORTHOA for pre-test planning, the "test" modes, ϕ_t , represent analytical modes that have been partitioned to the proposed set of test DOF.

It must be kept in mind that these techniques may not be completely accurate, since measurement locations are determined with a pre-test model that may not match reality. Nevertheless, they provide a rational basis for selecting measurement locations based on predicted behavior.

Example: Pre-Test Planning

Consider the two-dimensional cantilever beam shown in Figure 2, which is modeled with 11 grid points and 10 elements. Suppose that we want to set up a test to measure all modes (axial and bending) up to 2500 Hz. Suppose, too, that we have five biaxial accelerometer blocks and that we want to determine where to place these in order to accurately measure the mode shapes. The two accelerometers on each accelerometer block are aligned in the X and Y directions.

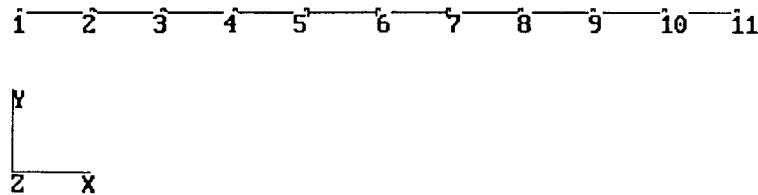
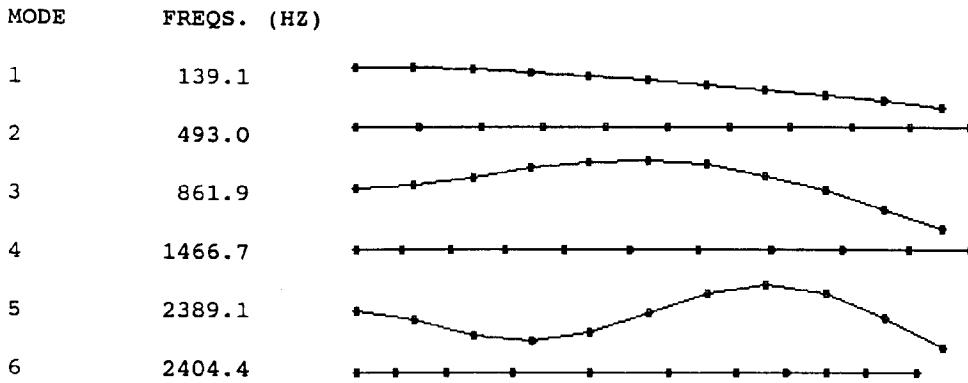


Figure 2: Two-Dimensional Cantilever Beam Model Geometry

Full Model. Results from the full model are shown below. There are three bending modes (modes 1, 3, and 5) and three axial modes (modes 2, 4, and 6) below 2500 Hz.

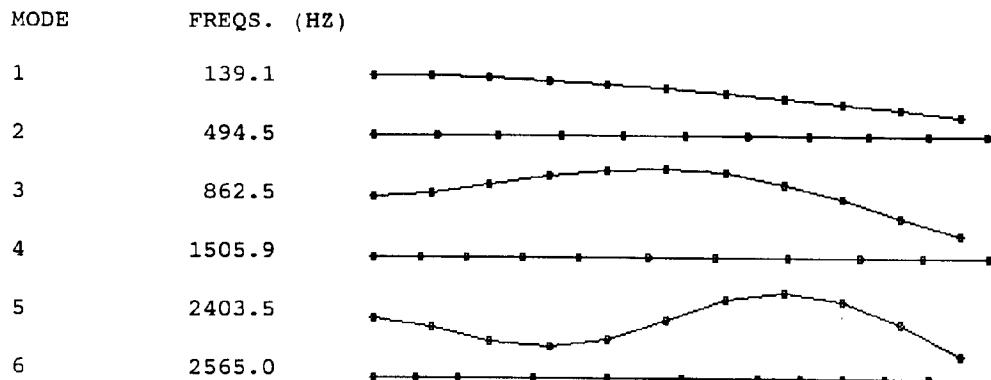


Reduced Model. Two proposed sets of accelerometer block locations were made:

- Set 1: Grid points 3, 5, 7, 9, 11
- Set 2: Grid points 7, 8, 9, 10, 11

Modes were computed for each set, and cross-orthogonality checks were made with respect to the full model. MAC and ORTHOA values less than 10^{-3} are shown as 0.0. Each column is an A-set analytical mode; each row represents a mode from the full model.

