

VIRTUAL MASS OF FLUID IN EGG-SHAPED DIGESTERS

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ABSTRACT

The MFLUID capability in MSC/NASTRAN-WS is used to calculate the virtual mass of fluid in an egg-shaped digester tank. In earthquake response calculations of this type of tank a finite element analysis of fluid/structure interaction is needed because simplified methods, such as for cylindrical tanks, are not available. Described are observations about the performance of the QUAD4 element for fluid/structure interaction and a verification of the virtual mass matrix. Useful enhancements are proposed.

INTRODUCTION

Egg-shaped digesters, built in Germany for several decades, are finding acceptance in the United States as wastewater treatment structures. Two such digesters were recently completed in Baltimore, Maryland, eight are under construction in Boston, Massachusetts and several are planned for Los Angeles, California. The last two locations are in areas where seismic accelerations must be considered in their design.

During an earthquake the contained fluid, up to three million gallons, exerts hydrodynamic pressure on the wall of the digester and induces significant stresses in the wall. Simplified design methods, such as used in the seismic design of cylindrical storage tanks, are not available for the design of egg-shaped digesters. Simplified representations of the virtual mass of fluid for use in finite element models of the structure of egg-shaped digesters give satisfactory results for tanks with thick walls, such as those constructed of prestressed concrete. However, virtual mass representations that give acceptable results for tanks with thin walls, such as those constructed of steel, could not be easily found. Hence, the fluid/structure interaction in a steel egg-shaped digester was computed using the virtual mass capability of MSC/NASTRAN-WS. The lack of consideration of gravitational effects in the virtual mass approach is not a limitation since sloshing in a full-contents egg-shaped digester is negligible.

Described are observations about the performance of the QUAD4 element in our fluid/structure interaction computations and a verification of the virtual mass matrix.

PROBLEM DEFINITION

Considered for analysis is the egg-shaped digester and its support shown in Figure 1. The tank is approximately 138 ft tall and has a maximum diameter of 84 ft. The tank wall is of steel with thicknesses varying from 3/8 in. to 3-3/16 in. The tank is supported by a conical steel skirt which in turn is supported by a cylindrical concrete ring wall. The steel tank weighs approximately 850,000 lbs and its full contents weigh approximately 27,000,000 lbs.

Because of symmetry, only one quarter of the fluid/structure system as cut by two mutually perpendicular vertical planes is included in the model. The portion of the fluid/structure system excluded from the model is represented by appropriate boundary conditions at the planes of symmetry. Since the interest here is in the lateral response of the digester,

symmetry conditions are enforced at the plane that aligns with the direction of the seismic acceleration and antisymmetry conditions at the plane normal to the acceleration.

The tank and its support structures are modeled with QUAD4 elements as shown in Figure 2. The discretization of the wetted surface and the structure has two distinct regions: a fine region near the support skirt where large discontinuity stresses are expected and a coarse region for the remainder of the tank. The elements subtend circumferential angles of 7.5 degrees and 10 degrees in the fine and coarse regions, respectively. The 7.5 degree spacing of grid points is dictated by the spacing of stiffeners in the conical support skirt. The aspect ratios of elements in the coarse region range from approximately 1.1 to 1.6 while those in the fine region are typically about 6. The ratio of element areas at the transitions from coarse to fine regions is approximately 5.

ANALYSIS

We verified the virtual mass matrix for this tank by computing the total mass acting in the direction of acceleration and the effective mass at each vertical level of grid points.

The total mass in the direction of acceleration is:

$$M_{\text{total}} = \left\{ U_a \right\}^T \left[M_{aa}^L \right] \left\{ U_a \right\}$$

where: $\left[M_{aa}^L \right]$ is the virtual mass matrix, a-set size

$\left\{ U_a \right\}$ is a column matrix with unity for the degrees of freedom coinciding with the direction of acceleration and zeroes elsewhere, a-set size

The effective mass at each level of grid points is obtained as outlined below. Compute

$$\left\{ M_a^1 \right\} = \left[M_{aa}^L \right] \left\{ U_a \right\}$$

Diagonalize $\left\{ M_a^1 \right\}$ and call the resulting matrix $\left[M_{aa}^2 \right]$. Then compute

$$\left\{ M_a^3 \right\} = \left[M_{aa}^2 \right] \left\{ U_a \right\}$$

Use a spread sheet to sum the elements of $\left\{ M_a^3 \right\}$ at each level of grid points. These sums are the effective masses at the grid point levels.

