Acoustic Sensitivity Analysis Using Boundary Elements and Structural Dynamic Response

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Abstract

Structural/acoustic sensitivity analysis can provide information on how structural design modifications can affect the noise emitted from vibrating structures. MSC/NASTRAN SOL108 and SOL11 can be used to compute structural dynamic frequency response sensitivities. This information can be coupled with Boundary Element (BE) results to derive the acoustic sensitivity with respect to structural design variables.

In this work two BE formulations, the Direct and Indirect, have been coupled to MSC/NASTRAN design sensitivity results to compute the acoustic sensitivity with respect to structural design variables. The theoretical background of the coupling algorithm is presented along with some examples of noise prediction in interior and exterior noise radiation.

I. Introduction

In MSC/NASTRAN an acoustics analysis and an acoustics sensitivity capability based on the acoustic finite element method is available. In this paper, the boundary element method is used to compute the noise radiated from a vibrating structure, or the noise generated inside an acoustic cavity, and the acoustic sensitivities are computed using the boundary element method. An acoustic boundary element analysis is a two step process. During the first step the vibration of the structure which generates the noise must be computed. MSC/NASTRAN [1] can be used to compute the forced frequency response. In the second step the computed vibration is used to generate the velocity boundary conditions for the acoustic analysis. The boundary element method [2] can be used to compute the emitted noise. There are two major advantages in using boundary elements for acoustic analysis:

(i) The method is applicable to both interior acoustics and radiation problems.

(ii) The modeling effort to generate a boundary element model using surface elements is significantly less then creating a solid finite element model.

This procedure can be used to predict the acoustic behavior of structural components during the design stage. The next step of this CAE process is to identify which structural modifications will reduce the emitted noise. A structural/acoustic sensitivity analysis can supply this information.

Similar to the acoustic analysis, a structural/acoustic sensitivity analysis is a two step process. During the first step the acoustic sensitivity, i.e. the information on how the acoustic response at a data recovery point will change with respect to the normal velocity boundary condition, is computed. Computation of the acoustic sensitivity is performed by the acoustic boundary element analysis software [3]. The structural vibration sensitivity can be computed using SOL108 or SOL111 in MSC/NASTRAN [4]. The acoustic sensitivity and the structural vibration sensitivity can be combined to show how the acoustic response at the data recovery point will change with respect to a change in a structural design variable. This comprises the structural/acoustic sensitivity.

In this paper the algorithm used to compute the acoustic sensitivity is described. Two boundary element formulations are available for acoustic analysis, namely the direct and the indirect. Algorithms for computing the acoustic sensitivities have been developed for both formulations. The results are combined with structural vibration sensitivities computed by SOL108 or SOL111 of MSC/NASTRAN. The final product is the structural/acoustic sensitivity. Two examples are presented: one for radiation and one for interior acoustics analysis.

II. Acoustic Sensitivity

The acoustic sensitivity indicates how the acoustic response at a data recovery point will change with respect to the normal velocity boundary condition [5], [6]. In the direct boundary element method, the primary variables are acoustic pressures and acoustic velocities on the boundary element model. The acoustic matrix equation is:

$$[A]\{p\} - [B]\{u_n\} = \{0\} \tag{1}$$

where $\{p\}$ = vector of acoustic pressure on the boundary, $\{u_n\}$ = vector of normal velocities on the boundary, [A] and [B] = acoustic matrices which depend on the geometry of the model and the frequency of the analysis. Once $\{p\}$ and $\{u_n\}$ have been computed the acoustic pressure at any data recovery point is

$$p_{dr} = \{A_{dr}\}^T\{p\} - \{B_{dr}\}^T\{u_n\} \tag{2}$$

where subscript "dr" indicates a data recovery point and vectors $\{A_{dr}\}$ and $\{B_{dr}\}$ depend on the geometry of the problem, the location of the data recovery point and the frequency of the analysis. The acoustic sensitivity for a data recovery point is defined as $\left\{\frac{\partial p_{dr}}{\partial u_{ni}}\right\}$ which describes how the acoustic pressure at the data recovery point p_{dr} will change with respect to the u_{ni} normal velocity at the i^{th} node of the model. In order to compute $\left\{\frac{\partial p_{dr}}{\partial u_{ni}}\right\}$ we first differentiate equation (1) with respect to u_{ni}. This results in

$$[A]\frac{\partial\{p\}}{\partial u_{ni}} - [B]\frac{\partial\{u_n\}}{\partial u_{ni}} = \{0\} \quad \Rightarrow$$

$$\frac{\partial\{p\}}{\partial u_{ni}} = [A]^{-1}[B]\{I_i\} \tag{3}$$

where $\{I_i\}$ = unit vector with only one non-zero component in the i^{th} position, and $\frac{\partial\{p\}}{\partial u_{ni}}$ = information how the acoustic pressure on the boundary will change with respect to changes in the normal velocity boundary condition. For the data recovery point equation (2) becomes:

$$\left\{\frac{\partial p_{dr}}{\partial u_{ni}}\right\}^T = \{A_{dr}\}^T[A]^{-1}[B] - \{B_{dr}\}^T \tag{4}$$