REDUCED MODEL 1 (POINTS 3,5,7,9,11)



MATRIX MAC

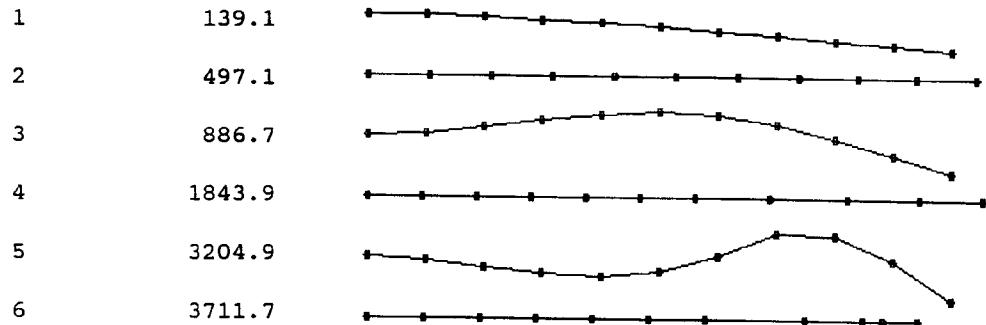
1.00	0.0	9.6E-2	0.0	1.0E-1	0.0
0.0	1.00	0.0	2.8E-2	0.0	2.8E-2
9.6E-2	0.0	1.0	0.0	1.1E-1	0.0
0.0	2.8E-2	0.0	1.0	0.0	2.8E-2
1.0E-1	0.0	1.1E-1	0.0	1.0	0.0
0.0	2.8E-2	0.0	2.8E-2	0.0	1.0

MATRIX ORTHOA

1.0	0.0	0.0	0.0	0.0	0.0
0.0	-1.0	0.0	0.0	0.0	0.0
0.0	0.0	1.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	0.0	0.0
1.4E-3	0.0	1.6E-3	0.0	1.0	0.0
0.0	0.0	0.0	0.0	0.0	1.0

REDUCED MODEL 2 (POINTS 7,8,9,10,11)

MODE FREQS. (HZ)



MATRIX MAC

1.00	0.0	1.8E-1	0.0	9.3E-3	0.0
0.0	1.00	0.0	1.9E-1	0.0	0.0
2.0E-1	0.0	1.0	0.0	5.0E-1	0.0
0.0	5.6E-1	0.0	8.5E-1	0.0	5.5E-2
6.8E-3	0.0	7.6E-1	0.0	8.9E-1	0.0
0.0	5.4E-3	0.0	8.4E-1	0.0	2.8E-1

MATRIX ORTHOA

-1.0	0.0	0.0	0.0	0.0	0.0
0.0	-1.0	0.0	1.6E-3	0.0	0.0
-2.1E-2	0.0	1.0	0.0	4.0E-3	0.0
0.0	4.8E-1	0.0	8.8E-1	0.0	-4.9E-2
-2.1E-1	0.0	-7.0E-1	0.0	6.8E-1	0.0
0.0	3.3E-1	0.0	-9.3E-1	0.0	-1.4E-1

It can be seen from the MAC and ORTHOA matrices that set 2 is a poor choice due to the large off-diagonal terms (in fact, mode 6 is apparently not represented at all, with a very low diagonal term of -0.14). Input and output files for set 1 are shown in the appendix.

Model Verification Calculations

Once the modal test data has been acquired and processed, it is compared to the MSC/NASTRAN baseline model results to assess the degree of correlation. If the match is sufficient, then the model can be used with confidence. If the results do not agree then it must be determined which is inaccurate: the model, the test, or both. If the model is inaccurate, then there are ways to refine it to match the test data [Ref. 2].

A rigorous evaluation method is to use cross-orthogonality in a manner similar to that described for choosing measurement locations (see the "Pre-Test Planning" section). The steps in this procedure are as follows:

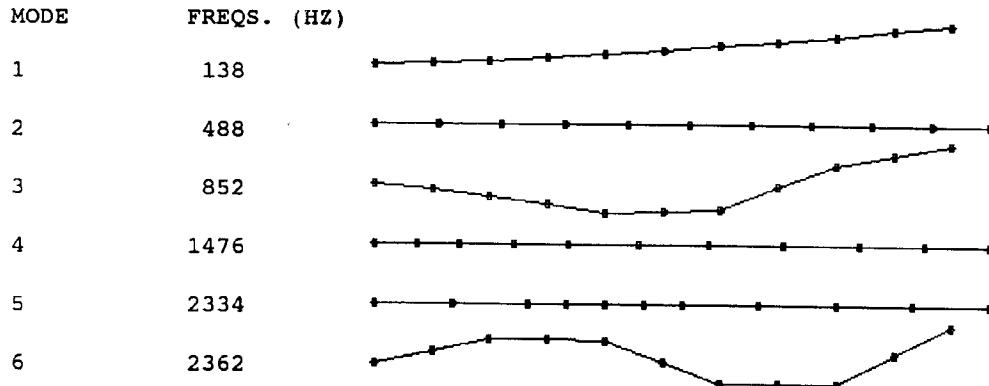
1. Obtain the measured modes; call these shapes ϕ_t (for test). Write these in DMIG format.
2. In the first MSC/NASTRAN run create an A-set that matches the test degrees of freedom. Compute the A-set modes, ϕ_a , and mass matrix, M_{aa} , and punch them in DMIG format for use in the second run. Note: Grid point numbers for the A-set must match grid point numbers for the DMIG test data. You may need to edit one, or both, of the DMIG files to ensure this.
3. In the second MSC/NASTRAN run read the test modes (ϕ_t), the A-set modes (ϕ_a) and mass matrix (M_{aa}), and compute MAC and ORTHOA. Grid point numbers corresponding to the test and A-set DOF must be defined in this run, too.

