SHAPE PARAMETERIZATION AND OPTIMIZATION USING THE BOUNDARY SHAPES CONCEPT

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ABSTRACT

The concept of boundary shapes for parameterization of design boundaries in shape optimization is introduced. In this concept the boundary definition of the finite element geometry is designed. In addition the requirement for use of shape basis vectors with low mesh distortion properties is ideally satisfied by interpolating domain point sensitivity by applying the boundary shapes or forms as enforced displacements and updating the shape basis vectors at each design cycle based on current geometry. This results in smooth mesh changes and minimizes the need for intermediate remeshing for small to moderate design changes. To support ease of use and provide flexibility in the prescription of boundary shapes the concept of auxiliary boundary models as been incorporated in MSC/NASTRAN as an integral part of the analysis model. The boundary shapes are generated with auxiliary boundary model analysis by exploiting available options in static analysis of applied loadings and multiple boundary conditions. Basic design examples demonstrating the power of the boundary shape approach are presented.

INTRODUCTION

The natural shape approach of Reference [1] has been adopted in MSC/NASTRAN for shape optimization. In this technique, a new shape is determined by defining the change in the finite element geometry or mesh as a linear combination of shape vectors. Thus the key to this technique is to provide a good set of low mesh distortion shape basis vectors. Prior to Version 68 of MSC/NASTRAN, the user interface for defining shape basis vectors for shape sensitivity was essentially manual and lacked ease of specification and flexibility. The shape basis vectors had to specified for both the boundary and interior points of the model. Also the shape basis vectors could not be updated based on the new model geometry, resulting in frequent user intervention for remeshing and the updating of shape basis vectors.

A shape optimization task typically requires variations of a few basic primitives of the design geometry. Thus the design task is to determine the 'best' structural shape of the geometry, given an initial configuration. In this context, the task is therefore to parameterize the given design boundary rather than its definition. Another requirement is that, as the design geometry is modified, it is essential that the underlying finite element mesh of the model vary smoothly between design cycles. As noted in Reference [2], distorted elements can lead to instability in the optimization procedure if continuity of the sensitivity gradients is not maintained. These requirements are satisfied by the boundary shapes concept [3], which is based on parameterizing a design boundary by applying a boundary shape as enforced displacement. Using linear static analysis for interpolating position changes of interior (domain) points results in a minimal mesh distortion solution. The design variables are the magnitudes of the boundary shapes. The resulting deformation pattern is used as the shape basis vectors in the natural shape approach. The finite element mesh in this context serves as a interpolation model between the boundary and the domain of the model.

Chargin and Raasch [3] developed a prototype DMAP for shape optimization that pioneered the concept of a boundary model while using superelements to characterize a design surface of a structure for prescribing boundary shapes. This approach suffers from the drawbacks and limitations of superelement definition in MSC/NASTRAN. Therefore, the concept of an auxiliary boundary model [4] for parameterization of design boundaries is introduced. The auxiliary boundary model is defined as an integral part of the analysis model for modeling ease and flexibility. The boundary shapes are generated by linear static analysis of the auxiliary model and then applied as enforced displacements on the analysis model. This enables easy generation of a multitude of candidate boundary shapes using combination of applied loadings, boundary conditions and material properties. The resultant deformation are the shape basis vectors which are updated every design cycle based on current geometry. This approach satisfies the requirement for low mesh distortion shape vectors and ensures smooth mesh changes to ensure continuity of shape gradients. Reference [5] describes use of external auxiliary models, in which the shape basis vectors are generated in an auxiliary analysis. However this approach precludes updating of the shape vectors, may not necessarily minimize the need for intermediate remeshing and requires a separate analysis to generate the shape vectors.

The modeling procedure and use of auxiliary boundary models is described in this paper and the capability is demonstrated on two design examples. The issue of mesh distortion is discussed from the viewpoint of minimum intermediate remeshing and some associated enhancements to the shape optimization capability are described.

SHAPE OPTIMIZATION PROBLEM

The design task is to determine the 'optimal' structural shape of given boundaries that will minimize the objective, typically weight, while satisfying the design constraints. The task can be formulated in a non–linear mathematical programming form as:

Find the set of shape variables x, that

$$\text{minimize} \quad F(x)$$

$$\text{subject to} \quad g_i(x, u), i = 1, m$$

$$x_j^l <= x <= x_j^u, j = 1, ndv$$

where F is the design objective to be minimized or maximized, $g_i(x, u)$ the design constraints, u is the displacement solution vector and x^l and x^u are side constraints on the design variable value.

SHAPE PARAMETERIZATION

In the natural shape approach the change in geometry is expressed as a linear combination of the shape basis vectors with the design variables as vector multipliers. The finite element geometry is updated using the following equation:

$$G_{new} = G_{old} + [T]\{X_{new} - X_{old}\}$$

where:

G_{new} is the vector of updated grid coordinates.

G_{old} is the vector of current grid coordinates.

$[T]$ is the shape vector matrix.

X_{new} is the updated design variable vector.