The indirect method models the acoustic medium on both sides of the boundary element model. This is achieved by using the difference of the acoustic pressure and the acoustic velocity between each side of the boundary element model as primary variables. The derivation of $\left\{\frac{\partial p_{dr}}{\partial u_{ni}}\right\}$ for the indirect method is similar to that of the direct method. It includes the boundary element matrices and vectors of the formulation. A key observation is that $\left\{\frac{\partial p_{dr}}{\partial u_{ni}}\right\}$ depends on the geometry of the problem and the frequency of analysis only. It is independent of the velocity boundary conditions. These computations have been implemented for both the direct and indirect boundary element methods in COMET/Acoustics [3].

III Structural Dynamic Sensitivity

The information on how the structural vibration will be modified with respect to a change in a structural design variable $\left\{\frac{\partial u}{\partial h}\right\}$ can be computed by MSC/NASTRAN. The direct approach is available in SOL108 and the modal approach in SOL111. The computation for the direct is based on equation [4]:

$$\left[-\omega^2[M]+i\,\omega[B]+[K]\right]\left\{\frac{\partial u}{\partial h}\right\}=-\left[-\omega^2\frac{\partial[M]}{\partial h}+i\,\omega\frac{\partial[B]}{\partial h}+\frac{\partial[K]}{\partial h}\right]\{u\} \tag{5}$$

where [M] = mass matrix, [B] = damping matrix, and [K] = stiffness matrix. The value of $\left\{\frac{\partial u}{\partial h}\right\}$ is a function of the mass, damping, and stiffness matrices and their derivatives. $\left\{\frac{\partial u}{\partial h}\right\}$ also depends on the applied load and on the baseline structural dynamic response. The modal approach available in SOL111 makes the assumption that the modal basis does not change with the structural modifications, and this assumption introduces an approximation in the solution.

IV Structural/Acoustic Sensitivity

The acoustic sensitivity results $\left\{\frac{\partial p_{dr}}{\partial u_{ni}}\right\}$ and the structural sensitivity results $\left\{\frac{\partial u}{\partial h}\right\}$ can be combined to derive the structural/acoustic sensitivity $\frac{\partial p_{dr}}{\partial h}$:

$$\frac{\partial p_{dr}}{\partial h} = \sum_{i=1}^{I} \frac{\partial p_{dr}}{\partial u_{ni}} \frac{\partial u_{ni}}{\partial h} \tag{6}$$

The sensitivity of the normal velocity with respect to a structural design variable is computed as:

$$\frac{\partial u_{ne}}{\partial h} = \frac{\int_{\Omega} \frac{\partial \vec{u}}{\partial h} \hat{n} \, d\Omega}{A} \tag{7}$$

where Ω = domain of the element, A = area of the element, u_{ne} = element normal velocity, and $\hat{n}$ = unit normal.

In order to compute the structural/acoustic sensitivity, three groups of data are required from the MSC/NASTRAN structural design sensitivity analysis. The first group of data is the locations (coordinates) of the nodes of the structural FEA model used in computing the structural sensitivities. The structural node coordinates are obtained from a MSC/NASTRAN bulk data file. The second group of data is the design sensitivity coefficient matrix, DSCM2, generated by the MSC/NASTRAN structural design sensitivity analysis. DSCM2 is obtained from a binary file created using the OUTPUT 4 format. The bulk data entry which specifies that DSCM2 should be output to a binary file is

PARAM,OPTEXIT,-4

The third group of data is the correlation table from the MSC/NASTRAN structural design sensitivity analysis. The correlation table gives the correlation between the column positions of DSCM2 and the design responses (which in our case are velocities at the nodes of the FEA model). The correlation table is obtained from the F06 file generated by the NASTRAN structural design sensitivity analysis.

Two groups of data are required from the COMET/Acoustics acoustic sensitivity analysis. The first group is the node coordinate and element connectivity data of the acoustic boundary element model used in the acoustic sensitivity analysis. This acoustic model data is normally obtained from the COMET input file that was used to perform the acoustic sensitivity analysis. The second group of data is the acoustic sensitivity results from the analysis. The acoustic sensitivities are obtained from a COMET results file.

