FINITE ELEMENT MODEL CORRELATION FOR STRUCTURES

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ABSTRACT

Agreement between measured response of a structure and numerical predictions using an initial finite element model (IFEM) is in general poor. An algorithm is developed, which produces an updated finite element model (UFEM) that is fully correlated with respect to modal measurements. An incremental nonlinear methodology based on large admissible perturbations in cognate space is used to produce the UFEM by postprocessing the results of the initial finite element analysis (FEA) using MSC/NASTRAN. No additional FEA requiring trial and error adjustment is required. The UFEM corresponds to a real structure and may differ from the IFEM in response and correlation variables by 100 - 300 percent depending on correlation measures and structural size. Two numerical applications for a structure are used to assess the strength, and limitations of the large perturbation methodology.

Introduction

Static and dynamic analysis of structures are usually performed by the Finite Element Method (FEM). However, numerical prediction of static and dynamic responses of structures are often inaccurate due to simplifying assumptions, uncertainty, and ignorance. The larger the manufacturing tolerances, the greater is the discrepancy between predicted and measured response. This problem is of particular importance and difficulty in Finite Element Analysis (FEA) of marine structures that have large manufacturing tolerances and structural imperfections so that finite element modelling and numerical predictions of response is highly inaccurate.

Finite Element Model Correlation is the process of finding corrected values of the correlation variables in the FE Model of a physical structure so that predictions by FEM match the response of the corresponding physical structure. Two structural FE Models are involved in this correlation problem : An Initial FE Model (IFEM) which fails to predict accurately the response of the modeled structure, and an Updated FE Model (UFEM) which must satisfy all measured response data. The response properties that have been measured on the physical structure are called correlation measures. Some properties of elements or group of elements have to be changed to satisfy the correlation measures. The element properties that are allowed to change are called correlation variables. In practice, an incomplete set of natural frequencies and mode shapes, and/or some static deflections are measured.

Nonlinear perturbation techniques are the most successful in solving not only the model correlation problem, but many other two-state structural problems. Specifically, they are capable of relating any two structural states that are modeled by the same FE model but are described by different design parameters. Only one FEA is required, that of the baseline structure even if the two structural states differ significantly(100-300 percent) in design variables and response. The problems of redesign, model correlation, failure mode identification, redundancy, and nondestructive testing can be addressed using nonlinear perturbation techniques. Linear perturbation methods were first developed by Stetson [1,2] for structural redesign, allowing for small differences in response, stiffness, and geometric properties between baseline and objective designs. Sandstrom and Anderson [3] improved that method. Nonlinear perturbation methods were developed by Hoff, Bernitsas et al. [4,5,6], Bernitsas et al. [7], and Kim and Anderson [8], with the main objective to improve the algorithm and make it applicable to large-scale structures and for larger differences between baseline and objective structures. Kim and Bernitsas [9] introduced static deflections as redesign objectives along with modal measurements. Bernitsas and Kang [10] forced admissibility on structural perturbations to increase allowable differences between the two structural states in numerical computations for redesign [11]. Bernitsas and Tawekal [12] solved the model correlation problem by introducing cognate space identification which reduces the modal basis for series expansion and the computational time.

The Finite Element model correlation procedure developed in this paper is illustrated in the diagram shown in Figure 1. The developed perturbation method and solution algorithm have the following major features :

- This algorithm should be used after all possible modifications of the FE model - related to the manufacturing tolerances and structural imperfections - which do not require trial and error have been made.
- Natural frequencies and mode shapes are measured and assumed to be exact.
- The set of measured modal properties may be incomplete.
- Differences between IFEM and UFEM in response and structural properties up to 100% to 300% are allowed depending on the scale of the structure and the correlation measurements.
- Both mass and stiffness matrices are updated.
- Properties allowed to change in the correlation process are fractional changes. Thus, the updated matrices represent mass and stiffness matrices of a real structure.
- IFEM and UFEM differ significantly in the values of elemental stiffness and mass, and in geometric properties. Nonetheless, the correlation problem is solved by postprocessing results of only one FEA, that of IFEM.
- A nonlinear perturbation algorithm is used which performs linear inadmissible predictions and nonlinear admissible corrections. Perturbations are defined as admissible if they satisfy both stiffness and mass orthogonality, thus ensuring that intermediate (incremental) modes correspond to the real structure calculated by the algorithm at each increment.
- No trial and error.
- At each increment the correlation problem is solved by minimizing the Euclidean norm of the correlation variables, or the error in the constraints depending on whether the problem is underdetermined or overdetermined, respectively. In general, the former yields better results.
- To reduce the number of constraints and make the problem underdetermined if possible – for a given number of correlation variables – the number of orthogonality constraints is reduced by identifying the space of modes cognate to the correlation modal objectives (measurements).