X_{old} is the current design variable vector

Auxiliary Boundary Model

The key idea in the boundary shapes approach is to apply prescribed displacements or shape forms on the design boundaries, with the design variables controlling the amplitudes of the boundary shapes. To provide maximum ease and flexibility in creating and/or specifying these boundary shapes, the concept of an auxiliary boundary model has been introduced in MSC/NASTRAN, Version 68. The auxiliary boundary model is used to characterize the design boundaries using the geometry of the analysis (primary) model. The boundary shapes are then generated using static analysis. The auxiliary models are integral to the analysis (primary) model and are defined with additional bulk data sections. Each auxiliary model has a case control section, from which a BULK data section is referenced. Each auxiliary model is independent of the primary model with the exception of the boundary GRID points and coordinate systems. The auxiliary model stiffness is used only for computing the boundary shapes. Functional forms such as maintaining a circular boundary may easily be enforced with the auxiliary boundary models. Thus through the different options of applied loading – mechanical and thermal, multiple boundary conditions and variable material elastic properties, a vast array of candidate boundary shapes can easily be generated. The boundary shapes may also be optionally scaled and combined to produce non–standard forms. As a simple illustration, consider the shape parameterization of the bottom edge of the plate (from Reference [3]), shown in Figure 1. The design boundary (bottom edge of plate) is modeled as an auxiliary boundary model (Figure 2). The four boundary shapes shown in Figure 3 are generated with a combination of applied loading and boundary conditions as described in the table below:

CONSTANT	– Unit enforced displacements
LINEAR	– Unit force at the tip with a hinged root boundary condition
QUADRATIC	– Unit moment at the tip with a clamped root boundary condition
CUBIC	– Unit force at the tip with a clamped root boundary condition

Modeling Procedure

The steps required to generate the boundary shapes include:

1. Identify design boundaries and surfaces (boundary grid points) of the primary model. These would include both fixed and free boundaries.
2. Model the design boundaries using 1–D (BAR, BEAMS) element types for edges and 1–D and 2–D (PLATES and SHELLS) for surfaces using the geometry of the primary model; i.e., the element topology is described with the boundary grid points of the primary model. If desired, additional grid points and coordinate systems may be defined for element coordinate systems, applied loading etc. It is not necessary to model the fixed design boundaries in the auxiliary boundary model.
3. Constrain the auxiliary model and apply fictitious loadings (enforced displacement, applied loadings, temperatures, boundary conditions etc.) to generate desired boundary shapes.
4. Associate the resulting boundary shape from the auxiliary model analysis with a design variable. List the boundary grid points to identify the fixed and free design boundaries. If desired, selected degrees of freedoms of the design boundary may be indentified for enforcing normal motion and have the tangential motion interpolated.

Auxiliary Boundary Model User Interface

CASE CONTROL	
AUXCASE	**Case Control Delimiter**
AUXMODEL	**Select Auxiliary Model ID**
BULK DATA	
BEGIN BULK [AUXMODEL=ID]	**Bulk Data delimiter**
DVBSHAP	**Associates design variable to a linear combination of boundary shapes**
BNDGRID	**Boundary grid identification numbers**

Table 1 – Auxiliary Boundary Model User Interface

VISUALIZATION OF SHAPE BASIS VECTORS

Generating the proper shape basis vectors is critical in the shape design process. A visual check is necessary to ensure the shape vectors possess the ability to affect the design objective (weight reducing for example) and/or design constraints (stress reducing). The shape basis vectors are generated as part of the linear static analysis in SOL 200. The shape basis vectors are accessible as additional subcases and can easily be viewed or plotted using standard MSC/NASTRAN post processing tools.

MESH DISTORTION ISSUES

The form of shape basis vectors used in the shape redesign task clearly impact the quality of the finite element mesh. Distorted elements will lead to discontinuity in the sensitivity gradients resulting in instability in the numerical optimization process. In the worst case mesh distortion errors may prevent the analysis from proceeding. Therefore it is essential to maintain mesh quality during the design process. Simply remeshing at every design cycle is not an attractive option because it is expensive and will likely result in topological changes – different number of elements and grids, causing discontinuous sensitivity gradient information. For small to moderate design changes the need for remeshing may be reduced to a minimum by using low distortion shape vectors producing smooth mesh changes. The boundary shapes concept produces low distortion shape vectors by using linear static analysis to interpolate movement of interior points of the finite element mesh with the boundary shapes applied as enforced displacements. Further to ensure smooth mesh changes the shape vectors are regenerated for each design cycle based on current geometry.

OTHER ENHANCEMENTS

There are several other related to shape optimization enhancements in Version 68 to improve robustness of the procedure [6,7]:

1. Individual move limits for design variables. This limits the amount of change in an individual design variable change and is very important to control mesh changes per design cycle.

2. Automatic selection of step size for each design variable, based on an energy norm method of Reference [3]. This removes the requirement on the user to specify a step size in the finite difference calculations.
3. Central differencing for computing more accurate pseudo loads in the semi–analytic approach for design sensitivities.

DESIGN EXAMPLES

To demonstrate the boundary shapes concept and modeling of auxiliary boundary model for generating the shape basis vectors, two design problems are optimized. The first design problem is presented in detail to clarify the modeling procedure and further explain the user interface. The second example demonstrates the technique on a 3–D model.

Plate Example

Consider the design example of a cantilevered plate of Figure 1 loaded with an inplane shear force. The design task is to minimize the weight of the plate, subject to a tip displacement constraint, by designing the bottom edge of the plate. To parameterize the plate boundary, an auxiliary boundary model shown in Figure 2, is used to characterize the design boundary using BAR element types. The design boundary is identified using the boundary grid points of the primary model. To generate the desired boundary shapes, four load conditions are applied to generate the desired shapes – constant, linear, quadratic and cubic. Each boundary shape in this example has a direct effect on the design objective – weight. The resulting shape vectors illustrated in Figure 4, are determined by applying the four different boundary shapes as enforced displacements on the analysis model for the boundary conditions specified on the *BNDGRID* entry. The top edge of the plate is fixed and the side edges are permitted to slide in the vertical (y–direction) and the bottom (design) surface is prescribed boundary motion in the normal direction. Note, only the normal motion (y–direction) is applied and movement of remaining degrees of freedom is interpolated. The appendix contains the input data for this example. First the primary model case control is listed followed by the auxiliary model case control. The AUXCASE entry serves to delimit the case controls and the entry *AUXMODEL=1*, associates a bulk data section with keyword *AUXMODEL=1*. Following the case control sections, the bulk data are listed, primary model followed by the auxiliary model. The auxiliary boundary model bulk data is tagged with a keyword *AUXMODEL=1* on the *BEGIN BULK* entry.