Since MSC/NASTRAN-WS does not output the needed matrices in a routine run, the following DMAP Alter was used in Solution 63 for acceleration in the T1 direction.

```
ALTER 748
MATGEN      /UXX/1/NOGSET $
DIAGONAL    UXX/DIAGXX $
MATMOD      DIAGXX,,,,/DIAGYY,/3/23456 $
UPARTN      USET,DIAGYY/UYU,UZZ,,/G/A/0/1 $
SMPYAD      UYU,MLAA,UYU,,,/MTOT/3////1 $
MATPRN      MTOT,,,,/ $
MPYAD       MLAA,UYU,/M1 $
MATMOD      M1,,,,/M2,/28 $
MPYAD       M2,UYU,/M3 $
MATGPR      GPL,USET,SIL,M3//A/A $
```

Note that the user's manual [1] describes Option 3 of MATMOD as applicable to g-size square or symmetric matrices. In the above alter, Option 3 produces the correct result for a g-size column matrix. To obtain the total mass acting in the T2 direction, when the acceleration is in the T1 direction, the third line of the DMAP Alter reads

```
MATMOD      DIAGXX,,,,/DIAGYY,/3/13456 $
```

However this produces a null DIAGYY. This can be circumvented by generating two column matrices and subtracting one from the other. Alternatively, the order of the DIAGONAL and MATMOD statements could be reversed.

DISCUSSION

The total virtual mass, corresponding to lateral acceleration, computed with the DMAP Alter was found to be 99% of the value hand calculated from the weight of the contents of the tank. The distribution of the virtual mass with elevation, obtained with the DMAP Alter and spread sheet as described above, is shown in Figure 3a. There the virtual mass value is normalized with respect to the mass of fluid in a conical frustrum centered on a level of grid points. The significant feature of the plot is that for most of the coarse region the ratio is nearly 1.0 but is significantly less than 1.0 in the fine region. The shape of the plot near the fine region suggests that the reason for the discrepancy is the large ratio of element areas, approximately 5, at the boundary between the coarse and fine regions. A contributing factor to the discrepancy could be the large aspect ratios, approximately 6, of elements in the fine region. Variation of virtual mass with aspect ratio is reported in Ref. [2].

The wetted elements in the fine region were replaced by two tiers of coarser elements subtending 7.5 degrees of the circumference. The ratio of element areas at the former boundary between the fine and coarse regions is now approximately 1.3. Also the aspect ratio of the replacing elements is approximately 1.3. The accuracy is significantly improved as shown in Figure 3b.

CONCLUSIONS

This verification exercise shows that the accuracy in calculating the virtual mass of fluid in an egg-shaped tank can deteriorate if the model has adjacent wetted elements with significantly different areas and, possibly, if the wetted elements have large aspect ratios.

The total virtual mass acting in each of the three coordinate directions is useful for checking the goodness of the model, but, unfortunately, it is not computed by MSC/NASTRAN-WS standard rigid formats. This check is similar to checking the resultant forces and moments on a statics model using the OLOAD summary.

Other desirable enhancements of the virtual mass method of fluid/structure interaction implemented in MSC/NASTRAN-WS include the following:

1. Better documentation is needed. The underlying theory of the method, now written up in internal MSC reports and memoranda [3,4,5,6] should be included in the Theoretical Manual [7].
2. A problem with a known solution should be included in the Verification Problem Manual [8].
3. The geometry limitations of the wetted elements, such as aspect ratio and the uniformity of element sizes, should be indicated. Apparently the performance of the QUAD4 element in virtual mass calculations is not as robust as it is in elastic modeling.