As with MAC and ORTHOA from the pre-test planning runs, values of the diagonal and off-diagonal terms are inspected. Values greater than 0.9 indicate good correlation while values less than 0.1 indicate poor correlation.

Note that the diagonal terms may be negative, which is simply a normalization difference between test and analysis; note, too, that modes could be switched (the third test mode is the fourth analytical mode), which will also be apparent from the terms in MAC and ORTHOA. Off-diagonal terms should be compared once these differences are taken into account.

Example: Model Verification

Consider the two-dimensional cantilever beam used in the previous example. Suppose that we ran a modal test using the accelerometer locations designated as set 1 (i.e., points 3, 5, 7, 9, and 11), and that we measured the lowest six modes (three bending and three axial) as given on the next page.



Mode shapes were computed only for grid points 3, 5, 7, 9, and 11 (the accelerometer locations), and mode shapes at the other grid points (2, 4, 6, 8, and 10) are plotted assuming a straight line between adjacent measurement locations. This is for plotting only; for calculations, only the measurement locations have been used.

Note that the third axial and bending test modes have "switched" with these same analytical modes. MAC and ORTHOA values less than 10^{-3} are listed as 0.0. Each column is an A-set analytical mode; each row represents a mode from the full model. Input and output files are shown in the appendix.

MATRIX MAC

1.00	0.0	9.2E-2	0.0	1.1E-1	0.0
0.0	1.00	0.0	2.8E-2	0.0	2.8E-2
2.1E-1	0.0	9.1E-1	0.0	4.5E-2	0.0
0.0	3.0E-2	0.0	1.0	0.0	2.8E-2
0.0	2.8E-2	0.0	2.8E-2	0.0	1.0
1.1E-1	0.0	9.6E-2	0.0	1.0	0.0

MATRIX ORTHOA

-1.00	0.0	5.6E-3	0.0	1.3E-2	0.0
0.0	1.00	0.0	0.0	0.0	0.0
-2.6E-1	0.0	-9.4E-1	0.0	-1.1E-1	0.0
0.0	9.4E-3	0.0	1.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	-1.0
-1.4E-2	0.0	2.3E-2	0.0	1.0	0.0

Note that analytical and test modes 5 and 6 (third bending and axial) have switched--test mode 5 is analytical mode 6, and vice-versa. This is denoted by the 1.0 terms in positions 5,6 and 6,5 (if they had not switched, these terms would have been in positions 5,5 and 6,6). Note, too, that there are -1.0 terms in ORTHOA, which occur because the test and analytical modes have a sign difference.

The largest off-diagonal term in ORTHOA is -0.26, with the second largest term -0.11. Therefore, there is relatively good agreement between the test and analytical modes, taking into account the fact that modes 5 and 6 are switched. If there had been numerous off-diagonal terms on the order of 0.25 or more, then the match between test and analysis would not have been good.

Implementation in Version 67.5

The DMAP alters for pre-test and post-test calculations are delivered with Version 67.5 as files premaca.v675 and postmaca.v675, respectively. (Due to their size, they are not reproduced here.) Sample files for the cantilever beam (A-set set 1) are also supplied. Portions of the input and output files are shown in this paper's appendix.

Guidelines

Proper location, alignment, and naming of test and analytical DOF is crucial; there must be a one-to-one match in order for the procedure to work as described. Editing and/or processing of test data or analytical results may be required to ensure this.

Summary

This writeup described implementation of cross-orthogonality calculations in MSC/NASTRAN for pre-test planning and model verification. The DMAPs and sample problems are supplied with Version 67.5.

References

1. Ewins, D., *Modal Testing: Theory and Practice*, Research Studies Press, Letchworth, Hertfordshire, England, 1984.
2. Blakely, K., "Updating MSC/NASTRAN Models to Match Test Data," Proceedings MSC World User's Conference, 1991.

APPENDIX: SELECTED INPUT AND OUTPUT FILES

The following Version 67.5 files are for pre-test calculations:

	Input File	Output File
Full (unreduced) model Alters to punch modes Print modes Punch modes	premac1.dat Include 'pchdispa.v675' premac1.f06 premac1.pch	
Reduced model (set 1) Alters to compute MAC, etc. Read previously-punched modes Print modes, MAC, ORTHOA Punch modes and mass (for use in post-test calculations--see below)	premac2.dat Include 'premaca.v675' Include 'premac1.pch' premac2.f06 premac2.pch	
Full model File premac1.dat		

```

GRID 3 20. 0.0 0.0
GRID 4 30. 0.0 0.0
GRID 5 40. 0.0 0.0
GRID 6 50. 0.0 0.0
GRID 7 60. 0.0 0.0
GRID 8 70. 0.0 0.0
GRID 9 80. 0.0 0.0
GRID 10 90. 0.0 0.0
GRID 11 100. 0.0 0.0
CBAR 1 1 1 2
CBAR 2 1 2 3
CBAR 3 1 3 4
CBAR 4 1 4 5
CBAR 5 1 5 6
CBAR 6 1 6 7
CBAR 7 1 7 8
CBAR 8 1 8 9
CBAR 9 1 9 10
CBAR 10 1 10 11
SPC 1 1 123456 0.0
PBAR 1 1 1.0 160.0
MAT1 1 3.+7 .3 160.0
EIGRL 100 0.0 10000. 6 7.7E-4
ENDDATA