The structural/acoustic sensitivity is a complex number and it needs to be interpreted appropriately. The baseline acoustic response (pressure) at a data recovery point can be written as $p_{dr} = p_{drR} + i\, p_{drI}$. The structural/acoustic sensitivity $\frac{\partial p_{dr}}{\partial h} = A + iB$ can be associated with the change in the acoustic response:

$$dp_{drR} = A\, dh$$
$$dp_{drI} = B\, dh \tag{8}$$

Based on the baseline acoustic response p_{dr}, it can be determined if a positive or a negative change must be implemented in the real and imaginary parts of the acoustic response. Then from the value of the sensitivities and equation (8) it can be determined if an increase $dh>0$ or a decrease $dh<0$ in the value of the design variable will reduce the noise.

V. Results

A structural/acoustic sensitivity analysis is performed for two example applications. The results demonstrate how the sensitivity computations can be used to reduce the noise, and the results are verified through reanalysis. The first example is noise reduction for a radiation problem. The indirect boundary element method was used for noise and acoustic sensitivity computations. The geometry is depicted in Figure 1. A square plate simply supported on all four edges is excited at frequency of 100Hz by a point force applied in the middle of the plate. The plate comprises the front side of a box with all other five sides rigid (Figure 1). A plane data recovery point mesh is defined in front of the vibrating plate. Results for the noise computed on the data recovery plane are displayed in Figure 2. Four zones of elements are defined on the plate. The thickness of each zone is used as a design variable. The structural/acoustic sensitivities were computed for the data recovery point in the middle of the data recovery mesh (baseline response, $p_{dr} = 0.126 - i\, 0.118$). This point was selected because it exhibits the highest response. The computed structural/acoustic sensitivities are summarized in Figure 1. It can be observed that the highest sensitivity is computed for the thickness of the outer zone of elements. Furthermore, by taking into account equation (8) and the baseline acoustic response, it can be determined that the reduction in the thickness of the outer zone of elements will reduce the radiated noise. A reanalysis was performed to verify the sensitivity results. The thickness of the elements in the outer zone is reduced and the vibration and the acoustic analyses are repeated. The acoustic results for the modified design are displayed in Figure 3. As it can be observed, there is a noise reduction according to the results of the sensitivity computations.

The second example is associated with an interior noise reduction problem (generic car interior). The direct boundary element method is used for noise and acoustic sensitivity computations. The acoustic model is depicted in Figure 4. Two data recovery planes are defined inside the acoustic cavity. Two antisymmetric forces are applied on the structural model. Using the structural vibration results as input for the acoustic analysis, the noise was computed for the baseline design (Figure 5). Three structural design variables were identified, corresponding to the thickness of three panels representing the front, the rear windshield, and the roof panel. Two interior data recovery points (points 23 and 7) were selected for sensitivity computations (Figure 5). The baseline response is $p_{23} = 0.535 - i\ 0.1178\ 10^{-3}$, $p_7 = -0.5287 + i\ 0.2\ 10^{-3}$. The results from the structural/acoustic sensitivity analysis are summarized in Figure 4. The results indicate that the highest noise reduction at the selected data recovery points can be achieved by reducing the thickness of the "roof" panel. This structural modification is implemented in the finite element model and a reanalysis is performed. The acoustic results are presented in Figure 6. Noise reduction is achieved according to the sensitivity computations. In fact, the noise reduction for point 23 is higher than the reduction for point 7. This was anticipated since the sensitivity for point 23 is higher than the sensitivity for point 7.

VI. Summary

In this paper an approach of performing structural/acoustic sensitivity analysis is presented. SOL108 of MSC/NASTRAN is utilized for computing the structural vibration sensitivities. The direct and the indirect boundary element methods are used to compute the noise and perform an acoustic sensitivity analysis. The structural/acoustic sensitivity can be computed and the results can be used to determine the structural modifications which will reduce the noise most effectively. This methodology can be applied both to radiation problems and interior acoustics. Two examples are presented in order to demonstrate the new capabilities. The acoustic sensitivity algorithms, and the computation of the structural/acoustic sensitivity by combining structural and acoustic results have been implemented under this work into COMET/Acoustics.

REFERENCES

[1] MSC/NASTRAN User's Manual, Version 67, The MacNeal-Schwendler Corporation, Los Angeles, CA, 1992.

[2] R. D. Ciskowski, and C. A. Brebbia, Boundary Element Methods in Acoustics, Computational Mechanics Publications, 1991.

[3] COMET/Acoustics User's Manual, Version 2.0, Automated Analysis Corporation, Ann Arbor, MI, 1994.

[4] G. J. Moore, MSC/NASTRAN Design Sensitivity and Optimization User's Guide, The MacNeal-Schwendler Corporation, Los Angeles, CA, 1992.

[5] N. Vlahopoulos, C. Dagg, and R. J. Bernhard, "Computation of Sensitivities for Passive Noise Control," Noise Con 93, Williamsburg, Virginia, pp. 529 - 534, 1993.