The final plate geometry is shown in Figure 5 and the design objective – weight was reduced by 30% in 7 design cycles.

Solid Beam Example

This 3–D example of a cantilevered solid beam (Figure 6) modeled with HEXA element type is designed for minimum weight subject to von–Mises stress constraints. The initial stress distribution is shown in Figure 7. The top and bottom surfaces of the beam shown in Figure 8 are modeled with QUAD4 plate elements as disjoint structures in a single auxiliary model. With the root end of the auxiliary plate structure fully constrained, a cubic shape is generated by applying a uniform tip loading. In addition, the component field on the BNDGRID entry is used to restrict motion in the x and z directions only on the remaining beam surfaces. The resulting shape vectors are shown in Figure 9. The problem converged in 5 design cycles to yield a final shape shown in Figure

10, with a 33% reduction in weight. Examination of the stress contours at the final configuration shown in Figure 11, indicate that the point of maximum stress has moved from the root to near the tip. This is to be expected because the root stress is not affected by the selected basis vectors.

CONCLUSIONS

The approach of shape parameterization and optimization using the boundary shapes concept has been described. Some of the key issues in generation of shape basis vectors and finite element mesh quality are addressed. Use of auxiliary boundary models for prescribing shape vectors is demonstrated to be effective in generating boundary shapes using static analysis. The power and generality of this capability has been demonstrated with simple design examples. User insight and ingenuity is required in the selection of auxiliary model loads and boundary conditions to produce the boundary shapes. This capability is available in Version 68 of MSC/NASTRAN and user feedback and suggestions are encouraged for possible future enhancements. Future plans call for extending the boundary shape concept to geometry based shape optimization.

ACKNOWLEDGMENT

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REFERENCES

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7. SHZ–03, "Enhancement for Shape Optimization – Robustness Part," Internal MSC memo, March 1993.