```

Results from full model File premac1.f06

MODE NO.	EXTRACTION ORDER	EIGENVALUE	REAL EIGENVALUES			GENERALIZED MASS	GENERALIZED STIFFNESS
			RADIANS	CYCLES	GENERALIZED MASS		
1	1	7.636173E+05	8.738520E+02	1.390779E+02	1.000005E+00	7.636173E+05	7.636173E+05
2	2	9.593501E+06	3.097338E+03	4.929566E+02	1.000000E+00	9.593501E+06	9.593501E+06
3	3	2.932654E+07	5.4153598E+03	8.618873E+02	1.000000E+00	2.932654E+07	2.932654E+07
4	4	8.492998E+07	9.215746E+03	1.466731E+03	1.000000E+00	8.492998E+07	8.492998E+07
5	5	2.253432E+08	1.501144E+04	2.389144E+03	1.000000E+00	2.253432E+08	2.253432E+08
6	6	2.282285E+08	1.510723E+04	2.404391E+03	1.000000E+00	2.282285E+08	2.282285E+08
EIGENVALUE = 7.636173E+05 CYCLES = 1.390779E+02							
POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	0.0	0.0	0.0	0.0	0.0	0.0
2	G	6.648145E-17	-1.200922E-01	0.0	0.0	0.0	-2.344345E-02
3	G	1.444976E-16	-4.573940E-01	0.0	0.0	0.0	-4.344439E-02
4	G	1.857187E-16	-9.776228E-01	0.0	0.0	0.0	-6.003818E-02
5	G	2.283181E-16	-1.647093E-00	0.0	0.0	0.0	-7.331271E-02
6	G	1.553089E-16	-2.433349E-00	0.0	0.0	0.0	-8.344875E-02
7	G	1.012660E-16	-3.305973E-00	0.0	0.0	0.0	-9.063621E-02
8	G	2.422960E-17	-4.237549E-00	0.0	0.0	0.0	-9.526662E-02
9	G	-1.741109E-17	-5.204722E+00	0.0	0.0	0.0	-9.784202E-02

Punched G-set modes from full model

File premac1.pch

ID	DMIG	UGVEXT	UGVEXT*	0	9	2	0
*	DMIG*	UGVEXT	UGVEXT*	2	1	6.6481447160-17	0
*	*	*	*	2	2	2-1.200921630D-01	
*	*	*	*	2	6-2.3443434371D-02		
*	*	*	*	3	1	1.4444975921D-16	
*	*	*	*	3	2-4.5739401460-01		
*	*	*	*	3	6-4.344435954D-02		
*	*	*	*	4	1	1.857186896D-16	
*	*	*	*	4	2-9.776227947D-01		
*	*	*	*	4	6-6.003817765D-02		
*	*	*	*	5	1 2.283180949D-16		
*	*	*	*	5	2-1.6470923584D+00		
*	*	*	*	5	6-7.331271061D-02		
*	*	*	*	6	1 1.553085975D-16		
*	*	*	*	6	2-2.433348689D+00		
*	*	*	*	6	6-8.342874830D-02		
*	*	*	*	7	1 0.012659685D-16		
*	*	*	*	7	2-3.305973102D+00		
*	*	*	*	7	6-9.063621155D-02		
*	*	*	*	8	1 2.422959542D-17		
*	*	*	*	8	2-4.237545942D+00		
*	*	*	*	8	6-9.528662506D-02		
*	*	*	*	9	1-1.741108975D-17		
*	*	*	*	9	2-5.204722201D+00		
*	*	*	*	9	6-9.784201806D-02		
*	*	*	*	10	1-4.097977165D-17		
*	*	*	*	10	2-6.189334130D+00		
*	*	*	*	10	6-9.888071523D-02		
*	*	*	*	11	1-1.0466867541D-16		
*	*	*	*	11	2-7.179607083D+00		
*	*	*	*	11	6-9.910058527D-02		
DMIG*	UGVEXT			2	0	1-7.972638657D-01	
*	etc.			...			
...	etc.			...			

