TOPOLOGY OPTIMIZATION USING MSC/NASTRAN

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Abstract

Recently, Bendsoe and Kikuchi developed a homogenization method which can be applied to find the optimal topology of a continuum in a fixed domain. The homogenization approach is based on an artificial but physical micro-structure whose properties are homogenized. Alternatively, it has been demonstrated that the solution of the optimum material distribution problem can be considerably simplified by employing a density-dependent isotropic material without a specific physical micro-structure. In this paper, topology optimization for minimum compliance under static loading and for maximum eigenvalue using this approach has been implemented using MSC/NASTRAN. Optimal topology for a plate under in-plane and bending loads is presented. Optimal material distribution for a plate to maximize the first frequency is also presented.

Introduction

In recent years, the shape optimization problem has been transformed to one of optimal material distribution by Bendsoe and Kikuchi [1]. They assumed that a structure is formed by a set of non-homogeneous elements which are composed of solid and void regions. The homogenization method is employed to obtain equivalent elastic constants for microstructures. Through an optimization process, they obtain optimum design under volume constraint. In their method, regions with dense cells are defined as structural shape, and those with void cells are areas of unnecessary material. Alternatively, it has been demonstrated that the optimal material distribution can be considerably simplified by employing a density-dependent isotropic material [2,3,4]. Both methods have the same advantage over conventional shape optimization. Remeshing of the structural domain and the evaluation of shape design sensitivity are avoided. The other advantage of density-dependent isotropic material is that the calculation of design sensitivity with respect to density can be implemented easily.

In recent research, sequential linear programming (SLP) is used [4,5] for topology optimization. SLP is a powerful method for structural optimization. When it is applied to the topology optimization problem, however, SLP has several drawbacks with regard to problem size and computational efficiency. In practice, the method of optimality criteria has often been used in topology optimization. In a recent paper, Mlejnek et al [2] proposed to use a separable power series to approximate the objective function explicitly. The new design can then be computed using Lagrange's multiplier method. This approach is based on an assumed value for the exponent in the power series.

In this paper, the objective function is expressed explicitly as a one-term posynomial. The Lagrange's multiplier can be solved explicitly. The new design can then be computed from the Lagrange's multiplier, previous design and sensitivities. The procedure is quite straightforward and efficient. This method can be used in both minimum compliance design and maximum eigenvalues design. Multiple loading for compliance minimization and multiple eigenvalues maximization are also considered in this research. The compliance and eigenvalues are calculated by using MSC/NASTRAN, the design sensitivity is implemented by post processing MSC/NASTRAN results in SOL 101 and SOL103. SOL 200 was not used.

Problem Statement

The objective of topology design is to either minimize the compliance or maximize the first natural frequency. The problem can be defined as: Find a set of design variables x to

$$\text{optimize } f(x) \tag{1}$$

$$\text{subject to} \quad g(x) = \int \rho(x)\, d\Omega - M_0 \le 0 \tag{2}$$

$$\text{and side constraints } 0 \le \rho(x) \le 1 \tag{3}$$

where $f(x)$ is the compliance to be minimized or the fundamental eigenvalue to be maximized, ρ is the density, Ω is the physical domain, and M_0 is the mass limit. The relation between density and Young's modulus is assumed as:

$$\frac{E_i}{E_0} = \rho^n \tag{4}$$

where n is an exponent, E_i and E_0 are intermediate and real material Young's moduli, respectively. Eq. (4) will penalize intermediate density and force the density to 0 or 1, when $n > 1$. In this paper, $n = 2$ is used for simplicity.

The objective function can be extended to multiple loading cases for compliance or multiple eigenvalues maximization. In minimization, the objective is to minimize the maximum compliance over all loading cases, and can be written as:

$$f = max_{l = 1, nl} C_l \tag{5}$$

where C_l is compliance for loading case l, and nl is the number of load cases. Since the maximum function in Eq (6) is not differentiable, it can be replaced by a differentiable approximation

[6]

$$f(x) = C_r(x) + \frac{1}{P}\log\sum_{l=1}^{nl} e^{P(C_i(x) - C_r(x))} \tag{6}$$

where $C_r(x)$ is the maximum compliance for all loading cases, and P is a chosen large number and we chose 1000 in this research.