FIGURES

Figure 1 – Cantilevered Plate Model

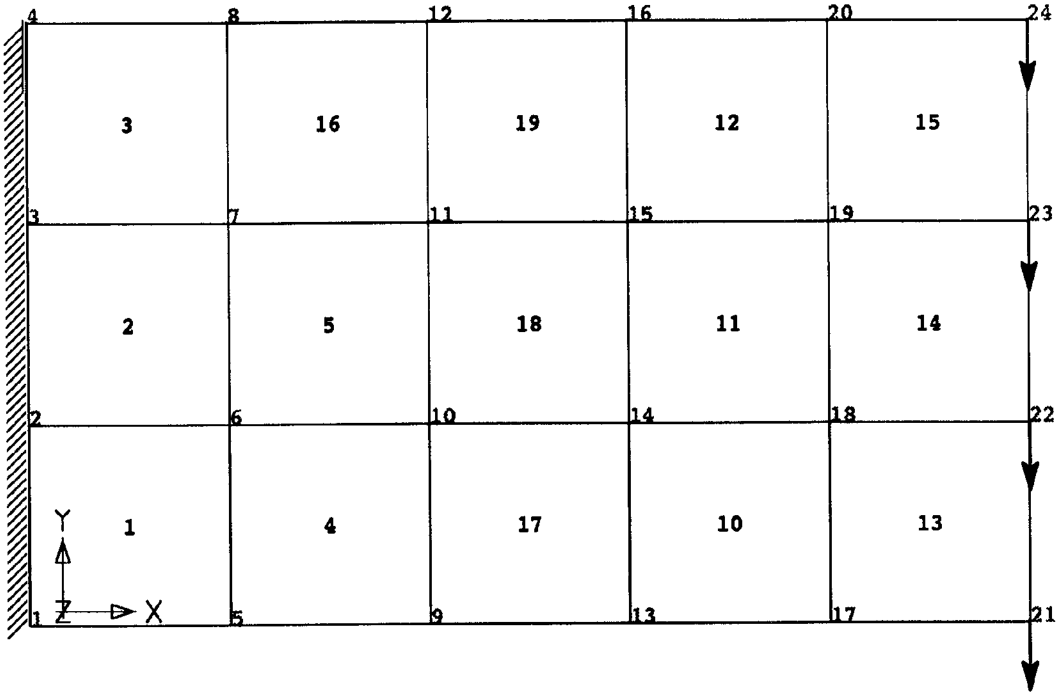


Figure 2 – Auxiliary Boundary Model

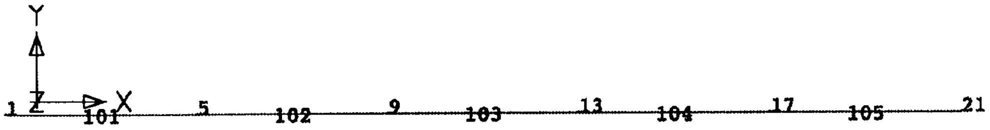


Figure 3 – Auxiliary Model Boundary Shapes

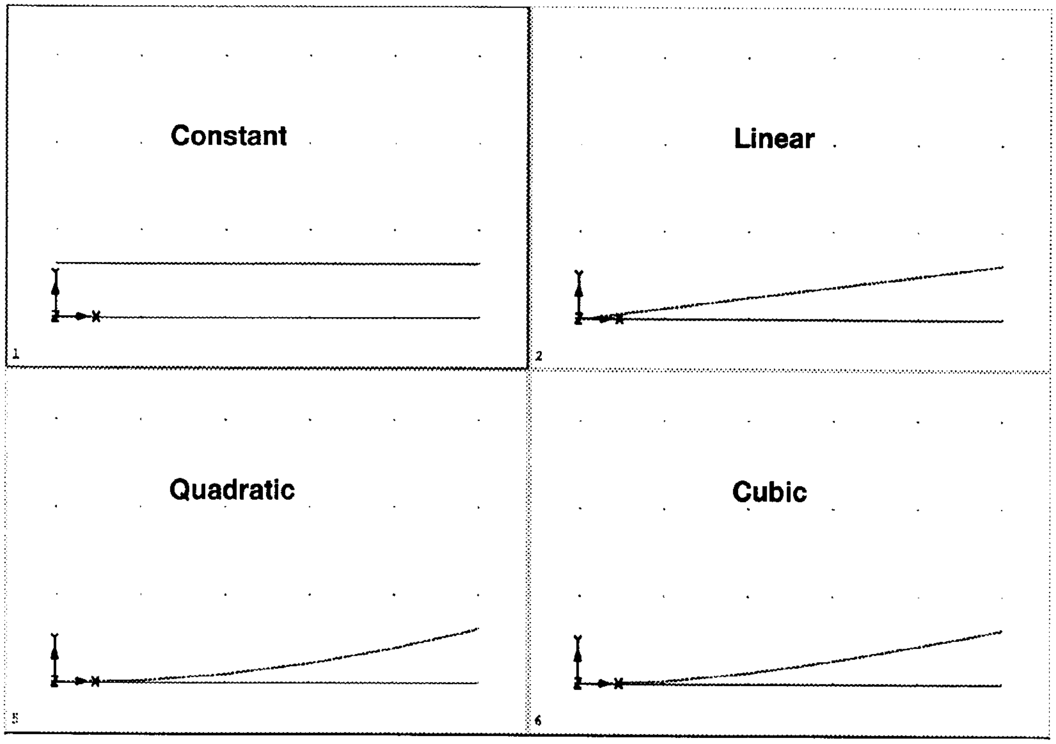