Reduced model (points 3,5,7,9,11)

File premac2.dat

ID	TEST-ANALYSIS, ANALYSIS
...	...

```

SOL 103
$ INCLUDE ALTER FOR CROSS-ORTHOGONALITY CHECKS
$ INCLUDE 'premaca.v675'
$ CEND
TITLE = TEST-ANALYSIS: ANALYSIS
SUBTITLE = REDUCED MODEL
LABEL = 5 POINTS
DISP = ALL
SPC=1
METHOD=100
BEGIN BULK
$ READ PCH FILE OF FULL MODEL
$ INCLUDE PUNCHED DISPLS. FROM PRIOR, FULL RUN
$ INCLUDE 'premac1.pch'
$ ... same as full model
$ REDUCE TO TEST DOF
ASET1, 12, 3, 5, 7, 9, 11
$ ENDDATA

```

Reduced model results (points 3, 5, 7, 9, 11)

File premac2.f06

MODE NO.	EXTRACTION ORDER	EIGENVALUE	REAL RADANS	REAL EIGENVALUES CYCLES	GENERALIZED MASS	GENERALIZED STIFFNESS
1	1	7.636484E+05	8.73898E+02	1.390807E+02	1.000000E+00	7.636484E+05
2	2	9.652556E+06	3.10656E+03	4.944714E+02	1.000000E+00	9.652552E+06
3	3	2.936994E+07	5.41904E+03	8.625249E+02	1.000000E+00	2.936994E+07
4	4	8.952774E+07	9.461910E+03	1.505910E+03	1.000000E+00	8.952774E+07
5	5	2.280674E+08	1.510190E+04	2.403542E+03	1.000000E+00	2.280674E+08
6	6	2.597403E+08	1.611646E+04	2.5653014E+03	1.000000E+00	2.597403E+08

... MODAL ASSURANCE CRITERIA FOR RESIDUAL STRUCTURE A-SET

... MODAL ASSURANCE CALCULATION IS PERFORMED FOR 6 CALCULATED MODES AND 6 INPUT MODES

... ANALYTICAL MODE 1 - MAC = 1.000000E+00

... ANALYTICAL MODE 2 - MAC = 3.581825E-34

... ANALYTICAL MODE 3 - MAC = 9.598562E-02

... ANALYTICAL MODE 4 - MAC = 8.548350E-33

... ANALYTICAL MODE 5 - MAC = 1.005941E-01

... ANALYTICAL MODE 6 - MAC = 3.805310E-28

```

    ... ANALYTICAL MODE          2 COMPARED TO INPUT MODE      1 - MAC = 6.273596E-35
    ... ANALYTICAL MODE          2 COMPARED TO INPUT MODE      2 - MAC = 9.999999E-01
    ... ANALYTICAL MODE          2 COMPARED TO INPUT MODE      3 - MAC = 2.522496E-35
    ... ANALYTICAL MODE          2 COMPARED TO INPUT MODE      4 - MAC = 2.777777E-02
    ... ANALYTICAL MODE          2 COMPARED TO INPUT MODE      5 - MAC = 5.880748E-29
    ... ANALYTICAL MODE          2 COMPARED TO INPUT MODE      6 - MAC = 2.777777E-02
    ... ANALYTICAL MODE          3 COMPARED TO INPUT MODE      1 - MAC = 9.622484E-02
    ... ANALYTICAL MODE          3 COMPARED TO INPUT MODE      2 - MAC = 2.298354E-32
    ... ANALYTICAL MODE          3 COMPARED TO INPUT MODE      3 - MAC = 9.999998E-01
    ... ANALYTICAL MODE          3 COMPARED TO INPUT MODE      4 - MAC = 5.611178E-31
    ... ANALYTICAL MODE          3 COMPARED TO INPUT MODE      5 - MAC = 1.059392E-01
    ... ANALYTICAL MODE          3 COMPARED TO INPUT MODE      6 - MAC = 4.078400E-28
    ... ANALYTICAL MODE          4 COMPARED TO INPUT MODE      1 - MAC = 1.313866E-31
    ... ANALYTICAL MODE          4 COMPARED TO INPUT MODE      2 - MAC = 2.777777E-02
    ... ANALYTICAL MODE          4 COMPARED TO INPUT MODE      3 - MAC = 7.057600E-31
    ... ANALYTICAL MODE          4 COMPARED TO INPUT MODE      4 - MAC = 1.000000E+00
    ... ANALYTICAL MODE          4 COMPARED TO INPUT MODE      5 - MAC = 5.673982E-29
    ... ANALYTICAL MODE          4 COMPARED TO INPUT MODE      6 - MAC = 2.777777E-02
    ... ANALYTICAL MODE          5 COMPARED TO INPUT MODE      1 - MAC = 1.016641E-01
    ... ANALYTICAL MODE          5 COMPARED TO INPUT MODE      2 - MAC = 5.887744E-30
    ... ANALYTICAL MODE          5 COMPARED TO INPUT MODE      3 - MAC = 1.071091E-01
    ... ANALYTICAL MODE          5 COMPARED TO INPUT MODE      4 - MAC = 8.352938E-30
    ... ANALYTICAL MODE          5 COMPARED TO INPUT MODE      5 - MAC = 9.999937E-01
    ... ANALYTICAL MODE          5 COMPARED TO INPUT MODE      6 - MAC = 2.530804E-27
    ... ANALYTICAL MODE          6 COMPARED TO INPUT MODE      1 - MAC = 2.231569E-30
    ... ANALYTICAL MODE          6 COMPARED TO INPUT MODE      2 - MAC = 2.777778E-02
    ... ANALYTICAL MODE          6 COMPARED TO INPUT MODE      3 - MAC = 2.159709E-30
    ... ANALYTICAL MODE          6 COMPARED TO INPUT MODE      4 - MAC = 2.777778E-02
    ... ANALYTICAL MODE          6 COMPARED TO INPUT MODE      5 - MAC = 1.693683E-27
    ... ANALYTICAL MODE          6 COMPARED TO INPUT MODE      6 - MAC = 1.000000E+00

...RESULTS OF CROSS-ORTHOGONALITY TEST
...EACH ROW REPRESENTS ONE ANALYTICAL MODE, EACH COLUMN REPRESENTS AN INPUT MODE

MATRIX ORTHOA (GINO NAME 101 ) IS A DB PREC   6 COLUMN X   6 ROW SQUARE MATRIX.

COLUMN 1 ROWS 1 THRU 6 ..... .
ROW 1) 1.00000D+00 1.4638D-17 2.9384D-07 -8.1914D-17 -1.3846D-08 1.0899D-17
COLUMN 2 ROWS 1 THRU 6 ..... .
ROW 1) 5.0629D-18 -1.00000D+00 -3.3754D-19 -4.9004D-12 9.5638D-18 3.4195D-11
COLUMN 3 ROWS 1 THRU 6 ..... .
ROW 1) -4.3714D-04 -2.6831D-17 1.00000D+00 -7.7664D-16 -2.4651D-05 1.5706D-16
COLUMN 4 ROWS 1 THRU 6 ..... .
ROW

