TEST/ANALYSIS CORRELATION FOR MULTIPLE CONFIGURATIONS

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ABSTRACT

This paper extends the applicability of an existing sensitivity-based test/analysis correlation method, which permits the refinement of a finite element model by correlating with dynamic test results, to permit the simultaneous correlation with test results of multiple configurations. It also demonstrates a technique to overcome the limitation of most commercial FE programs in handling the integrated analysis task for a structure of multiple configurations in mass distributions, boundary conditions, and structural add-ons. Some promising features of this application has been revealed through a numerical example.

INTRODUCTION

Test/Analysis correlation, in particular the refinement of finite element (FE) models to accord with test results of the modeled structure, is an emerging field in the aerospace industry. Until now, most of the work has been limited to correlations with test data of a single configuration of a structure at a time [1-5]. In practice, however, test articles are often tested in different configurations (different payloads and boundary conditions, with and without add-on structures, etc.). The test data from all of these configurations are valid, and it is usually desirable to include them all in the correlation process. Correlating configurations individually runs the same risk that correlating individual modes did in the past. A change which improves one configuration may worsen the correlation of another. The obvious way to avoid this is to target the dynamic properties of the different configurations simultaneously. This results in a unique FE model which correlates with the test results of all the configurations in a weighted average sense.

In MSC/NASTRAN [6], multiple configurations are handled differently depending on what makes the configuration different. Multiple boundary conditions of most kinds are considered as multiple subcases of the same model. Multiple mass distributions, some boundary conditions (e.g., soft of rigid-element supports), and of course any structural additions, are all treated as distinct models and analyzed in separated runs. In this paper, multiple configurations of the same structure are defined in a broad sense and include any changes which involve boundary conditions, non-structural masses, and additional attached structures (provided that the latter two contain no design variables).

A typical sensitivity-based correlation algorithm requires the computation of design sensitivities for each configuration based on the corresponding analysis data. This can cause some complexities to the data base in the multiple configuration case. In order to avoid inconvenience in using the correlation procedure, an efficient and effective data structure had to be carefully designed. This paper presents an approach to effectively organize the data during the process of analyses for multiple configurations and then performs the test/analysis correlation in a very similar fashion as in the single configuration case.

In our basic correlation procedure [5,7], the design-sensitivities computation and the correlation equation were implemented as a DMAP program which largely depends upon the data base generated in the preceding analysis run. In order to correlate test/analysis results of a structure with multiple configurations, the basic correlation procedure had to be changed. The change primarily involves the reorganization of the data structure which, in turn, influences the user interaction. However, it was desirable to make the change as seamless as possible to the end user of the program. Hence the procedural enhancement has been designed and accomplished with the idea of minimizing user input and inconvenience.

METHODOLOGY

Consider the correlation equation as follows:

$$\mathbf{T}^T\mathbf{T}\Delta\mathbf{x} = \mathbf{T}^T\Delta\mathbf{y} \quad , \tag{1}$$

where **T** and $\Delta\mathbf{y}$ are the assembled sensitivity matrix and error vector, respectively, of m configurations, and $\Delta\mathbf{x}$ is the vector of design-variable changes, necessarily common to all the configurations. **T** and $\Delta\mathbf{y}$ are assembled in the forms described below. The assembled error vector takes the form of:

$$\Delta\mathbf{y} = \begin{Bmatrix} \{\Delta\mathbf{y}^1\} \\ \{\Delta\mathbf{y}^2\} \\ \vdots \\ \{\Delta\mathbf{y}^m\} \end{Bmatrix} \tag{2}$$

where each $\{\Delta\mathbf{y}^i\}$ represents the corresponding error vector of the i-th configuration. It, in turn, can be assembled in the following form:

$$\{\Delta\mathbf{y}^i\} = \begin{Bmatrix} \{\Delta\bar{\lambda}\}^i \\ --- \\ \{\Delta\mathbf{MAC}\}^i \end{Bmatrix} \tag{3}$$