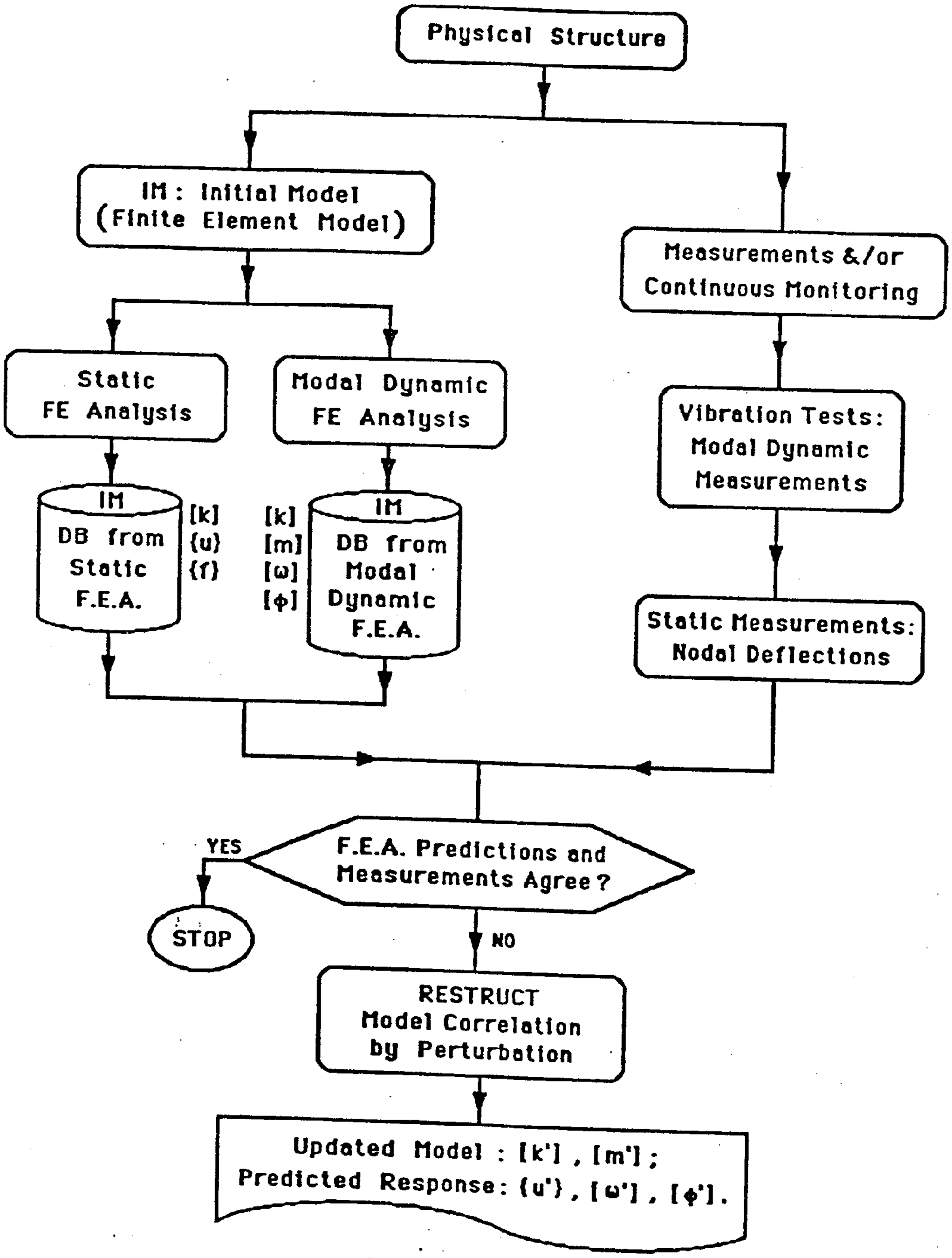

Figure 1: FEA-predictions, response measurements, model correlation

Problem Definitions

The IFEM and UFEM are related through the following perturbation relations :

$$[k'] = [k] + [\Delta k] \tag{1}$$

$$[m'] = [m] + [\Delta m] \tag{2}$$

$$[\ \omega'^2\ ] = [\ \omega^2\ ] + [\ \ \Delta(\omega^2)\ \ ] \tag{3}$$

$$[\phi'] = [\phi] + [\Delta\phi] \tag{4}$$

where primed and unprimed quantities refer to the UFEM and IFEM respectively, the prefix Δ indicates the total difference between an UFEM quantity and its IFEM counterpart, $[\phi] = [\{\psi\}_1, \{\psi\}_2, \ldots, \{\psi\}_n]$, and $[\ \omega^2\ ]$ is the diagonal matrix of the eigenvalues. In order to ensure that UFEM represents a real structure, changes in the global stiffness and mass matrices are expressed as the sum of changes in structural components [3,4,5]