REFERENCES

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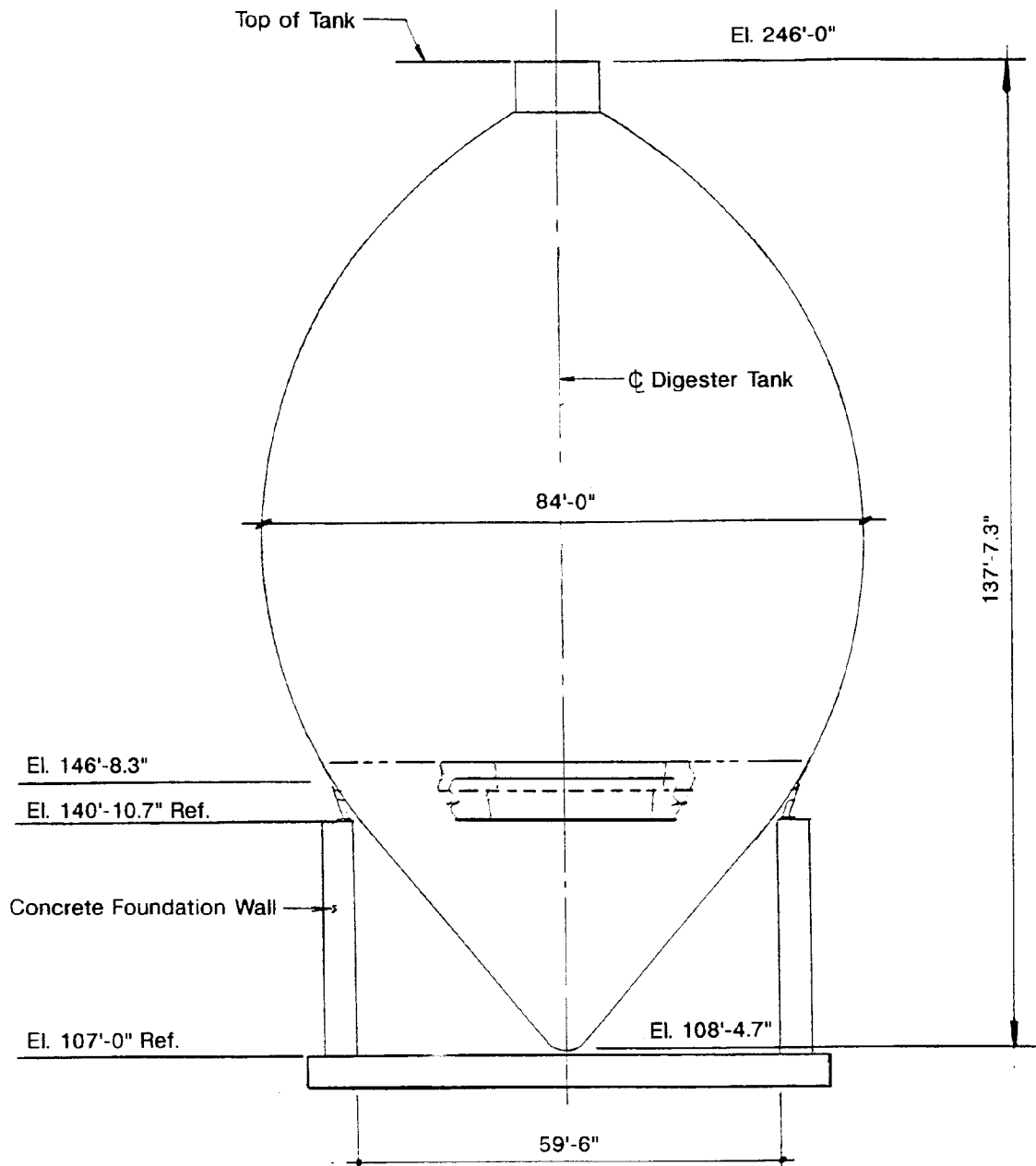