It consists of the normalized eigenvalue (or frequency) error vector and the MAC (Modal Assurance Criterion [4,8]) error vector for the modes of the i-th configuration. The normalized eigenvalue error and MAC error for the j-th mode of the i-th configuration are defined, respectively, as:

$$\Delta\bar{\lambda}_j^i = \frac{\Delta\lambda_j^i}{\lambda_{tj}^i} = \frac{\lambda_{tj}^i - \lambda_{aj}^i}{\lambda_{tj}^i} \tag{4}$$

and

$$\Delta\mathrm{MAC}_j^i = 1 - \mathrm{MAC}_j^i = 1 - \left(\frac{(\boldsymbol{\varphi}_t{}^T\boldsymbol{\varphi}_a)^2}{(\boldsymbol{\varphi}_t{}^T\boldsymbol{\varphi}_t)(\boldsymbol{\varphi}_a{}^T\boldsymbol{\varphi}_a)}\right)_j^i \tag{5}$$

where the subscripts t and a represent test and analysis, respectively.

Correspondingly, the sensitivity matrix becomes:

$$\mathbf{T} = \begin{bmatrix} [\mathbf{T}^1] \\ [\mathbf{T}^2] \\ \vdots \\ [\mathbf{T}^m] \end{bmatrix} \tag{6}$$

where a typical $[\mathbf{T}^i]$ can be expressed as:

$$[\mathbf{T}^i] = \begin{bmatrix} \left[\dfrac{\partial \bar{\lambda}^i}{\partial \mathbf{x}}\right] \\ --- \\ \left[\dfrac{\partial \mathbf{MAC}^i}{\partial \mathbf{x}}\right] \end{bmatrix} \tag{7}$$

It consists of two submatrices, the sensitivity matrices of the normalized eigenvalues and of the MACs [8] of the i-th configuration, with respect to design variables **x**.

The assemblage of **T** and $\Delta\mathbf{y}$ is done in a sequential order such that all the analyses and computations of $\{\Delta\mathbf{y}^i\}$ and $[\mathbf{T}^i]$ are performed separately and sequentially. For instance, the i-th configuration is analyzed separately, followed by the computation of $\{\Delta\mathbf{y}^i\}$ and $[\mathbf{T}^i]$. After $\{\Delta\mathbf{y}^i\}$ and $[\mathbf{T}^i]$ are formed, a merging process is performed such that:

$$\mathbf{T}^I = \begin{bmatrix} [\mathbf{T}^{I-1}] \\ --- \\ [\mathbf{T}^i] \end{bmatrix} \tag{8}$$

where

$$\mathbf{T}^{I-1} = \begin{bmatrix} [\mathbf{T}^1] \\ [\mathbf{T}^2] \\ \vdots \\ [\mathbf{T}^{i-1}] \end{bmatrix} \tag{9}$$

and

$$\Delta\mathbf{y}^I = \begin{Bmatrix} \{\Delta\mathbf{y}^{I-1}\} \\ --- \\ \{\Delta\mathbf{y}^i\} \end{Bmatrix} \tag{10}$$

where

$$\Delta\mathbf{y}^{I-1} = \begin{Bmatrix} \{\Delta\mathbf{y}^1\} \\ \{\Delta\mathbf{y}^2\} \\ \vdots \\ \{\Delta\mathbf{y}^{i-1}\} \end{Bmatrix} \tag{11}$$

When the assemblage of $\mathbf{T}^M$ and $\Delta\mathbf{y}^M$ is completed (where $\mathbf{T}^M$ and $\Delta\mathbf{y}^M$ include all of the m configurations), they become:

When the assemblage of $\mathbf{T}^M$ and $\Delta\mathbf{y}^M$ is completed (where $\mathbf{T}^M$ and $\Delta\mathbf{y}^M$ include all of the m configurations), they become:

$$\mathbf{T} = \mathbf{T}^M \tag{12}$$

$$\Delta\mathbf{y} = \Delta\mathbf{y}^M \tag{13}$$

Now Eq.(1) can be solved in the usual manner.

Since the objective here is to perform the correlation based on Eq.(1), we need only to store a very small amount of data at the end of each configuration process. Eq.(1) will be performed only when the assemblage of **T** and $\Delta\mathbf{y}$ is completed. This means that the actual design changes are computed after the analysis and data processing of the last configuration. In other words, one correlation iteration consists of a configuration-by-configuration analysis and **T** and $\Delta\mathbf{y}$ assemblage process and, at the end, the computation of the design changes. The basic iterative correlation process, based on Eq.(1), remains the same as for a single configuration.