$$[\Delta k] = \sum_{e=1}^{p} [\Delta k_e] = \sum_{e=1}^{p} [k_e]\alpha_e \tag{5}$$

$$[\Delta m] = \sum_{e=1}^{p} [\Delta m_e] = \sum_{e=1}^{p} [m_e]\,\alpha_e \tag{6}$$

where p is the number of element properties or substructures that are allowed to change, and α_e is the fractional change in property e. Some α_e 's may represent the fractional change of only $[k_e]$ or $[m_e]$, and several α_e 's may refer to the same element but different properties like bending, torsion, stretching, etc.

The relation between energy balance equations of the IFEM and UFEM is

$$[K'] = [M']\,[\omega'^2]$$

or

$$[K] + [\Delta K] = \{[M] + [\Delta M]\} \{[\omega^2] + [\Delta\omega^2]\} \tag{7}$$

where [K], [M], and [K′],[M′] are the generalized stiffness and mass matrices for IFEM and UFEM respectively.

Introducing perturbation relations (1) - (6) into (7) produces the following n^2 scalar equations. These are the general dynamic perturbation equations :

$$\begin{aligned} \sum_{e=1}^{p} (\{\psi'\}_i^T[k_e]\{\psi'\}_i \; - \; \omega_i'^2 \{\psi'\}_i^T[m_e]\{\psi'\}_i)\alpha_e & \\ = \omega_i'^2 \{\psi'\}_i^T[m]\{\psi'\}_i \; - \; \{\psi'\}_i^T[k]\{\psi'\}_i & \end{aligned} \tag{8}$$

$$\sum_{e=1}^{p} \{\psi'\}_j^T[k_e]\{\psi'\}_i\alpha_e = - \{\psi'\}_j^T[k]\{\psi'\}_i \tag{9}$$

$$\sum_{e=1}^{p} \{\psi'\}_j^T [m_e] \{\psi'\}_i\alpha_e = - \{\psi'\}_j^T [m] \{\psi'\}_i \tag{10}$$

for $i = 1,2,\ldots,n$, $\;j = i+1,i+2,\ldots,n$.

Equation (8) represents the n diagonal terms in Eq. (7) for the UFEM, that is the Rayleigh quotients for $\omega_i'^2$. Eqs. (9) and (10) are equivalent to the orthogonality conditions of the UFEM modes $\{\psi'\}$ with respect to $[k']$ and $[m']$. Theoretically, orthogonality of modes with respect to one of $[k']$ or $[m']$ implies orthogonality with respect to the other. Numerically, however, both conditions must be forced if $\{\psi\}_j$, $j = 1,2,\ldots,n$, are to represent modes of a real structure. In the model correlation process, Eqs. (8) - (10) are used to impose modal measurements on the UFEM. It should be emphasized that in general an incomplete set of measured modes and natural frequencies are available. Further, modes measured may not be complete, i.e., deflections of a particular mode may be measured while slopes may not. Consequently, the role of the modal dynamics general perturbation Eqs. (8) - (10) is three-fold : to force a measured quantity on UFEM ; to make measured modes part of the orthogonal modal basis $\{\psi'\}_i$, $i = 1,2,\ldots,n$; and to complete incompletely measured modes.

Incremental Solution Algorithm

The objective of the correlation process is to compute $[\Delta k]$ and $[\Delta m]$, that is the stiffness and mass matrix perturbations to define UFEM. Correlation variables are selected as fractional changes α_e, $e = 1, 2, \ldots, p$, as defined by Eqs. (5) and (6), where p properties of elements or substructures are allowed to change. Such properties may be mass, bending stiffness, torsional stiffness, etc., or geometric properties like linear dimensions of a beam cross section [5], plate thickness [9], or a tube is internal and external diameters [7]. Several elements may be linked together in one group to ensure that they remain identical in the UFEM [9]. More than one property may be allowed to change in each element or substructure.

Measured modal properties may be used as correlation measures (objectives) defined by the following equations :

$$\omega_i'^2 = \omega_i^2 + \Delta\omega_i^2 = c_{\omega_i}, \quad i = 1, 2, \ldots, S_\omega \tag{11}$$

$$\phi'_{ki} = \phi_{ki} + \Delta\phi_{ki} = c_{\phi_{ki}}, \quad \text{no. of } (k, i) = S_\phi \tag{12}$$

where the right-hand side represents the measured properties and S_ω, S_ϕ are the number of measured frequencies and modal dofs. Measured properties are assumed to be exact. It should be stressed again that in practice, the set of measured modes usually is incomplete and measured mode shapes are incomplete as well.