```

```

1) 1.4433D-16 4.9249D-11 8.0096D-16 1.0000D+00 4.3369D-16 -3.5882D-11
COLUMN 5 ROWS 1 THRU 6 -----
ROW 1) 1.3560D-03 -5.3830D-17 1.6108D-03 -4.8274D-16 1.0000D+00 -3.9773D-14
COLUMN 6 ROWS 1 THRU 6 -----
ROW 1) 7.7438D-17 2.5674D-16 5.0998D-17 6.0368D-16 5.3247D-14 1.0000D+00
-----  

THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN = 6
THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.
...
--PHIEXT (NODES) AND MAAEXT (MASS) OUTPUT AS DMIG

```

Punched output from run (points 3,5,7,9,11)

```

File premac2.pch
DMIG PHIEXT 0 9 2 0
DMIG* PHIEXT 3 1-3.959362338D-16
* 3 2-4.57413218D-01
* 5 1-5.925740855D-16
* 5 2-1.64716187D+00
* 7 1-3.508410468D-16
* 7 2-3.306109152D+00
* 9 1-3.735076484D-16
* 9 2-5.204924215D+00
* 11 1-7.71240439D-16
* 11 2-7.179897556D+00
* DMIG* PHIEXT 3 2 0
* 1 1.584620896D+00
...
etc.
...
DMIG* PHIEXT 3 6 0
* 3 1-5.884898924D+00
* 5 2 3.087113898D-14
* 5 1-5.765180000D-14
* 7 2 3.304301277D-14
* 7 1 5.884898924D+00
* 9 2-3.121114478D-14
* 9 1 3.753941602D-15
* 9 2-1.215684212D-14
* 11 1-5.884898924D+00
* 11 2 3.408354688D-14
* MAAEXT 0 1 2 0
* MAAEXT 3 3 1
* DMIG* MAAEXT 3 1 1.154999976D-02
* 5 3 1 1.924999960D-03
* DMIG* MAAEXT 2

```

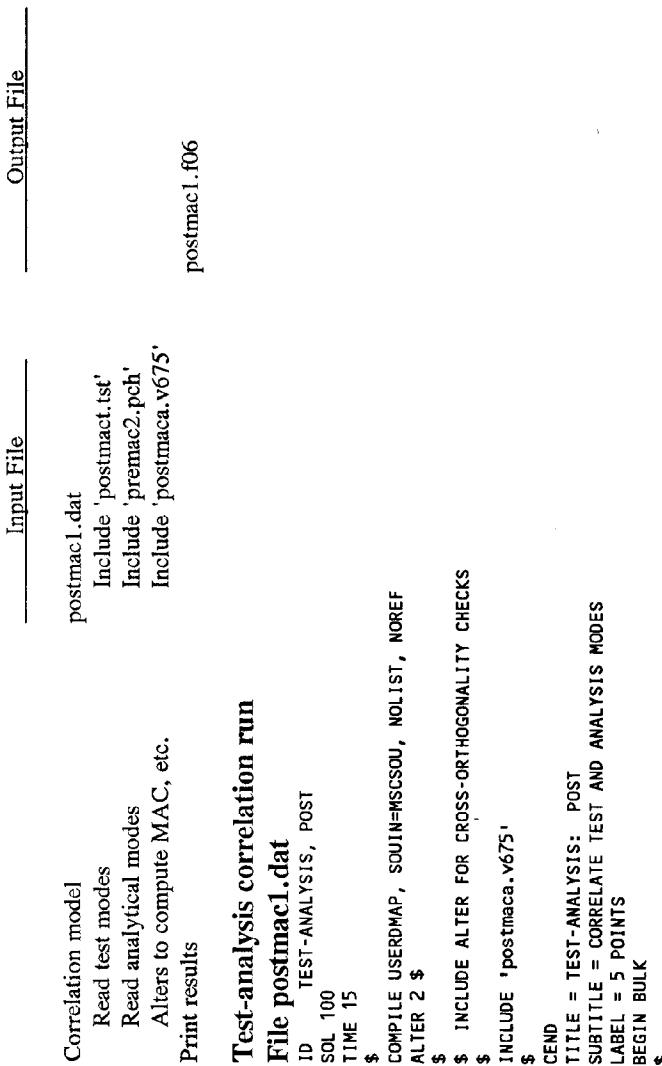
```

*          3          2 1.268066034D-02
*          5          2 1.872194081D-03
*          7          2-9.553313958D-04
*          9          2 3.465158578D-04
*          11         2-1.016234984D-04
*          1          5 1.924999960D-03
*          ...
etc.

*          ...        11          2
*          DMIG*      MAAEXT      3          2-1.016234984D-04
*          *          5          2 2.749199878D-04
*          *          7          2-7.467960985D-04
*          *          9          2 1.899324162D-03
*          *          11         2 5.123875189D-03