The multiple-configuration test/analysis correlation capability is implemented into a MSC/NASTRAN's DMAP program, PAREDYM [9]. PAREDYM refines FE model iteratively and determines the model that predicts natural frequencies and mode shapes of the structure to best match the test results.

In order to ease the control of the automated iteration process as well as the identification of the configuration data, changes in user input became inevitable. However, the change in user input was minimized to add only one PARAM card in the bulk data deck of MSC/NASTRAN. The new PARAM,CONFIG card has the format as shown:

```
PARAM   CONFIG   I
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where I is an integer value corresponding to the configuration identification number. The configuration numbers are ordered numbers which identify the configuration being analyzed (from 1 to m). The I for the PARAM,CONFIG should be equal to the positive configuration number when it is less than m and negative m when the configuration number is m. This tells PAREDYM not to perform the correlation computation of Eq.(1) until the last configuration is analyzed and **T** and $\Delta\mathbf{y}$ are fully assembled.

NUMERICAL EXAMPLE

A simple beam was used as a structure tested in three different configurations to demonstrate the effectiveness of the method. These three configurations are modeled by three individual finite element models as shown in Fig.1. Each model consists of 5 beam elements and 5 lumped mass elements at nodes. Principal area moments of inertia in the vertical bending plane of the three far left elements, i.e., I_1, I_2, I_3, and two far right masses, i.e., M_5 and M_6, were designated as design parameters. Their baseline values as well as their target values are listed in Table 1. The target values of the design parameters were used to generate mock test data and it is thus expected to have the parameters updated from their baseline values to these values at the convergence of the correlation process.

TABLE 1 Design Parameter Values

Design Parameter	Baseline Value	Target Value	Normalized Target Value*
I_1	4.5	3.	.66667
I_2	4.5	3.	.66667
I_3	4.5	3.	.66667
M_5	3.33333	5.	1.5
M_6	3.33333	5.	1.5