Figure 4 – Shape Basis Vectors for Plate Model

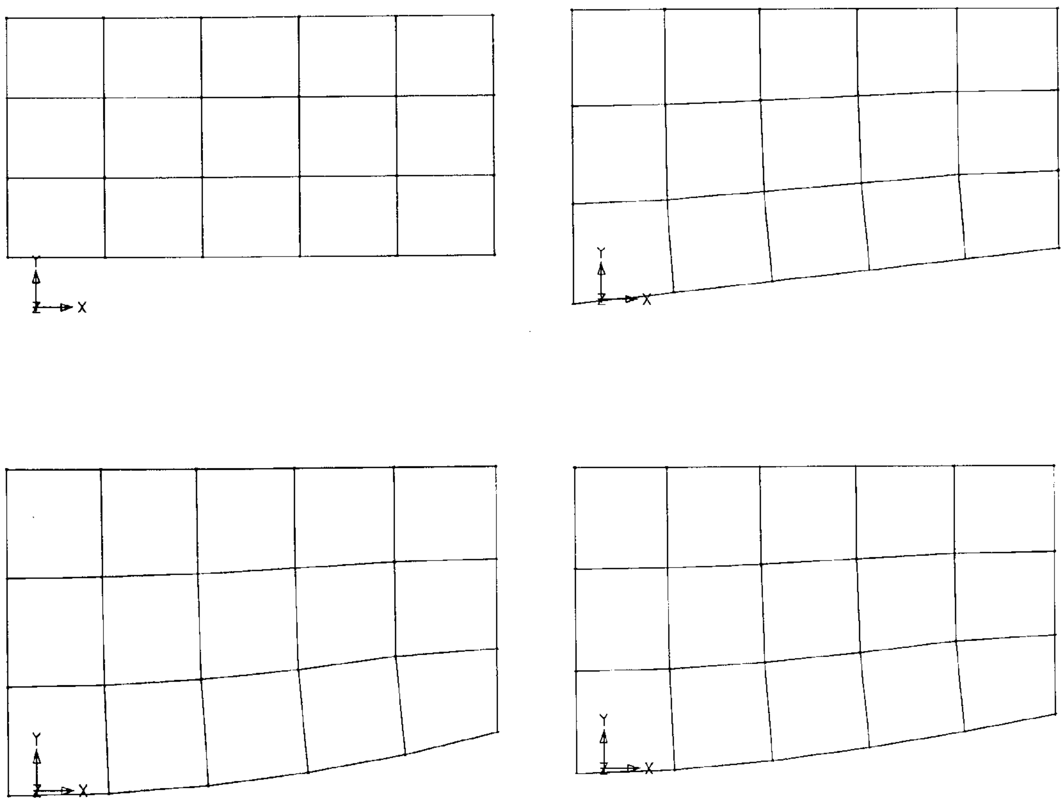


Figure 5 – Final Shape for Plate Model

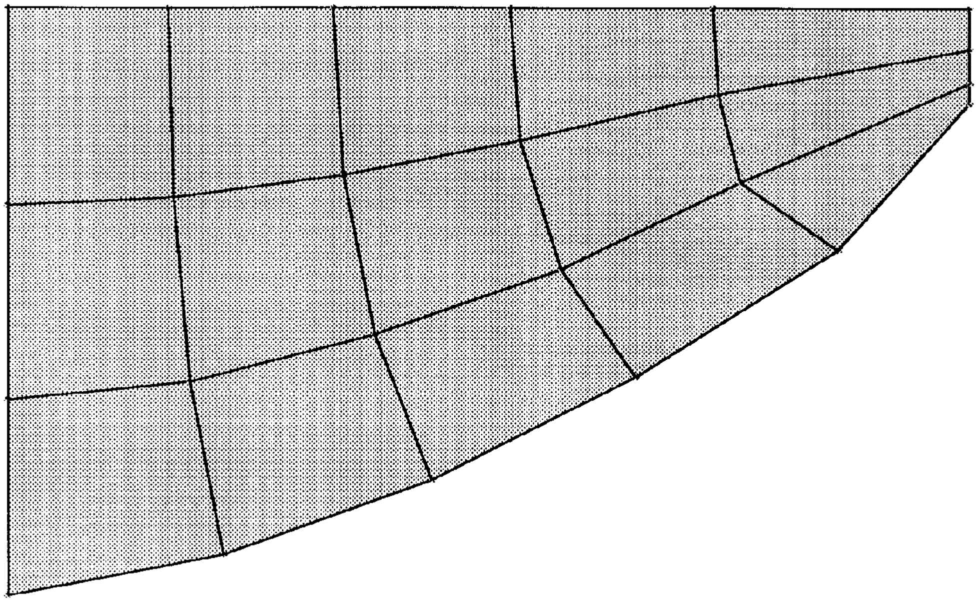


Figure 6 – Cantilevered Solid Beam Model