In the eigenvalues (frequency) case, the objective function f to be maximized is given by the minimum eigenvalue of the structure

$$f = min_{j=1,n\lambda}\lambda_j(x) \tag{7}$$

where $n\lambda$ is the desired number of eigenvalues to be maximized, and λ's are eigenvalues in ascending order. It can be also replaced by:

$$f(x) = \lambda_1(x) - \frac{1}{P}\log\sum_{j=1}^{n\lambda} e^{P(\lambda_1(x) - \lambda_j(x))} \tag{8}$$

Note that Eq.(6) and (8) are the well-known K-S function [7] written in a special form to avoid numerical overflow for large P.

Design Sensitivity of Compliance and Eigenvalue

The compliance can be expressed as follows:

$$f(x) = \int_{\Gamma} F^i z^i d\Gamma \tag{9}$$

where F^i is the force, z^i is the displacement, and Γ is the loaded boundary.

The sensitivity of the compliance can be written as [4]:

$$f'(x) = -\int_{\Omega} \varepsilon^{ij}(z)\,(D^{ijkl})'\varepsilon^{kl}(z)\,dz \tag{10}$$

where ε^{ij} is the strain tensor, z is the displacement vector, and D^{ijkl} is the elasticity tensor and the prime indicates a partial derivative with respect to the design variables (density).

Sensitivity of eigenvalues can be written as [4]:

$$\lambda' = \int_{\Omega} [\varepsilon^{ij}(z)\ (D^{ijkl})'\varepsilon^{kl}(z) - \lambda z^i z^i]\, d\Omega \tag{11}$$

where the first term is the derivative of the dynamic strain energy, and the second term is the derivative of the kinematic energy with respect to the design variables (density).

Solution of the Optimum Design Problem

The topology optimization problem is characterized by an implicit relationship between the design variables and objective function through appropriate governing equations. Our solution to this problem proceeds as following: (a) Formulate an explicit design problem by using a one-term posynomial to approximate the objective function. (b) Find a search direction using Lagrange's multiplier technique. Since there is only one constraint, this procedure is very efficient. (c) Choose step size by a quadratic approximation of the objective function in the search direction. These steps are summarized below.

(a) Formulation of explicit design problem by the approximating objective function f can be approximated as a one-term posynomial:

$$f = C\prod_{i=1}^{n} x_i^{e_i} \tag{12}$$

where

$$e_j = \frac{x_j \partial F}{F\, \partial x_j} \tag{13}$$

The explicit design problem is then to optimize f given by Eq. (12) subject to the constraints (2) and (3).

(b) Find search direction. The solution to the above explicit design problem is

$$x_{jt} = \frac{x_j \partial F}{\mu \partial x_j} \left(\frac{\partial G}{\partial x_j}\right)^{-1} \tag{14}$$

where

$$\mu = \sum_{j=1}^{n} \frac{\partial F}{\partial x_j} x_j \tag{15}$$

The direction from the current design to x_{jt} is the search direction.

(c) Update design. The updated design is

$$x_{jnew} = (1 - \alpha) x_j + \alpha x_{jt} \tag{16}$$

where

α= step size.

where α is between 0 and 1. α can be computed by assuming a quadratic variation of the objective function in the search direction.

Once x_{inew}'s are solved, the next design step can be replaced by x_{inew}. The new designs are to be checked for side constraint violation. Any design variable that violates the bound will be set as passive. The above procedure will be used iteratively.

The above formulation has been implemented in a FORTRAN program. This program reads output from MSC/NASTRAN SOL 101 or SOL 103. It then performs sensitivity analysis and proceeds to find an updated design. The program also modifies input data for MSC/NASTRAN for the new design. The process is then repeated until a convergent solution is obtained.

Numerical Examples

The described optimization method is demonstrated by several examples in this section. We will present minimum compliance design for beam structure under transverse loading, bridge structure under three point loading, and simplified truck frame under twisting. We will also present maximum eigenvalue design for simplified truck frame and clamped-clamped beam.

Minimum Compliance Design.

Example 1: Beam Structure with Transverse Loading.

The first example is a cantilever beam modelled as a two-dimensional plane stress problem. The left side of beam is clamped and the vertical load is applied at the middle of the free end as shown in Figure 1.

The problem is to minimize the compliance with 25% of the volume constraint imposed on the design domain. Using consistent units, the Young's modulus E, the poisons ratio ν, thickness t and the load are as follows:

$E = 2.07 \times 10^5$

$\nu = 0.3$

$F = 300$

$t = 1$

A 32×20 mesh with CQUAD4 is used in the finite element model. The density of each element is the design variable. After 30 iterations, an optimal material distribution is obtained and shown in Figure 2. The result is similar to that reported in the literature [8].