The problem of finite element model correlation by perturbation can be defined now as follows : Compute the values of correlation variables α_e, $e = 1, 2, \ldots, p$, subject to S_ω natural frequency constraints (11) and S_ϕ modal node constraints (12).

An incremental perturbation algorithm which solves the model correlation problem and handles successfully the aforementioned difficulties is developed in the foregoing discussion. The strategy is illustrated diagrammatically in Figure 2. The major aspects of the algorithm are explained as follows :

(a) Modal objectives are achieved incrementally. That is, the $S_\omega c_{\omega_i}$'s and the $S_\phi c_{\phi_{ki}}$'s, are achieved in increments no larger than 7 percent. Thus, incremental correlation variables ${}_l\alpha_e$, $e = 1, 2, \ldots, p$, remain small in each increment l, $l = 1, 2, \ldots, N$; where N indicates the total number of increments required to achieve all correlation measures. Small values of α_e 's are necessary in the prediction phase as explained in the next paragraph. If an incremental difference is denoted by δ — as opposed to a total difference indicated by Δ —

then the incremental correlation problem will be defined by the incremental counterparts of Eqs. (8) - (10), (11) - (13) and

$$-1 < {}_l\alpha_e^{\,-} \le \alpha_e \le {}_l\alpha_e^{\,+}, \quad e = 1, 2, \ldots, p \tag{13}$$

where ${}_l\alpha_e^{\,-} = -\,0.15$, ${}_l\alpha_e^{\,+} = +\,0.15$, and $(1+\alpha_e) = \prod_{l=1}^{N}(1 +_l \alpha_e)$.

(b) In the prediction phase of the algorithm in each increment, inadmissible linear perturbations are performed. This part of the algorithm is based on the small perturbation method developed by Stetson [1,2] and improved by Sandstrom and Anderson [3]. Incremental modal changes are expressed in terms of the incremental matrix of admixture coefficients ${}_l[c]$, as $[\delta_l\phi] =_l [\phi]_l[c]^T$, where ${}_lc_{ii} = 0, {}_l c_{ij}, i, j = 1, 2, \ldots, n_r$ are small, and n_r is the number of extracted modes used in the algorithm. Applying that method to the incremental counterpart of the energy balance Eq. (7), diagonal and off-diagonal terms yield, respectively,

$$\begin{aligned}\delta_l\omega_i^2 &= \frac{1}{{}_lM_i}\left[\sum_{e=1}^{p}\left({}_l\{\psi\}_i^T[k_e]_l\{\psi\}_i \;-\; {}_l\omega_i^2\,{}_l\{\psi\}_i^T[m_e]_l\{\psi\}_i\right)_l\alpha_e\right] \\ &= \delta_l c_{\omega_i}, \qquad \mathrm{i} = 1, 2, \ldots, S_\omega\end{aligned} \tag{14}$$

$${}_lc_{ij} = \sum_{e=1}^{p}\frac{1}{{}_lM_j\left({}_l\omega_i^2 - {}_l\omega_j^2\right)}\left({}_l\{\psi\}_j^T[k_e]_l\{\psi\}_i - {}_l\omega_i^2\,{}_l\{\psi\}_j^T[m_e]_l\{\psi\}_i\right)_l\alpha_e \tag{15}$$

Further, by definition, measured modal changes are

$$\delta_l\phi_{ki} = \sum_{j=1, j\neq 1}^{n_r} {}_l\phi_{kj\,l}c_{ij} = \delta_l c_{\phi_{ki}}, \qquad \text{no.of}\,(k,i) = S_\phi \tag{16}$$

The advantages of the linear perturbation method are several. First, the explicit expressions (14) and (16) for modal incremental differences between IFEM and UFEM result in the development of linear relations for the incremental correlation variables ${}_l\alpha_e$, $e = 1, 2, \ldots, p$. Second, partially measured modes may be computed using eq. (17). A major disadvantage is that linear predictions are inadmissible in the sense that orthogonality conditions with respect to $[k]$ and $[m]$ are satisfied only approximately in the form of eq. (15).

(c) In the correction phase of the algorithm in each increment, correction into the admissible space is performed. At the end of the prediction phase, after computing approximate values of the ${}_l\alpha_e$'s, all extracted modes ${}_l\{\psi'\}$ are updated. Then, the incremental counterparts of Eqs. (8) - (10) are used to solve for the corrected values of the correlation variables ${}_l\alpha_e$, $e = 1, 2, \ldots, p$, subject to upper and lower bounds specified in (13). Finally, values of frequencies that have not been measured are predicted using the Rayleigh quotient.