```

The following Version 67.5 files are for test-analysis correlation.



```

$ READ PCH FILE OF ANALYSIS MODEL
$ FROM PRE-TEST RUN
$ INCLUDE 'premac2.pch'
$ READ PCH FILE OF TEST DATA FROM EXTERNAL FILE
$ INCLUDE 'postmact.tst'
$ DEFINE GRIDS FOR TEST-ANALYSIS
$ GRID 3
$ GRID 5
$ GRID 7
$ GRID 9
$ GRID 11
$ ENDDATA

```

Test data

File postmact.tst

DMIG*	PHITEST	0	9	2	0
*	PHITEST	3	1	0.00+00	0
*	PHITEST	5	2	0.05D+00	
*	PHITEST	7	1	0.D+00	
*	PHITEST	9	2	0.25D+00	
*	PHITEST	11	1	0.D+00	
*	PHITEST	11	2	0.45D+00	
*	PHITEST	9	1	0.D+00	
*	PHITEST	9	2	0.70D+00	
*	PHITEST	11	1	0.D+00	
*	PHITEST	11	2	1.00D+00	
*	PHITEST	3	1	3.00D+00	0
*	PHITEST	3	2	0.D+00	
*	PHITEST	5	1	0.60D+00	
*	PHITEST	7	2	0.D+00	
*	PHITEST	7	1	0.80D+00	
*	PHITEST	9	2	0.D+00	
*	PHITEST	9	1	0.95D+00	
*	PHITEST	11	2	0.D+00	
*	PHITEST	11	1	1.0D+00	
*	PHITEST	3	2	0.D+00	
*	PHITEST	3	1	0.D+00	0
*	PHITEST	5	2	0.30D+00	
*	PHITEST	5	1	0.D+00	
*	PHITEST	7	1	0.70D+00	

```

*          7      2-0.60D+00
*          9      1      0.D+00
*          9      2      0.50D+00
*          11     1      0.D+00
*          11     2      1.00D+00
DMIG*   PHITEST    4      0
*          *          3      1-0.80D+00
*          *          3      2      0.D+00
*          *          5      1-0.95D+00
*          *          5      2      0.D+00
*          *          7      1-0.30D+00
*          *          7      2      0.D+00
*          *          9      1      0.60D+00
*          *          9      2      0.D+00
*          *          11     1      1.00D+00
*          *          11     2      0.D+00
DMIG*   PHITEST    5      0
*          *          3      1      1.00D+00
*          *          3      2      0.D+00
*          *          5      1      0.00D+00
*          *          5      2      0.D+00
*          *          7      1-1.00D+00
*          *          7      2      0.D+00
*          *          9      1      0.00D+00
*          *          9      2      0.D+00
*          *          11     1      1.00D+00
*          *          11     2      0.D+00
DMIG*   PHITEST    6      0
*          *          3      1      0.D+00
*          *          3      2      0.65D+00
*          *          5      1      0.D+00
*          *          5      2      0.60D+00
*          *          7      1      0.D+00
*          *          7      2      0.45D+00
*          *          9      1      0.D+00
*          *          9      2      0.-45D+00
*          *          11     1      0.D+00
*          *          11     2      1.00D+00

```

Results from test-analysis correlation

File postmac1.f06

```

--> INPUT MATRICES HAVE BEEN READ
--> THERE ARE      6 TEST MODES AND      6 ANALYTICAL MODES
--> THERE ARE      20 NULL COLUMNS IN THE MASS MATRIX - THESE WILL BE REMOVED
--> IF YOU DO NOT WANT NULL COLUMNS REMOVED, SET PARAM,NUCOL,-1)
--> MODAL ASSURANCE CRITERIA BETWEEN TEST AND ANALYSIS
--> MODAL ASSURANCE CALCULATION IS PERFORMED FOR      6 ANALYSIS MODES AND      6 TEST MODES
--> ANALYTICAL MODE      1 COMPARED TO TEST MODE      1 - MAC =  9.993510E-01
--> ANALYTICAL MODE      1 COMPARED TO TEST MODE      2 - MAC =  4.974734E-34
--> ANALYTICAL MODE      1 COMPARED TO TEST MODE      3 - MAC =  2.091194E-01