[6] D. C. Smith, and R. J. Bernhard, "Computation of Acoustic Shape Design Sensitivity Using a Boundary Element Method," Numerical Techniques in Acoustic Radiation, Winter Annual Meeting of ASME, San Francisco, California, pp. 109 - 116, 1989.

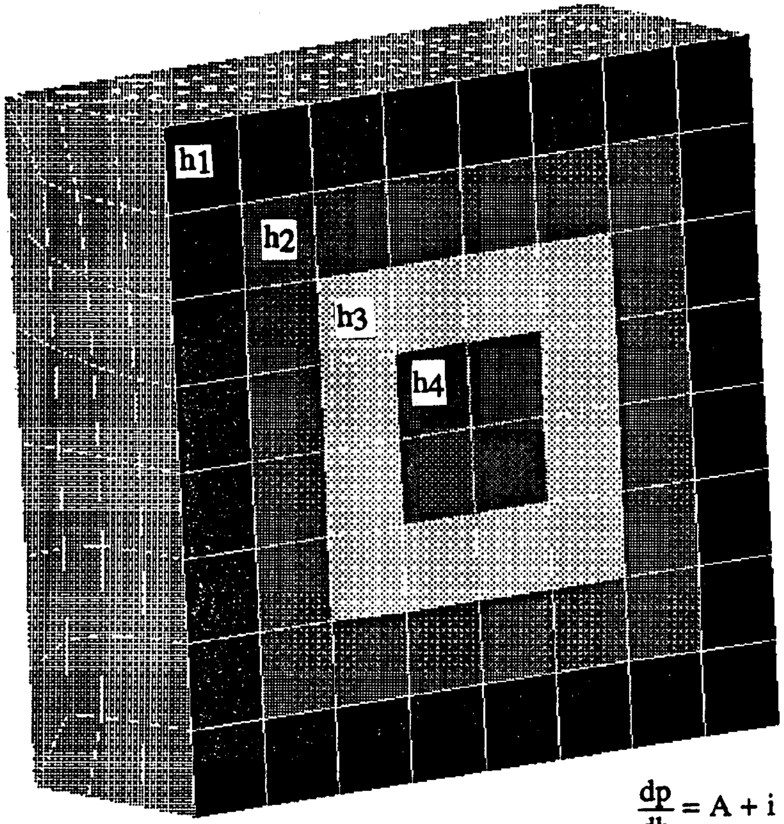
$$p_f = 0.126 - i\,0.118$$

$$\frac{\partial p_f}{\partial h_1} = \underline{22.6} - i\,\underline{19.9}$$

$$\frac{\partial p_f}{\partial h_2} = -0.93 + i\,0.79$$

$$\frac{\partial p_f}{\partial h_3} = -4.26 + i\,3.93$$

$$\frac{\partial p_f}{\partial h_4} = -5.82 + i\,5.36$$



$$\frac{dp}{dh} = A + i\,B \Rightarrow dp_r = A\,dh \;\&\; dp_i = B\,dh$$

$$\begin{matrix} dp_r < 0 \\ dp_i > 0 \end{matrix} \Rightarrow dh_1 < 0$$

Figure 1. Acoustic Model for Plate Radiation Problem, and Structural/Acoustic Sensitivity Results (Indirect Method)