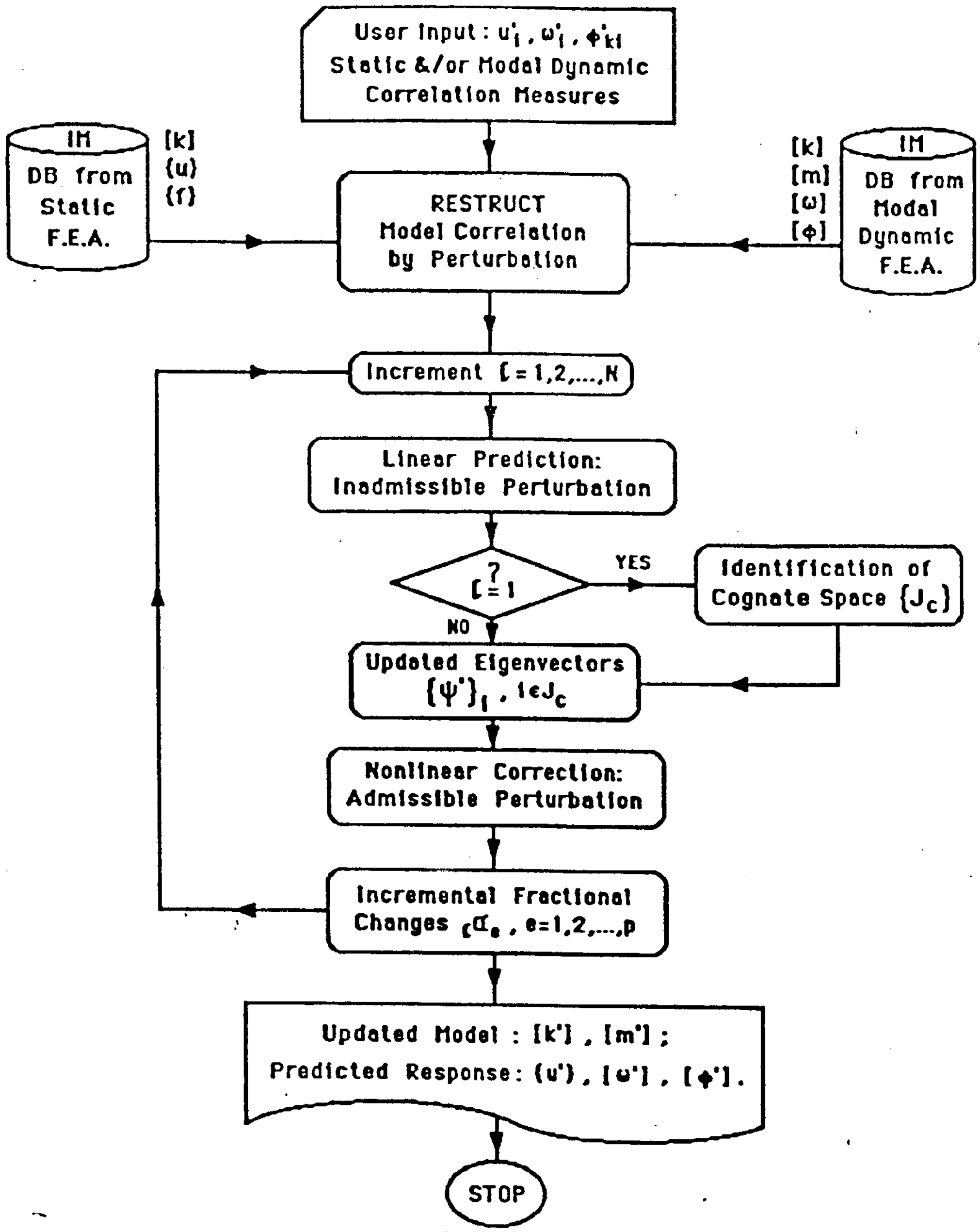

Figure 2: Incremental Algorithm of Structural Model Correlation Using Large Admissible Perturbations

(d) Depending on the number of correlation measures α_e 's, and the number of constraints, the problems in both phases – prediction and correction – may be overdetermined or underdetermined. In the former case, a generalized inverse algorithm is used [13,9] to produce a minimum error solution in satisfaction of all equality constraints. In the latter case, which in general provides better results, an optimization algorithm with a criterion of minimum change between IFEM and UFEM is used. Two forms of the minimum change criterion are available in the algorithm : (*i*) local (stepwise) criterion which requires minimization of the Euclidean norm of the ${}_l\alpha_e$'s in every increment ; (*ii*) global criterion which requires minimization of the Euclidean norm of the α_e 's of each increment, where $\alpha_e = \Pi_{q=1}^{l}(1 +_q \alpha_e) - 1$. Quadratic programming by QPSOL is used when all constraints are linear with respect to the α_e 's [14]. If the constraints are nonlinear, as in problems for plate [9] or tubular [7] elements, a nonlinear programming solver [10] like NPSOL must be used.