** Normalized Target Value = Target Value/Baseline Value*

In order to examine the effect of simultaneously correlating multiple sets of test/analysis data, correlation of three different cases has been performed for comparison purposes. These three cases involve correlation with different combinations of configuration:

(CASE I) Configuration 1
(CASE II) Configurations 1 and 2
(CASE III) All three configurations

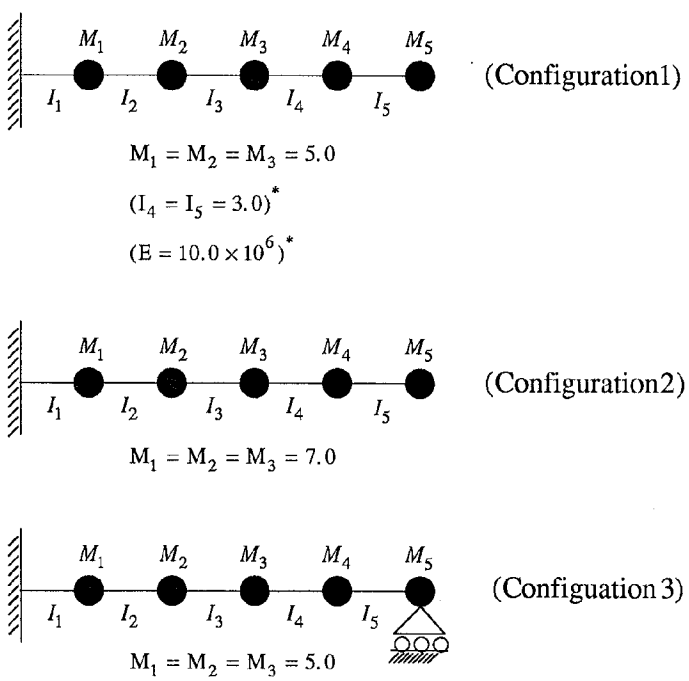

Fig. 1 Three Configurations of a Beam

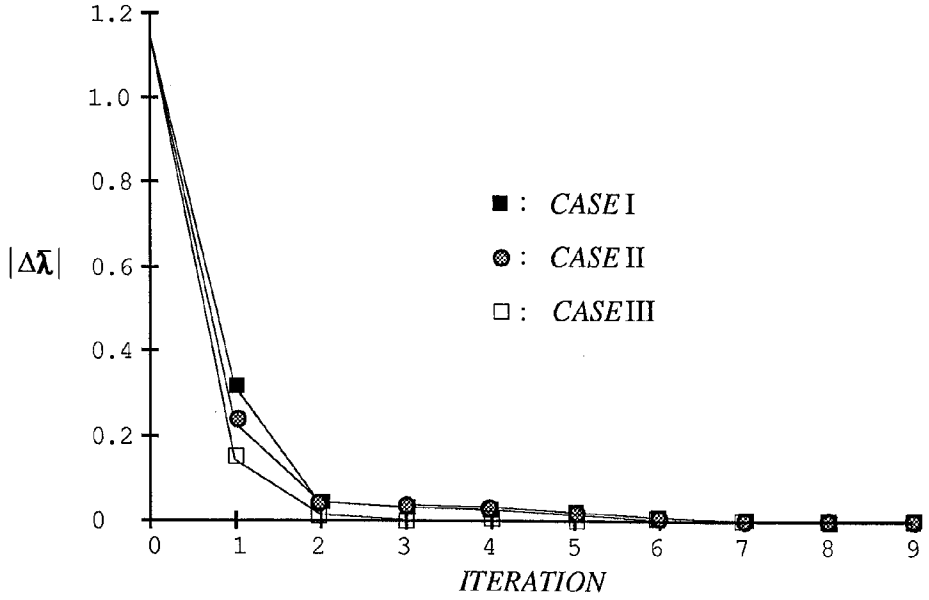

Fig. 2 Iteration History of the Configuration I Eigenvalue Error Norm

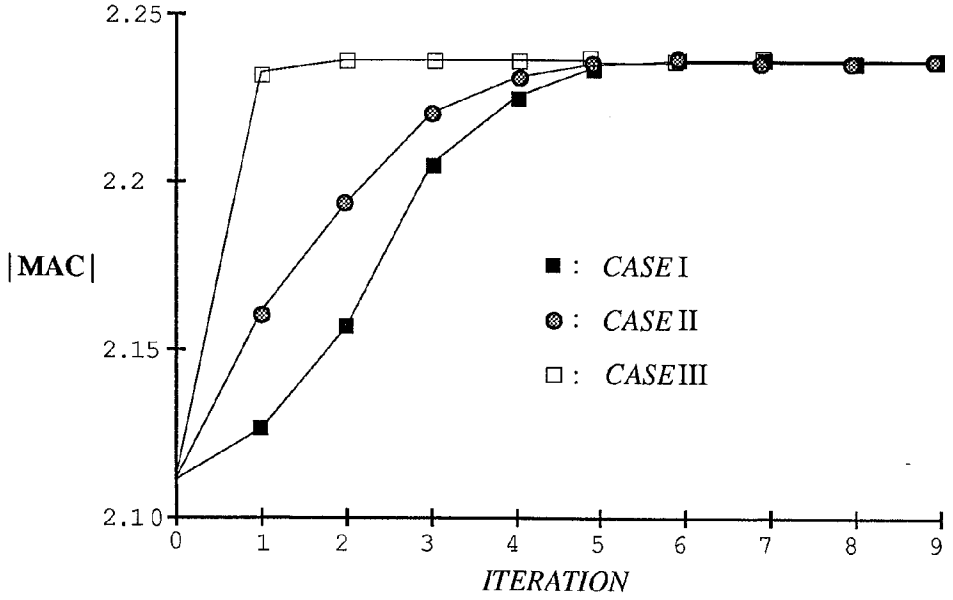

Fig. 3 Iteration History of the Configuration I MAC Norm

TABLE 2 Comparison of Correlation Results

CASE NO.	DESIGN VAR.	ITERATION 1	2	3	4	5
I	1	.244	.321	.400	.494	.609
	2	.734	.777	.738	.711	.682
	3	.717	.691	.698	.687	.676
	4	.962	.889	1.183	1.423	1.572
	5	.974	.750	.950	1.142	1.377
II	1	.283	.372	.460	.556	.646
	2	.727	.766	.722	.700	.675
	3	.715	.686	.690	.679	.670
	4	.923	1.051	1.318	1.509	1.556
	5	1.017	.901	1.077	1.261	1.453
III	1	.638	.666	.667	.667	.667
	2	.697	.668	.667	.667	.667
	3	.669	.666	.667	.667	.667
	4	1.357	1.505	1.500	1.500	1.500
	5	1.203	1.457	1.499	1.500	1.500

Natural frequencies and MAC coefficients corresponding to the lowest five modes of each configuration were selected as correlation data. Table 2 lists the iteration histories of all the normalized design variables for the three different correlation cases. It shows clearly that the performance of PAREDYM seems to be improved as more configurations are included in the correlation process. Figures 2 and 3 depict iteration histories of the norms of the normalized eigenvalue errors and the MAC values, respectively, for the Configuration 1 of all the correlation cases. The norms used here are defined as:

$$|\Delta\bar{\lambda}| = \sqrt{\sum_{i=1}^{NMODE} \Delta\lambda_i} \tag{14}$$

and

$$|\mathbf{MAC}| = \sqrt{\sum_{i=1}^{NMODE} MAC_i} \tag{15}$$

for eigenvalue errors and MAC values, respectively, of NMODE normal modes.

CONCLUSIONS

The methodology for simultaneously correlating multiple-configuration test/analysis data of the same structure has been developed in this paper. A numerical example has demonstrated the effectiveness of the method and has also revealed the benefit and the potential of correlating with multiple-configuration data.