```

```

    ... ANALYTICAL MODE          1 COMPARED TO TEST MODE          4 - MAC =
    ... ANALYTICAL MODE          1 COMPARED TO TEST MODE          5 - MAC =
    ... ANALYTICAL MODE          1 COMPARED TO TEST MODE          6 - MAC =
    ... ANALYTICAL MODE          2 COMPARED TO TEST MODE          1 - MAC =
    ... ANALYTICAL MODE          2 COMPARED TO TEST MODE          2 - MAC =
    ... ANALYTICAL MODE          2 COMPARED TO TEST MODE          3 - MAC =
    ... ANALYTICAL MODE          2 COMPARED TO TEST MODE          4 - MAC =
    ... ANALYTICAL MODE          2 COMPARED TO TEST MODE          5 - MAC =
    ... ANALYTICAL MODE          2 COMPARED TO TEST MODE          6 - MAC =
    ... ANALYTICAL MODE          3 COMPARED TO TEST MODE          1 - MAC =
    ... ANALYTICAL MODE          3 COMPARED TO TEST MODE          2 - MAC =
    ... ANALYTICAL MODE          3 COMPARED TO TEST MODE          3 - MAC =
    ... ANALYTICAL MODE          3 COMPARED TO TEST MODE          4 - MAC =
    ... ANALYTICAL MODE          3 COMPARED TO TEST MODE          5 - MAC =
    ... ANALYTICAL MODE          3 COMPARED TO TEST MODE          6 - MAC =
    ... ANALYTICAL MODE          4 COMPARED TO TEST MODE          1 - MAC =
    ... ANALYTICAL MODE          4 COMPARED TO TEST MODE          2 - MAC =
    ... ANALYTICAL MODE          4 COMPARED TO TEST MODE          3 - MAC =
    ... ANALYTICAL MODE          4 COMPARED TO TEST MODE          4 - MAC =
    ... ANALYTICAL MODE          4 COMPARED TO TEST MODE          5 - MAC =
    ... ANALYTICAL MODE          4 COMPARED TO TEST MODE          6 - MAC =
    ... ANALYTICAL MODE          5 COMPARED TO TEST MODE          1 - MAC =
    ... ANALYTICAL MODE          5 COMPARED TO TEST MODE          2 - MAC =
    ... ANALYTICAL MODE          5 COMPARED TO TEST MODE          3 - MAC =
    ... ANALYTICAL MODE          5 COMPARED TO TEST MODE          4 - MAC =
    ... ANALYTICAL MODE          5 COMPARED TO TEST MODE          5 - MAC =
    ... ANALYTICAL MODE          5 COMPARED TO TEST MODE          6 - MAC =
    ... ANALYTICAL MODE          6 COMPARED TO TEST MODE          1 - MAC =
    ... ANALYTICAL MODE          6 COMPARED TO TEST MODE          2 - MAC =
    ... ANALYTICAL MODE          6 COMPARED TO TEST MODE          3 - MAC =
    ... ANALYTICAL MODE          6 COMPARED TO TEST MODE          4 - MAC =
    ... ANALYTICAL MODE          6 COMPARED TO TEST MODE          5 - MAC =
    ... ANALYTICAL MODE          6 COMPARED TO TEST MODE          6 - MAC =

```

...RESULTS OF CROSS-ORTHOGONALITY TEST
...EACH ROW REPRESENTS ONE ANALYTICAL MODE, EACH COLUMN REPRESENTS AN INPUT MODE

MATRIX ORTHOA (GINO NAME 101) IS A DB PREC		6 COLUMN X		6 ROW SQUARE MATRIX.	
COLUMN	ROW	1	ROWS	1 THRU	6
ROW 1)	-9.9996D-01	3.3553D-19	5.6116D-03	4.4947D-17	1.3249D-02
ROW 1)	-1.1417D-18	9.9996D-01	-3.2065D-18	-8.2329D-04	5.8557D-17
ROW 1)	-2.5890D-01	4.8917D-20	-9.4181D-01	7.9554D-16	-1.0785D-01
ROW 1)	-2.5890D-01	4.8917D-20	-9.4181D-01	7.9554D-16	-1.0785D-01

COLUMN ROW	4	ROWS	1 THRU	6
1)	1.1872D-16	9.3829D-03	9.0667D-16	9.9994D-01	3.3362D-16 -3.1712D-11
COLUMN ROW	5	ROWS	1 THRU	6
1)	2.3146D-17	6.4163D-11	8.6138D-17	3.1668D-11	7.7386D-15 -1.0000D+00
COLUMN ROW	6	ROWS	1 THRU	6
1)	-1.4140D-02	3.0717D-19	2.3385D-02	-3.6948D-16	9.9962D-01 7.0435D-15

THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN = 6
 THE DENSITY OF THIS MATRIX IS 100.00 PERCENT.