FIGURE 1 - GEOMETRY OF DIGESTER TANK AND SUPPORT

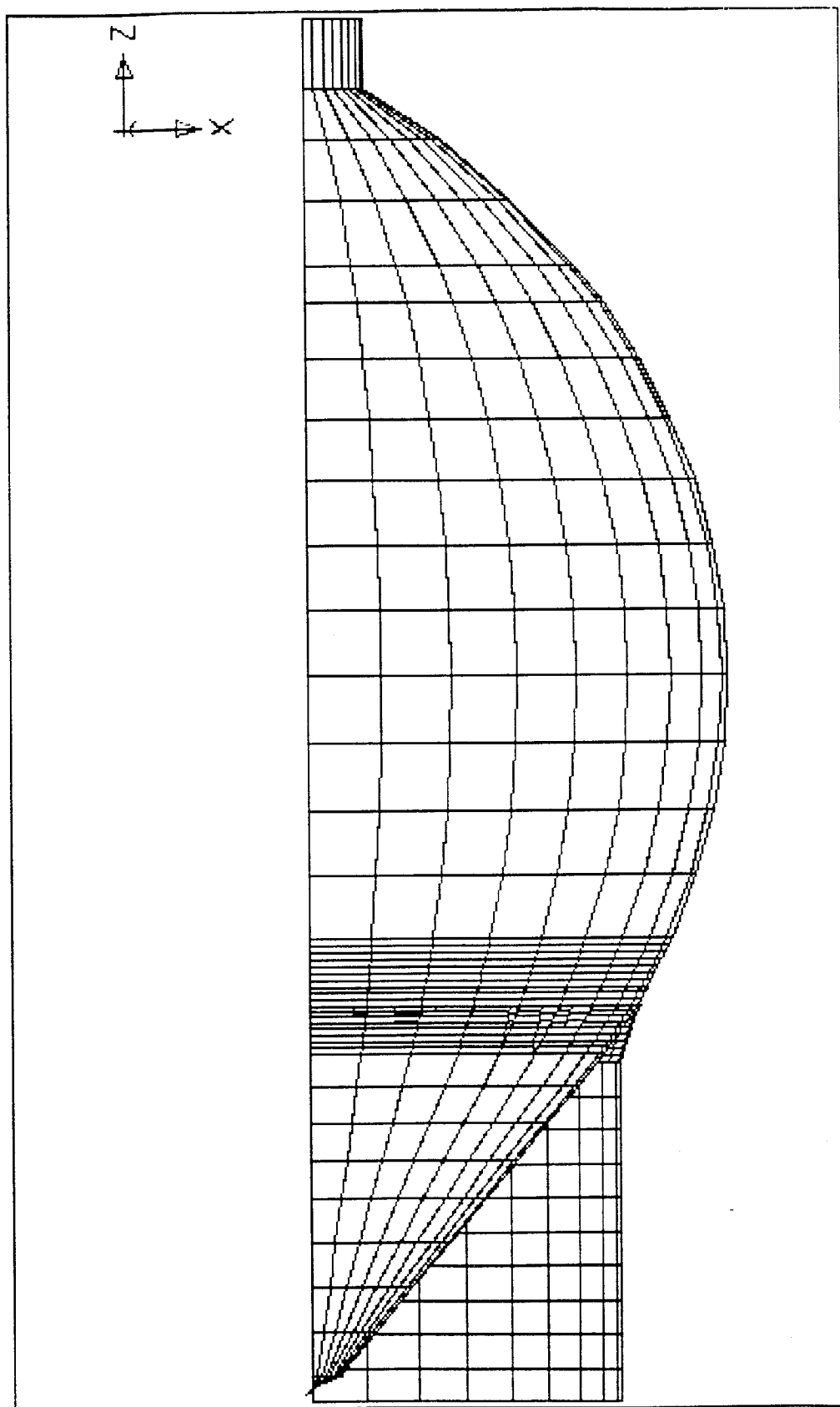
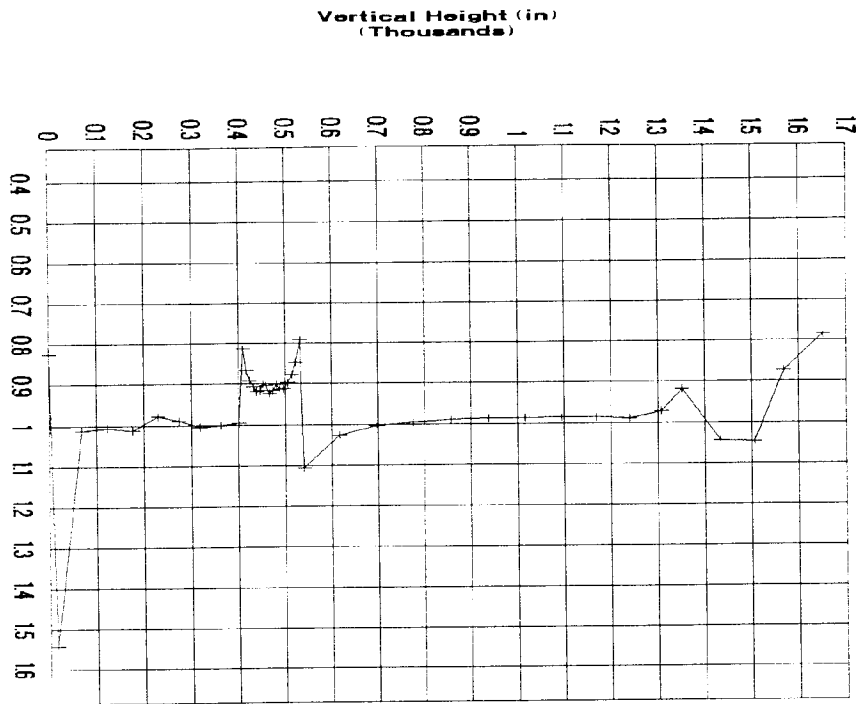