The results of our previous work [7,9,10] had led us to believe that the correlation process becomes more difficult to converge when there are more test data to be correlated simultaneously. However, the numerical example in this paper shows an interesting result which contradicts our earlier belief. Here the more test data are available for the correlation, the better the convergence rate is.

It should also be noted that all of the correlation data have been transformed and normalized into the well-conditioned functions, $\bar{\lambda}_j^i$ and MAC_j^i. The effect of such a transformation normally provides better numerical conditioning [8].

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REFERENCES

[1] Berman, A., Flannelly, W. G., "Theory of Incomplete Models of Dynamics Structures," *AIAA Journal*, **9** (8), pp. 1481-1487, 1971.

[2] Chen, J. C., Garba, J. A., "Analytical Model Improvement Using Modal Test Results," *AIAA Journal*, **18** (6), pp. 684-690, 1980.

[3] Collins, J. D., Hart, G. C., Hasselman, T. K., "Statistical Identification of Structures," *AIAA Journal*, **12** (2), pp. 185-190, 1974.

[4] Ewins, D. J., *Modal Testing: Theory and Practice*, Research Studies Press, Taunton, Somerset, England, 1984.

[5] Ting, T., Ojalvo, I. U., "Dynamic Structural Correlation via Nonlinear Programming Techniques," *Finite Elements in Analysis and Design*, **5**, pp. 247-256, 1989.

[6] *MSC/NASTRAN User's Manual,* Version 67, Vol. II, The MacNeal Schwendler Corporation, Los Angeles, CA, 1991.

[7] Ting, T., Ojalvo, I. U., Chen, T. L. C., "A Robust Modal Correlation Procedure for Large-Scale Structures," *Proc. of the 7th IMAC*, Las Vegas, NA, vol. 1, pp. 650-656, 1989.

[8] Ting, T., T.L.C. Chen, and W. Twomey, "Correlating Mode Shapes Based on Modal Assurance Criterion," Proc. of MSC 1992 Users Conference, Los Angeles, CA, May 1992. (to appear in Journal of Finite Element in Analysis and Design)

[9] Ojalvo, I.U., T. Ting, and D. Pilon, "PAREDYM - A Parameter Refinement Computer Code for Structural Dynamic Models," The Int. J. of Analytical and Experimental Modal Analysis, Vol. 5, No. 1, Jan. 1990, pp. 43-49.

[10] Twomey, W. J., et al., "A General Method for Modifying a Finite-element Model to Correlate with Modal Test Data," *Journal of the American Helicopter Society*, Vol. **36** (3), pp. 48-58, 1991.