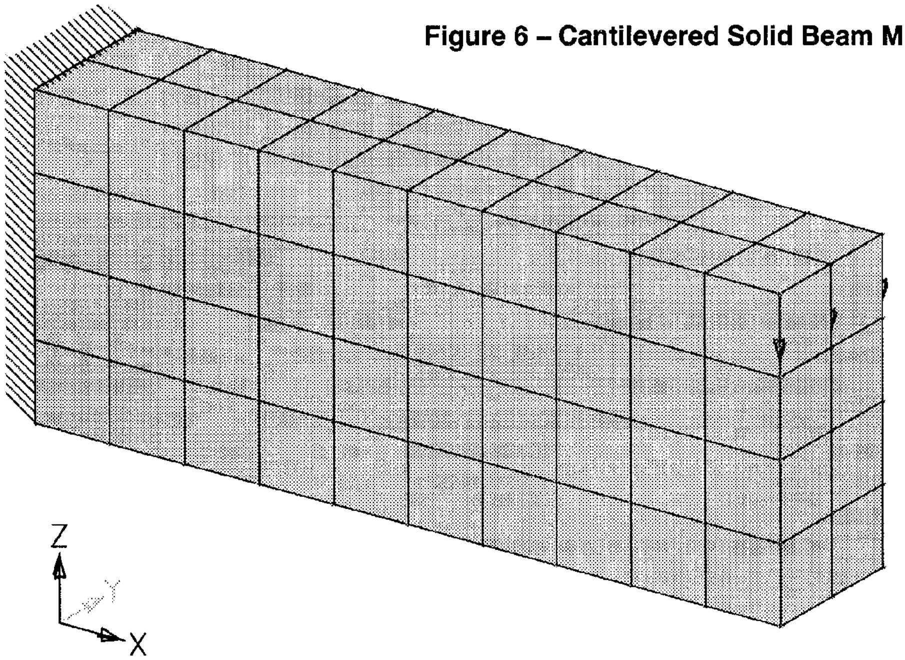


Figure 7 – Initial Stress Distribution for Solid Beam Model

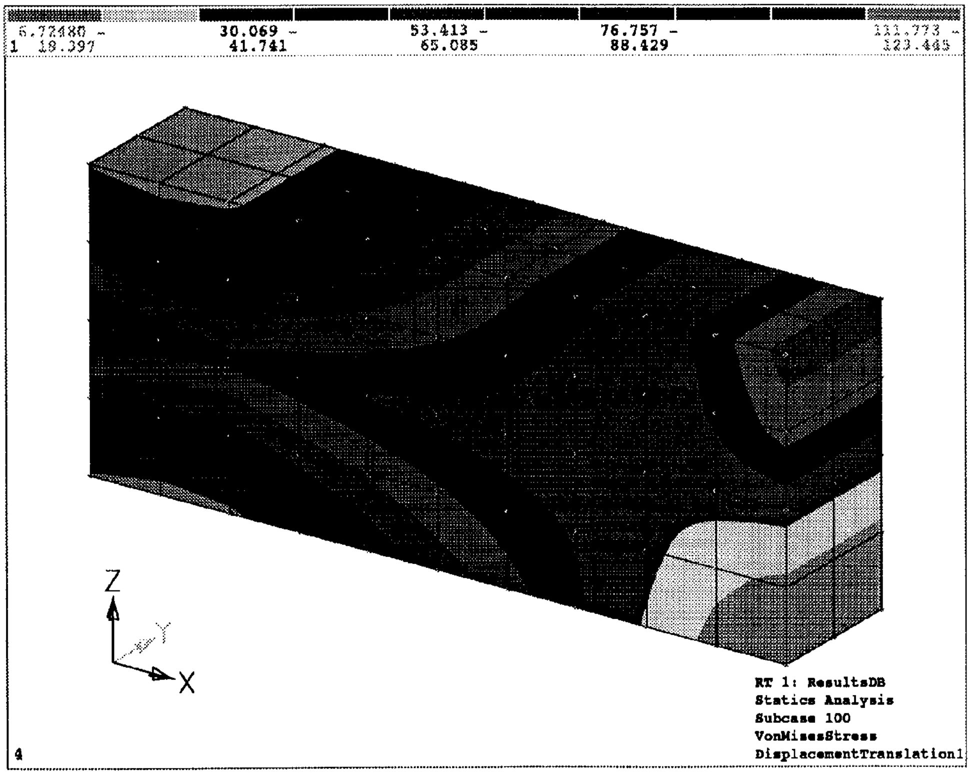


Figure 8 – Auxiliary Boundary Model for Solid Beam Model

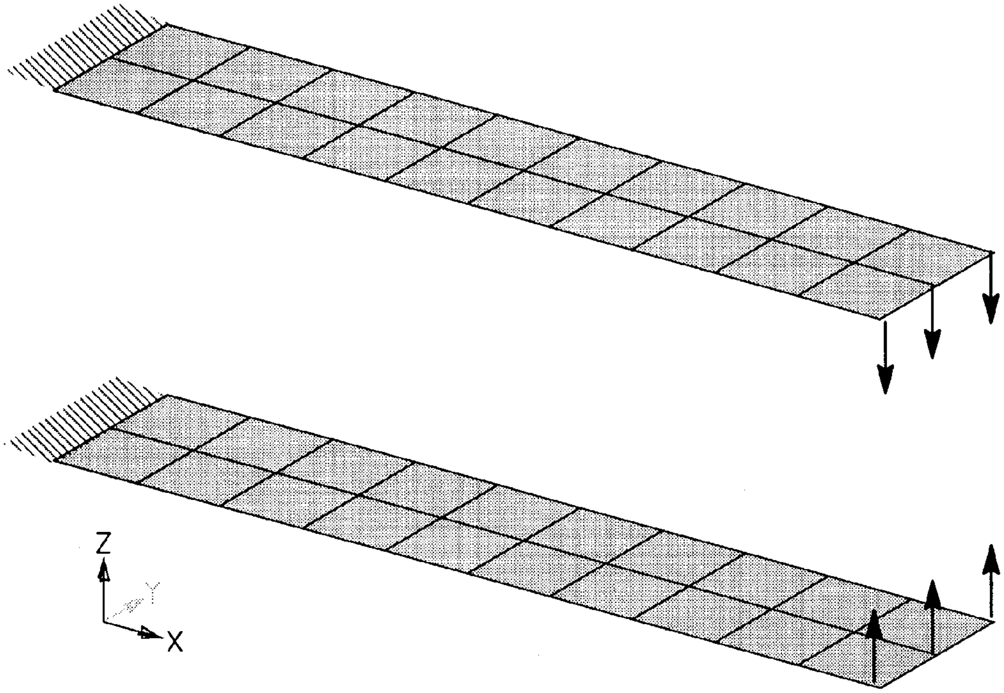