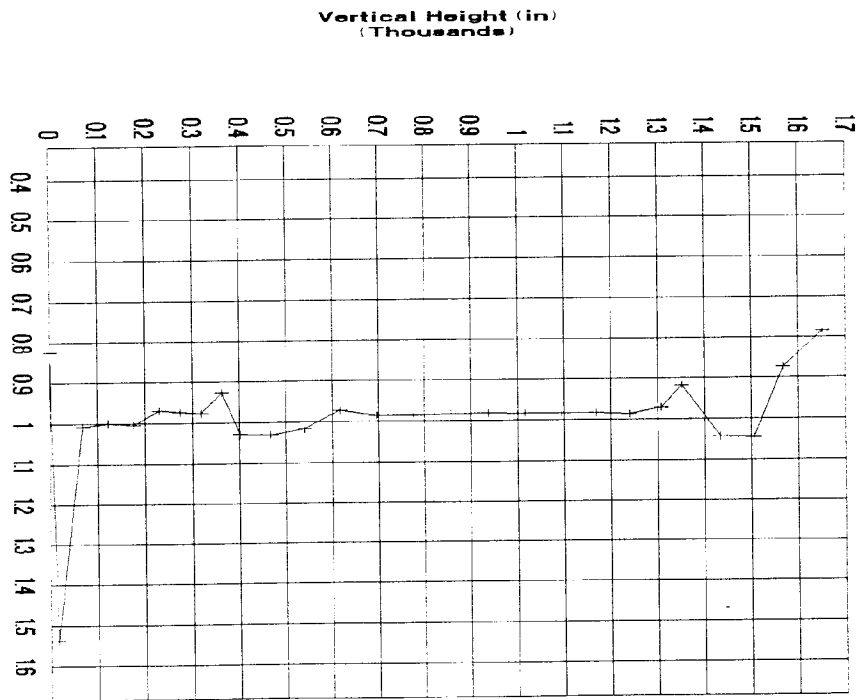
FIGURE 2 - FINITE ELEMENT MODEL OF DIGESTER TANK AND SUPPORT

Fine Mesh



(a)

Coarse Mesh



(b)

FIGURE 3 - DISTRIBUTION OF VIRTUAL MASS WITH ELEVATION