(e) Computer code RESTRUCT (REdesign of STRUCTures) [11] is used to implement the algorithm already described and summarized in Fig. 3. RESTRUCT was initially developed to solve the problem of structural redesign [9]. It is currently about 22000 Fortran 77 commands. RESTRUCT may serve as postprocessor to any special or general purpose FE code, including MSC/NASTRAN and performs computations with concentrated mass, spring, rod (truss), bar, beam, triangular and quadrilateral plate, and marine riser tubular elements.

Analysis

The problem of structural finite element model correlation is studied here using a 5 - element, 9 - dof, clamped - hinged beam numerical application. The beam properties is shown in Figure 3. From dynamic analysis, the first natural frequency and its normal mode was computed. The first natural frequency is $f_1 = {}^{\omega_1}/_{2\Pi} = 24.283$ Hz. The first normal mode deflections are $\phi_{9,1} = 0.30271$, $\phi_{15,1} = 0.80156$, $\phi_{21,1} = 1.000$, $\phi_{27,1} = 0.68719$, where MSC/NASTRAN dof numbering is used. The database was stored using the DMAP commands shown in Figure 4. Two applications of correlation with respect to measured values of f_1' and the corresponding mode is considered with local (stepwise) optimum criterion and global optimum criterion in the optimization algorithm. In these applications, 8 mode are extracted, $n_r = 8$; 10 correlation variables are used, the moment of inertia and cross section area of each element. The results are shown in Table 1 and Table 2. In both cases, the correlation measure ratio $r_{\omega^2} = {}^{\omega_i'^2}/_{\omega_1^2} = 2.0$. The available measurements reflect deflections only. That is, only the first mode is available for correlation, and it has been partially measured. Since the number of constraints $S = S_A + 1 = 9$, is less than number of correlation variables $p = 10$, the problem is underdetermined and is solved by optimization.