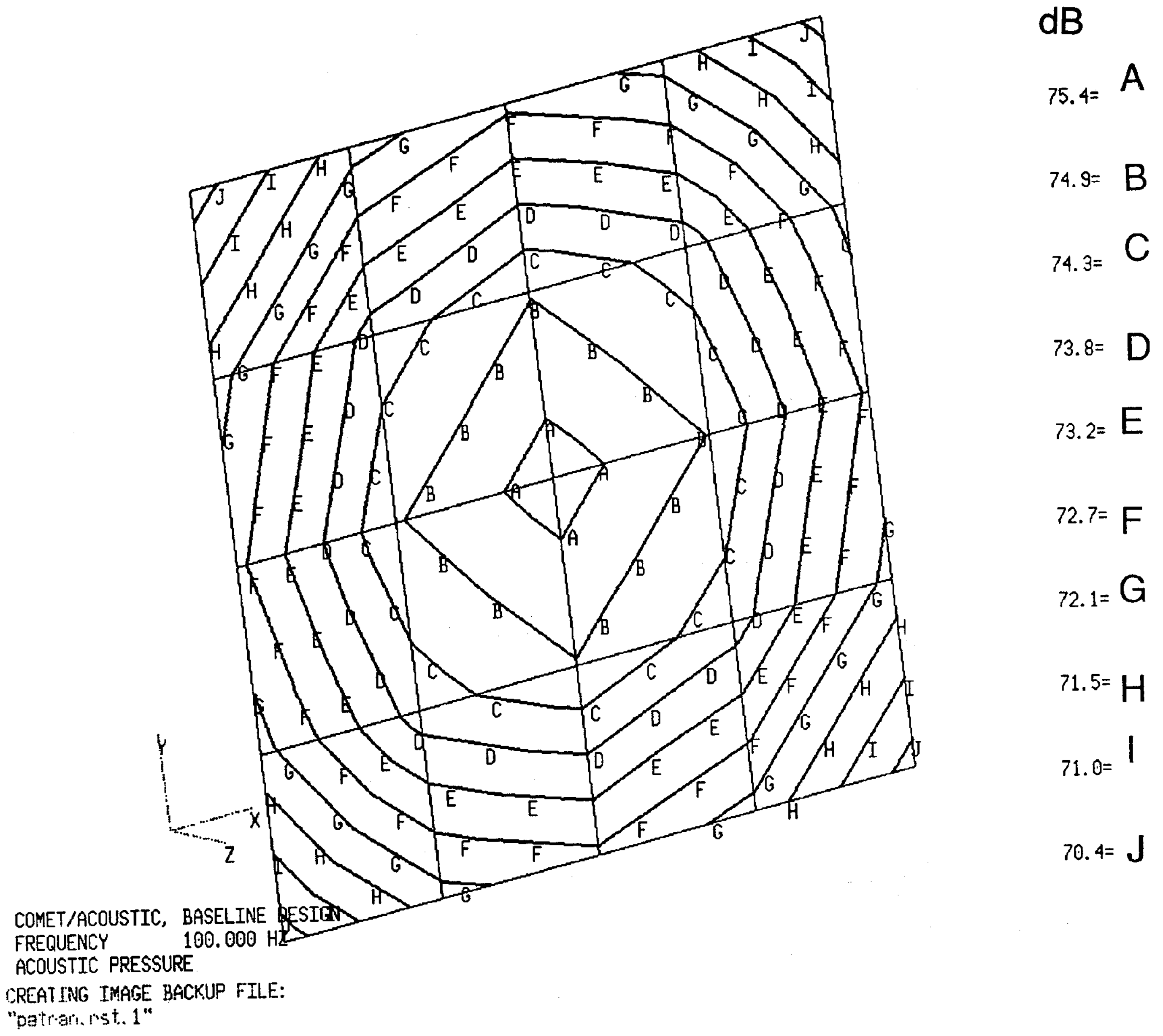


Figure 2. Acoustic Results for Baseline Design

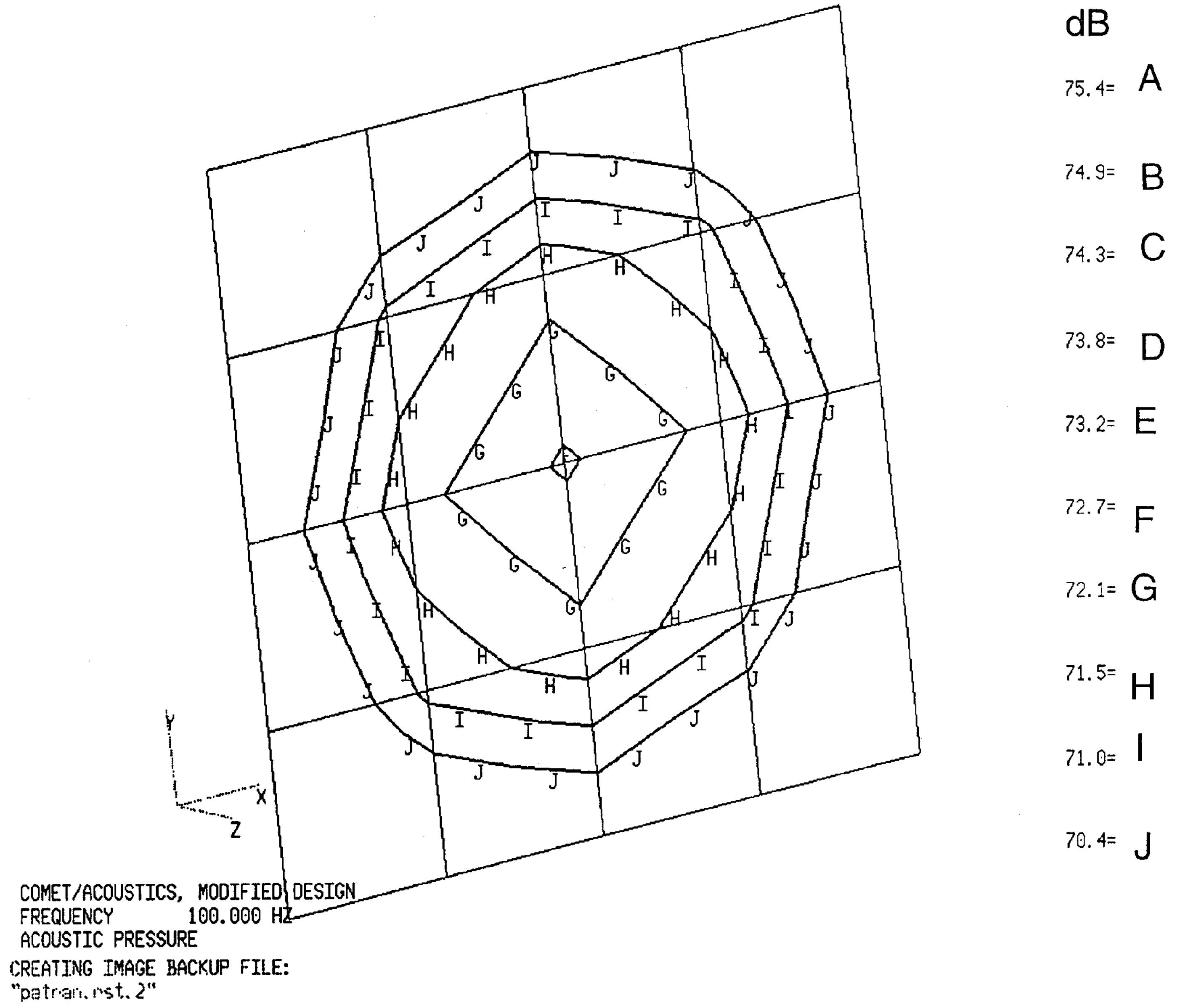