Figure 9 – Shape Basis Vectors for Solid Beam Model

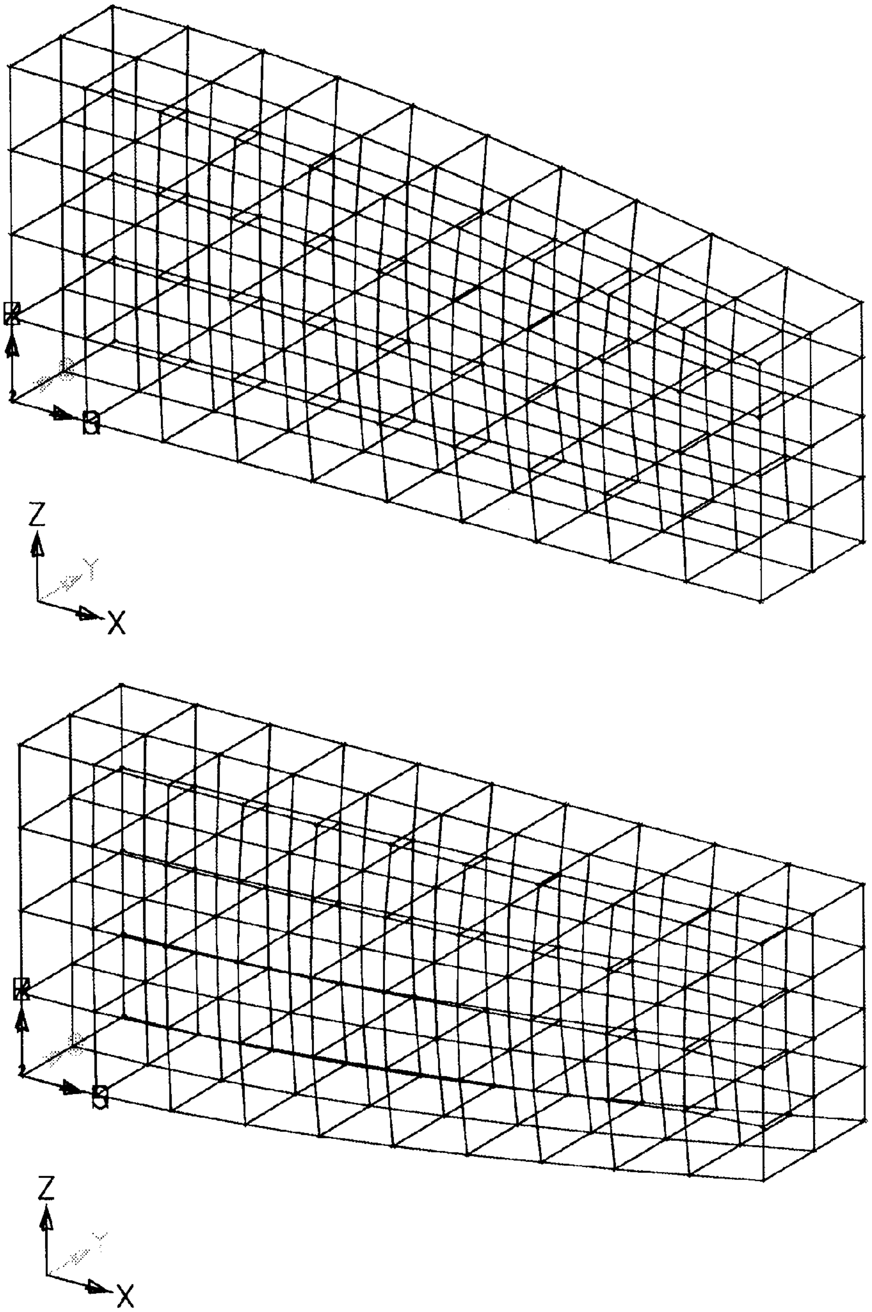


Figure 10 – Final Shape for Solid Beam Model

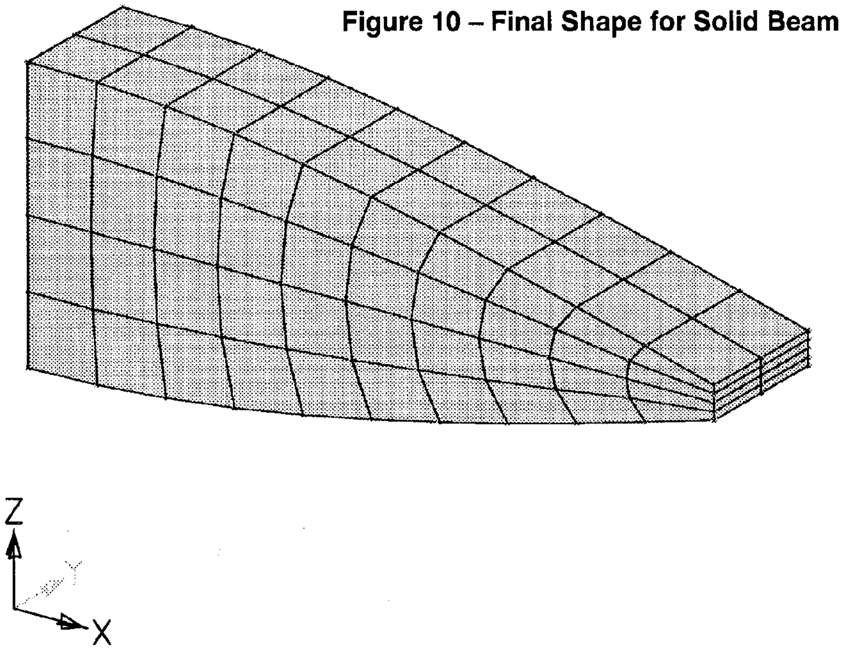
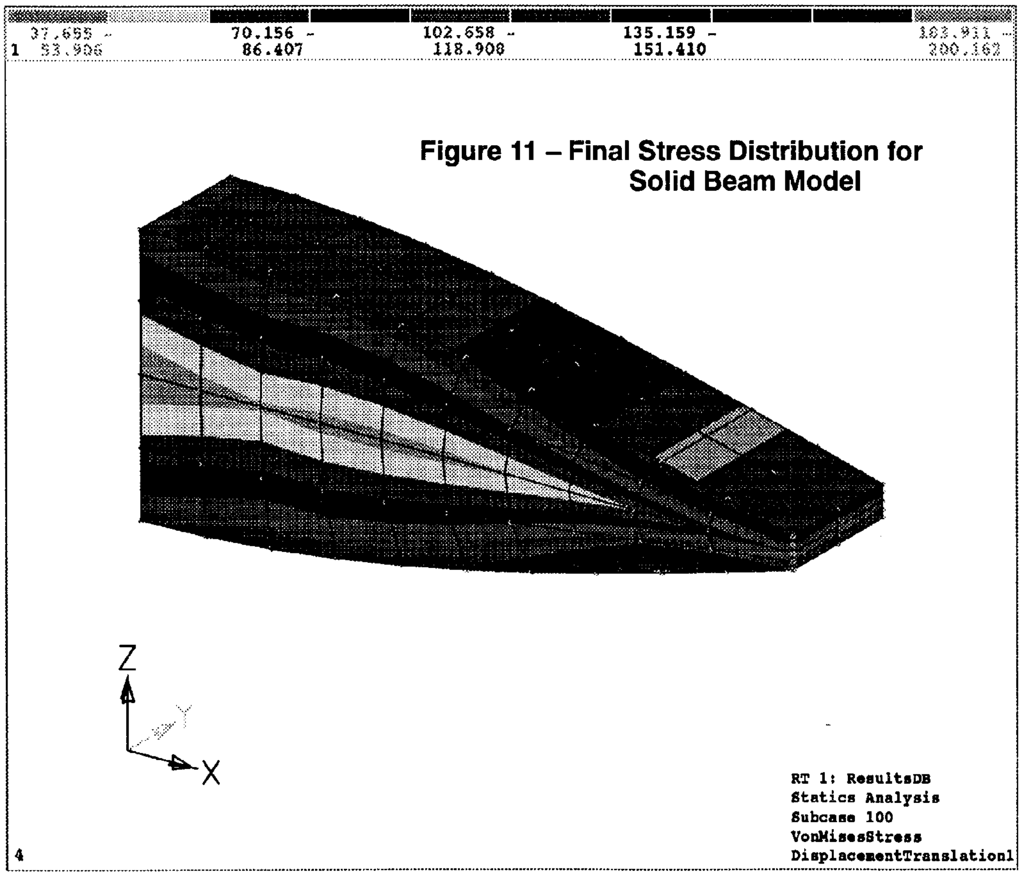