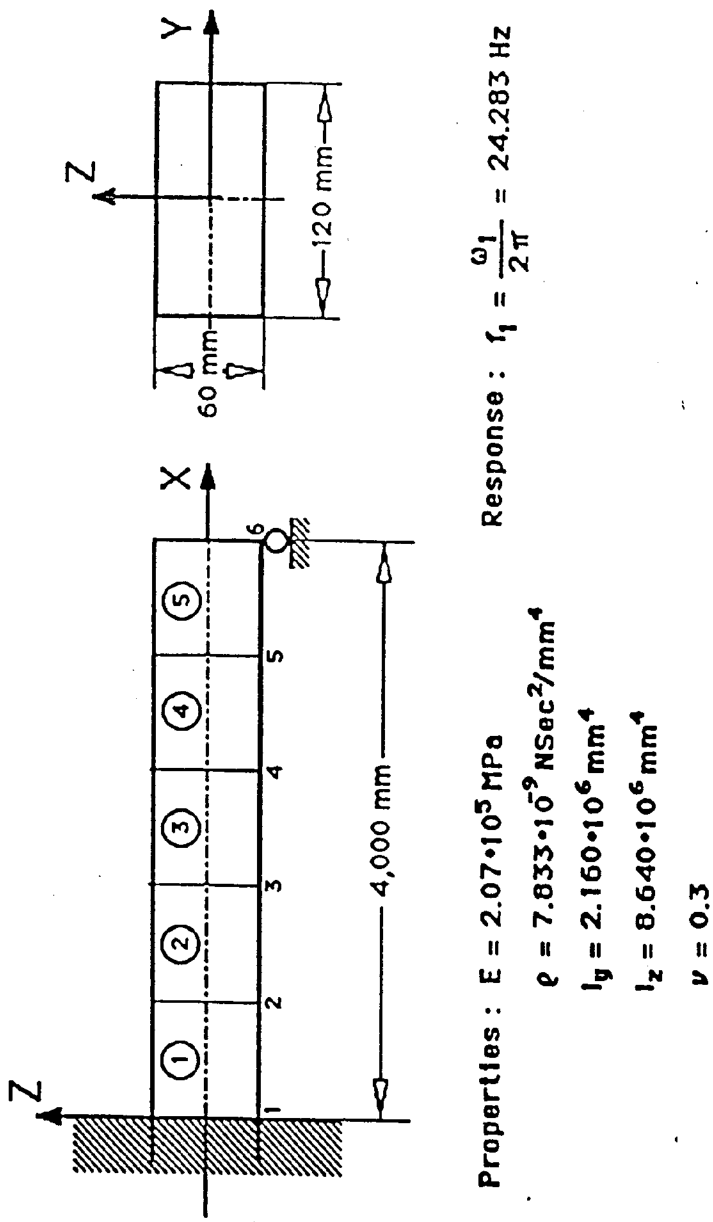

Figure 3: Clamped-Hinged Beam Model : 5 elements, 9 dofs

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ID TAWEKAL, RICKY L. $
TIME 1
SOL 3   $NORMAL MODES
$DIAG 14 VERSION 3.0
ALTER 509
OUTPUT4 KELM,MELM,UGV//0/8
OUTPUT2 KDICT,MDICT,LAMA,EQEXIN,GPDT//0/8/ $
OUTPUT2 VELEM//0/8   $
ENDALTER
CEND
TITLE = C-H 5-ELEMENT BEAM DYNAMIC ANALYSIS
SUBTITLE = 10 Modes. MODIFIED GIVEN'S METHOD
LABEL = BASELINE STRUCTURE
METHOD=75
DISPLACEMENT=ALL
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GRID,2, , 500. ,      0. ,     0. , ,1246
GRID,3, ,1000. ,      0. ,     0. , ,1246
GRID,4, ,1500. ,      0. ,     0. , ,1246
GRID,5, ,2000. ,      0. ,     0. , ,1246
GRID,6, ,2500. ,      0. ,     0. , ,12346
$ Reference Point for Coordinate System
GRID,7, ,    1. ,     1. ,     0. , ,123456
CBAR            1       23      1       2       7
CBAR            2       23      2       3       7
CBAR            3       23      3       4       7
CBAR            4       23      4       5       7
CBAR            5       23      5       6       7
PBAR,23,25,5000.,4.17+06,1.042+06
MAT1,25,2.07+05,8.0+04,0.3,7.833-09
PARAM,COUPMASS,1
EIGR,75,MGIV,0.,10000.,10,10,,1.E-05,+NEXT
+NEXT,MAX
ENDDATA
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Figure 4: MSC/NASTRAN Version 64 Input File

Table 1 : Beam Model ; Frequency and Mode Shape Measures ; Stepwise Optimum

	Initial Model	Correlation measures	RESTRUCT prediction	Updated Model	Error (%)
f_1^*	24.2832	34.3416	34.34	34.3391	-0.0071
$\phi_{9,1}^{**}$	0.30271	0.29520	0.2952	0.29521	0.0034
$\phi_{11,1}$	-8.3266E-04	-	-8.1277E-04	-8.1339E-04	-0.08
$\phi_{15,1}$	0.80156	0.78908	0.78908	0.78905	-0.0038
$\phi_{17,1}$	-6.8707E-04	-	-6.9072E-04	-6.9258E-04	-0.27
$\phi_{21,1}$	1.00000	1.00000	1.00000	1.00000	0.0
$\phi_{23,1}$	8.6207E-05	-	5.2097E-05	4.8915E-05	-6.04
$\phi_{27,1}$	0.68719	0.70453	0.70453	0.70458	0.0071
$\phi_{29,1}$	9.1521E-04	-	9.0343E-04	8.9972E-04	-0.41
$\phi_{35,1}$	1.2643E-03	-	1.3217E-03	1.3181E-03	-0.27
f_2^*	78.8804	-	102.9	102.93	0.029
$\phi_{9,2}$	-7.7108E-01	-	-7.3710E-01	-7.3540E-01	0.23
$\phi_{11,2}$	1.5231E-03	-	1.4660E-03	1.4596E-03	-0.44
$\phi_{15,2}$	-9.4673E-01	-	-9.0943E-01	-9.0894E-01	0.054
$\phi_{17,2}$	-1.1855E-03	-	-1.1027E-03	-1.1019E-03	0.072
$\phi_{21,2}$	3.0329E-01	-	2.7394E-01	2.7235E-01	-0.58
$\phi_{23,2}$	-2.2984E-03	-	-2.2114E-03	-2.2222E-03	0.49
$\phi_{27,2}$	1.0000E+00	-	1.0000E+00	1.0000+00	0.00
$\phi_{29,2}$	3.6773E-04	-	2.2526E-04	2.0840E-04	-7.48
$\phi_{35,2}$	2.3881E-03	-	2.5069E-03	2.4952E-03	-0.47

Number of extracted modes : $n_r = 8$
Number of correlation variables : $p = 10$
Total CPU time ; t : $t = 25,377msec$
Number of admissibility constraints : $S_a = 8$
* Correlation measure ratio : $\frac{\omega_2'^2}{\omega_1'^2} = 2.000$
** Only deflection of modal node measured