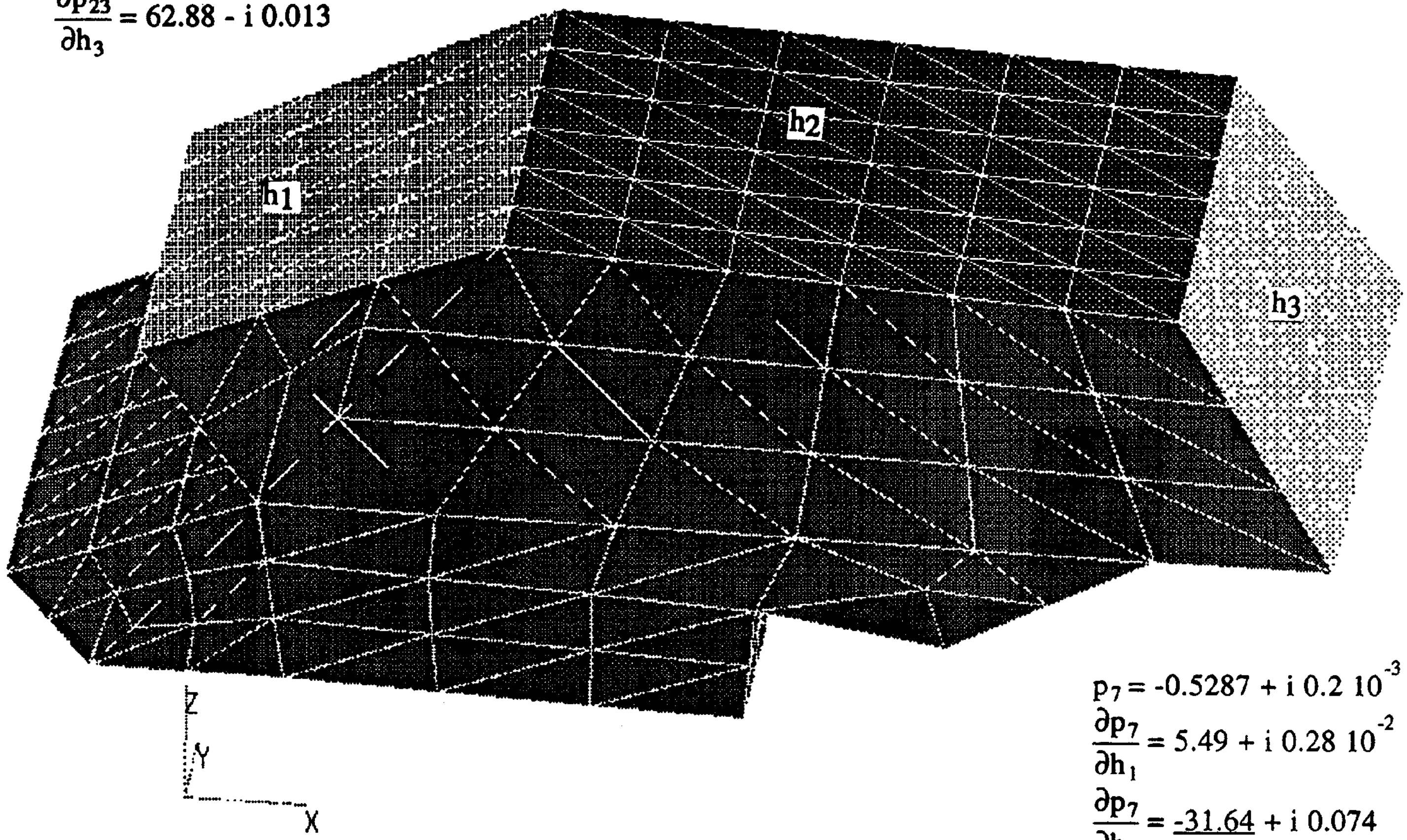
Figure 3. Acoustic Results for Modified Design