Figure 11 – Final Stress Distribution for Solid Beam Model



APPENDIX A

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SOL 200
CEND
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SUBCASE 10
DESOBJ = 10
DESSUB = 2
SPC     = 1
LOAD    = 1
DISP    = ALL
ANALYSIS = STATICS
$
AUXCASE
    TITLE = AUXILIARY MODEL 100
    AUXMODEL=100
    SUBCASE 100      $
    SPC   = 100      $
    LOAD  = 100      $
    DISP        = ALL      $
    LABEL = CONST    $
    SUBCASE 200      $
    SPC   = 200      $
    LOAD  = 200      $
    DISP        = ALL      $
    LABEL = LINEAR   $
    SUBCASE 300      $
    SPC   = 300      $
    LOAD  = 300      $
    DISP        = ALL      $
    LABEL = QUADRATIC  $
    SUBCASE 400      $
    SPC   = 300      $
    LOAD  = 400      $
    DISP        = ALL      $
    LABEL = CUBIC  $
BEGIN BULK
$
PARAM,NASPRT,1
PARAM,AUTOSPC,YES          $
$
GRID           1       0      0.      0.      0.      0
GRID           2       0      0.     10.      0.      0
GRID           3       0      0.     20.      0.      0
GRID           4       0      0.     30.      0.      0
GRID           5       0     10.      0.      0.      0
GRID           6       0     10.     10.      0.      0
GRID           7       0     10.     20.      0.      0
GRID           8       0     10.     30.      0.      0
GRID           9       0     20.      0.      0.      0
GRID          10       0     20.     10.      0.      0
GRID          11       0     20.     20.      0.      0
GRID          12       0     20.     30.      0.      0
GRID          13       0     30.      0.      0.      0
GRID          14       0     30.     10.      0.      0
GRID          15       0     30.     20.      0.      0
GRID          16       0     30.     30.      0.      0
GRID          17       0     40.      0.      0.      0
GRID          18       0     40.     10.      0.      0
GRID          19       0     40.     20.      0.      0
GRID          20       0     40.     30.      0.      0
GRID          21       0     50.      0.      0.      0
GRID          22       0     50.     10.      0.      0
GRID          23       0     50.     20.      0.      0
GRID          24       0     50.     30.      0.      0
GRDSET                                                    345
CQUAD4         1       1       1       2       6       5
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CQUAD4         2       1       2       3       7       6
CQUAD4         3       1       3       4       8       7
CQUAD4         4       1       5       6      10       9
CQUAD4         5       1       6       7      11      10
CQUAD4        16       1       7       8      12      11
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CQUAD4        12       1      15      16      20      19
CQUAD4        13       1      17      18      22      21
CQUAD4        14       1      18      19      23      22
CQUAD4        15       1      19      20      24      23
$
$
$--------------------    P S H E L L  ------------------------------------
$PSHELL.. PID  .. MID1 ..  T   .. MID2 ..12I/T .. MID3 .. TS/T .. NSM    ..
PSHELL         1    1000    1.00
$
$--------------------    M A T 1  ----------------------------------------
$MAT1  .. MID  ..  E   ..  G   ..  NU  .. RHO  ..ALPHA ..TREF  ..        ..
MAT1        1000  75000.            0.3     2.7-9
$
$--------------------    S P C 1  ---------------------------------------
$SPC1  .. SID  ..  C   ..  G1  ..  G2  ..  G3  ..  G4  ..  G5  ..  G6  ..
SPC1           1  2          4
SPC1           1  1          1       2       3       4
SPC1           1  6          1      THRU     24
$
FORCE          1       21       0     100.      0.     -1.      0.
FORCE          1       22       0     100.      0.     -1.      0.
FORCE          1       23       0     100.      0.     -1.      0.
FORCE          1       24       0     100.      0.     -1.      0.
$
DESVAR         1 CONST     1.000   -100.   100.0    0.4
DVBSHAP       1           100           1            1.
$
DESVAR        2           LINEAR        1.000        -100.        100.0        0.4
DVBSHAP       2           100           2            1.
$
DESVAR         3 QUADRAT   1.000   -100.   100.0    0.4
DVBSHAP       3           100           3            1.6E5
$
DESVAR         4 CUBIC     1.000   -100.   100.0    0.4
DVBSHAP       4           100           4            4.8E3
$
BNDGRID       1           1            2            3            4
BNDGRID       1           21           22           23           24
BNDGRID       2           4            8            12           16           20           24
BNDGRID       2           1            5            9            13           17           21
$
DRESP1  1       WEIGHT  WEIGHT
DRESP2  10      WE1000  1
+       DRESP1  1
DEQATN  1       F(A)=1.0E+8*A
DOPTPRM      DESMAX      7            P1           1            P2           15
DRESP1  2       DISP    DISP                       2               24
DCONSTR 2       2       -.2        .2
$
BEGIN BULK AUXMODEL=100
PARAM,AUTOSPC,YES
PARAM,POST,0
$ BOTTOM
CBAR       101       2       1       5                       1.
CBAR       102       2       5       9                       1.
CBAR       103       2       9      13                       1.
CBAR       104       2      13      17                       1.
CBAR       105       2      17      21                       1.
$
PBAR         2       2       1.     1000.   1000.
```

```
MAT1        2     2.+5             .3       0.0
SPC1             100     126      1      21
SPC1         100          126          4            24
$
$ CONSTANT
FORCE      100      1              0.              1.
SPCD       100      1       2      1.
SPCD       100     21       2      1.
$
SPC1        200     12       1
SPC1        200     2       21
SPC1    200     126     4       24
$
$ LINEAR
FORCE      200     21              1.              1.
SPCD       200     21       2      1.
$
SPC1        200     12       1
SPC1        200     2       21
SPC1    200     126     4       24
$
$ QUADRATIC
SPC1        300     126      1
SPC1    300     126     4       24
MOMENT     300     21             +1.        0.      0.       1.
$
SPC1        300     126      1
SPC1    300     126     4       24
$ CUBIC
FORCE      400     21              1.        0.     1.        0.
$
ENDDATA
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