Table 2 : Beam Model ; Frequency and Mode Shape Measures ; Global Optimum

	Initial Model	Correlation measures	RESTRUCT prediction	Updated Model	Error (%)
f_1^*	24.2832	34.3416	34.34	34.3391	0.003
$\phi_{9,1}^{**}$	0.30271	0.29520	0.29520	0.29522	0.007
$\phi_{11,1}$	-8.3266E-04	-	-8.1214E-04	-8.1213E-04	0.001
$\phi_{15,1}$	0.80156	0.78908	0.78908	0.78910	0.002
$\phi_{17,1}$	-6.8707E-04	-	-6.9592E-04	-6.9473E-04	0.17
$\phi_{21,1}$	1.00000	1.00000	1.00000	1.00000	0.0
$\phi_{23,1}$	8.6207E-05	-	6.3349E-05	6.3491E-05	0.22
$\phi_{27,1}$	0.68719	0.70453	0.70453	0.70422	0.016
$\phi_{29,1}$	9.1521E-04	-	8.9904E-04	8.9921E-04	0.019
$\phi_{35,1}$	1.2643E-03	-	1.3176E-03	1.3186E-03	0.076
f_2^*	78.8804	-	98.32	98.318	-0.002
$\phi_{9,2}$	-7.7108E-01	-	7.8850E-01	7.3540E-01	0.18
$\phi_{11,2}$	1.5231E-03	-	-1.5733E-03	-1.5707E-01	0.16
$\phi_{15,2}$	-9.4673E-01	-	1.0000E+00	1.0000E+00	0.000
$\phi_{17,2}$	-1.1855E-03	-	1.0633E-03	1.0675E-03	0.39
$\phi_{21,2}$	3.0329E-01	-	-1.3308E-01	-1.3269E-01	0.29
$\phi_{23,2}$	-2.2984E-03	-	2.0902E-03	2.0918E-03	0.076
$\phi_{27,2}$	1.0000E+00	-	-8.3449E-01	-8.3610-01	0.19
$\phi_{29,2}$	3.6773E-04	-	-1.2921E-04	-1.1528E-04	10.78
$\phi_{35,2}$	2.3881E-03	-	-2.1113E-03	-2.1188E-03	0.36

Number of extracted modes : $n_r = 8$
Number of correlation variables : $p = 10$
Total CPU time ; t : $t = 25,315\ msec$
Number of admissibility constraints : $S_a = 8$
* Correlation measure ratio : $\frac{\omega_2^2}{\omega_1^2} = 2.000$
** Only deflection of modal node measured

Discussion

For this small scale structure the relative error in the results are very small. The relative error in results predicted by RESTRUCT and reanalyzed by MSC/NASTRAN for the second mode that has not been constrained are also presented in Table 1 and Table 2. When the stepwise optimum criterion is used, a relatively large error occurs for $\phi_{23.1}$ which is rotation at node #4 of the first mode and for $\phi_{29.2}$ which is rotation at node #5 of the second mode. In the case where the global optimum criterion is used, large error occurs only for $\phi_{29.2}$ which is rotation at node #5 of the second mode. In general, results generated by the global optimum criterion are better then those generated by stepwise optimum criterion. The final fractional changes α_e 's are summarized in Table 3 and Table 4.

Table 3 : Updated Beam Model ; Stepwise Optimum ; Frequency and Mode Shape Measures

Element Property	Element #				
	1	2	3	4	5
I_2	0.18736	0.08827	0.04046	0.16554	0.08047
Area	0.01236	-0.27256	-0.71074	-0.29400	-0.04378
$\Sigma_{e=1}^{p}\ \alpha_1^2 = 0.74636$ Total CPU time = 25315 msec					

Table 4 : Updated Beam Model ; Global ; Frequency and Mode Shape Measures

Element Property	Element #				
	1	2	3	4	5
I_2	0.41914	0.33602	0.29623	0.24685	0.12390
Area	-0.02112	-0.20407	-0.41512	-0.38547	-0.11544
$\Sigma_{e=1}^{p}\ \alpha_1^2 = 0.82896$ Total CPU time = 25377 msec					

Conclusions

The problem of correlating a FE Model to a real structure for which modal vibration measurements are available was studied. The set of modal measurements may be incomplete and only parts of specific modes may have been measured. Measurements are assumed to be exact. Is was assumed in the problem definitions that large differences may exist between measured response and predictions using IFEM. Differences as large as 100 % were used in the numerical application.

A perturbation based methodology was developed to solve the problem of updating the FEM without trial and error and without additional FE Analysis. Both mass and stiffness matrices were updated to produce an updated (correlated) finite element model (UFEM). Correlation variables were stiffness, mass, or geometric properties of the structural element or group of elements, and not individual terms in the stiffness and mass matrices. Thus, the mass and stiffness matrix distributions of the updated model correspond to the real structure.

References

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