$p_{23} = 0.535 - i\,0.1178\ 10^{-3}$

$\frac{\partial p_{23}}{\partial h_1} = 32.15 - i\,0.48\ 10^{-2}$

$\frac{\partial p_{23}}{\partial h_2} = \underline{805.11} - i\,0.187$

$\frac{\partial p_{23}}{\partial h_3} = 62.88 - i\,0.013$



$p_7 = -0.5287 + i\,0.2\ 10^{-3}$

$\frac{\partial p_7}{\partial h_1} = 5.49 + i\,0.28\ 10^{-2}$

$\frac{\partial p_7}{\partial h_2} = \underline{-31.64} + i\,0.074$

$\frac{\partial p_7}{\partial h_3} = 5.18 + i\,0.43\ 10^{-2}$

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Figure 4. Generic Acoustic Car Model, and Structural/Acoustic Sensitivity Results (Direct Method)

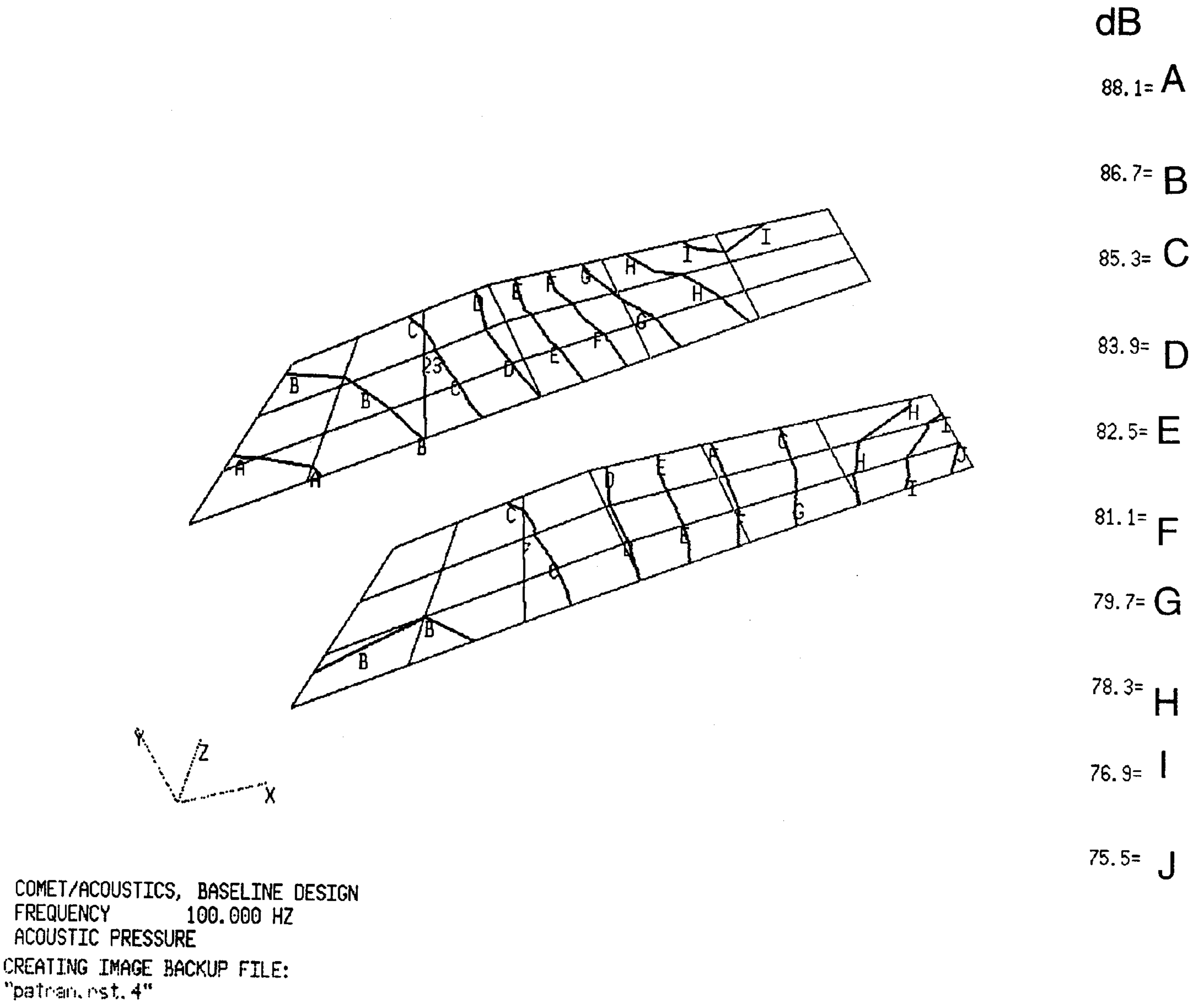
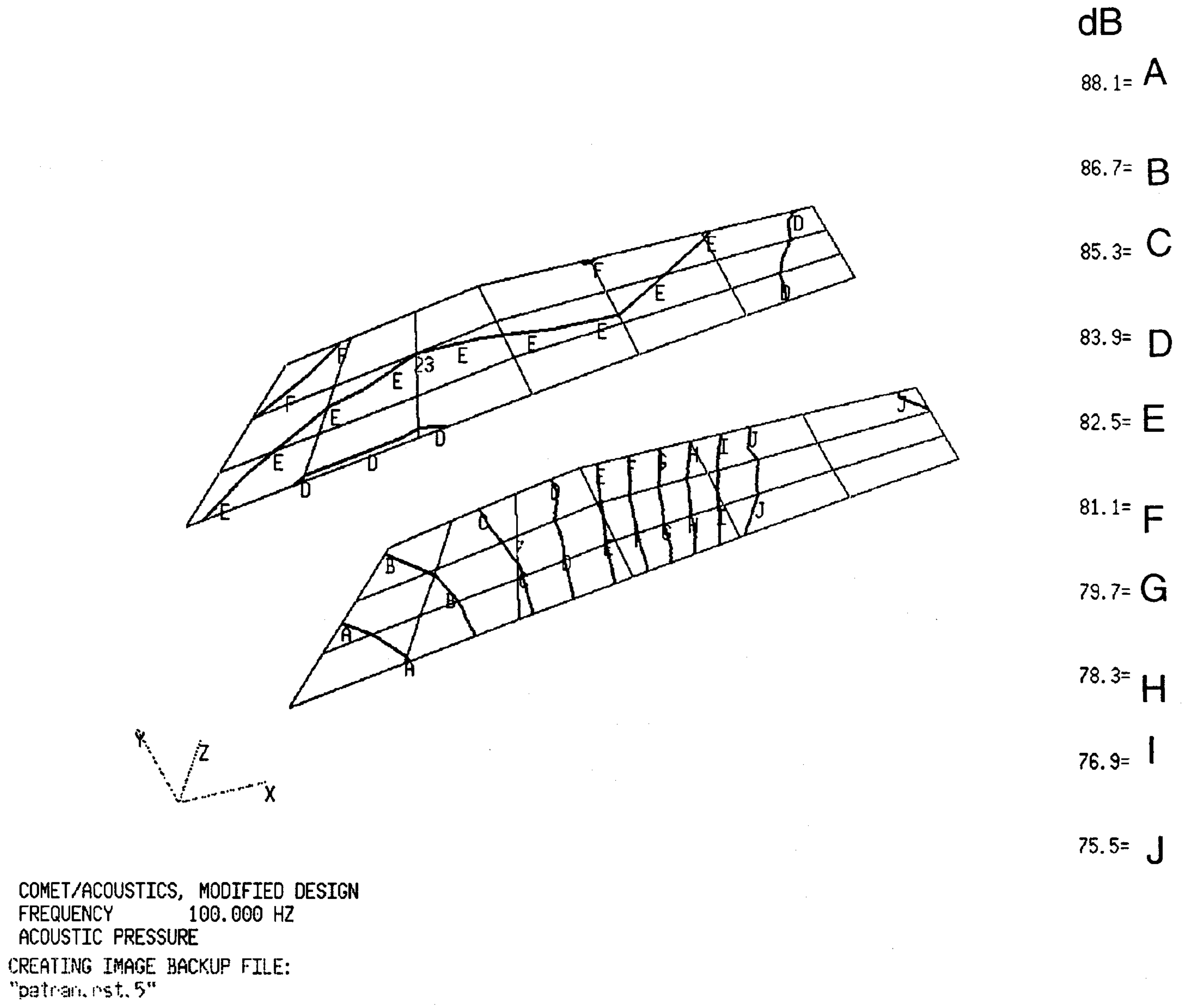


Figure 5. Acoustic Results for Baseline Design



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COMET/ACOUSTICS, MODIFIED DESIGN
FREQUENCY       100.000 HZ
ACOUSTIC PRESSURE
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Figure 6. Acoustic Results for Modified Design