Example 2. Beam with Simple Supports.

Consider a simple supported beam modelled as two-dimensional plane stress problem. The vertical load is applied at 1/4, 1/2, 3/4 of the length as shown in Figure 3. The structure is to be optimized for minimum compliance with 25% material-usage constraint imposed. Material constants, and the forces are as follows:

$E = 1 \times 10^8$

$\nu = 0.3$

$F_1 = 500$

$F_2 = 1000$

$F_3 = 500$

$t = 0.1$

The finite element model contains a 32×24 mesh of CQUAD4. The density is the design variable. An optimal topology design is obtained after 15 iterations. The optimal material distribution is like the truss structure shown in Figure 4.

Example 3. 3-D Simplified Truck Frame with Twisting Load Case.

The geometry of the simplified truck frame is shown in Figure 5. The boundary conditions at node A, B, C, D is in xyz, z, z, y direction respectively. The thickness is 3. The load is 1000 and -1000 applied at node 5,6 in z direction respectively. The objective is to minimize compliance with 50% material usage constraint imposed. The material constants are the same as example 1.

The structure is modeled as a set of shell elements. The number of elements is 1360. The optimal material distribution is obtained after 15 iterations and is shown in Figure 5.

Maximum Eigenvalue Design.

Example 4. Maximize Fundamental Eigenvalue of 3-D Simplified Truck Frame.

The geometry, boundary conditions and material constants are the same as example 3. The true material density is 7.93×10^{-6}. The objective is to maximize eigenvalues with a 25% material usage constraint imposed. In [9], the author suggested that the single eigenvalue maximization results in substantial oscillations of the structural eigenvalues during the iterations. We have also observed this phenomenon. To circumvent this difficulty the first two eigenvalues maximization is used in this research. The optimal material distribution is obtain after 60 iterations and is shown in Figure 6. During the optimization process, the second mode switches from a twisting mode to a local mode which is shown in Figure 7.

Example 5. Maximize Fundamental Eigenvalue of Clamped-Clamped Beam.

Consider a clamped-clamped beam with a concentrated mass at the center as shown in Figure 8.

The objective is to maximize the fundamental eigenvalue with a 50% material usage constraint imposed. The true material density is 7.93×10^{-6}. The concentrated mass is 350. The optimal mass distribution is like a truss structure shown in Figure 9.

Concluding Remarks

Two-dimensional static and vibrational topology optimization was accomplished by using density-dependent isotropic material and a one-term posynomial optimization methods. In the static case, the results converge very fast even without step size search. In the dynamic case, the results are dependent on step size. Large step size may lead to divergence. However, in all cases tested, if a small step size was used (0.1), the algorithm always lead to a convergent solution.

Since MSC/NASTRAN is a versatile analysis tool used in many industries, the proposed method can be applied to find the optimal topology for many products. Currently the design program is written in FORTRAN. Further development is needed to write a DMAP program for topology optimization.

Acknowledgement

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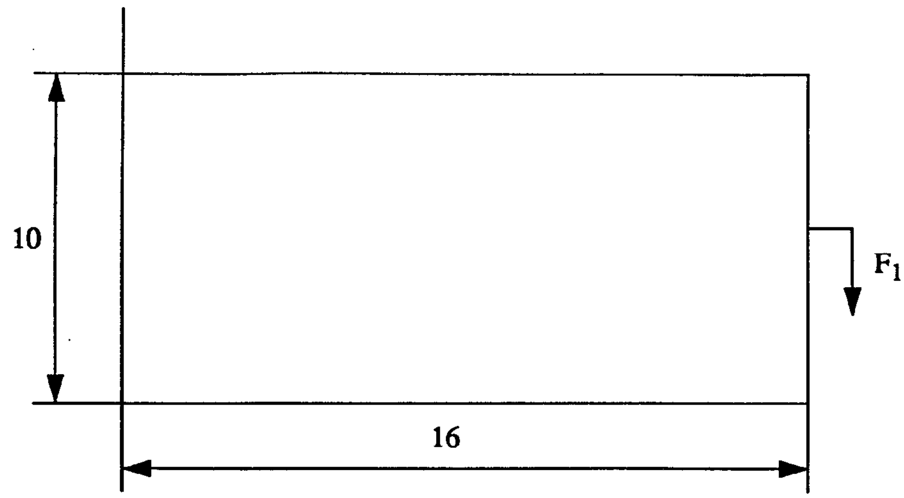

Figure 1. Example 1. Cantilever Beam

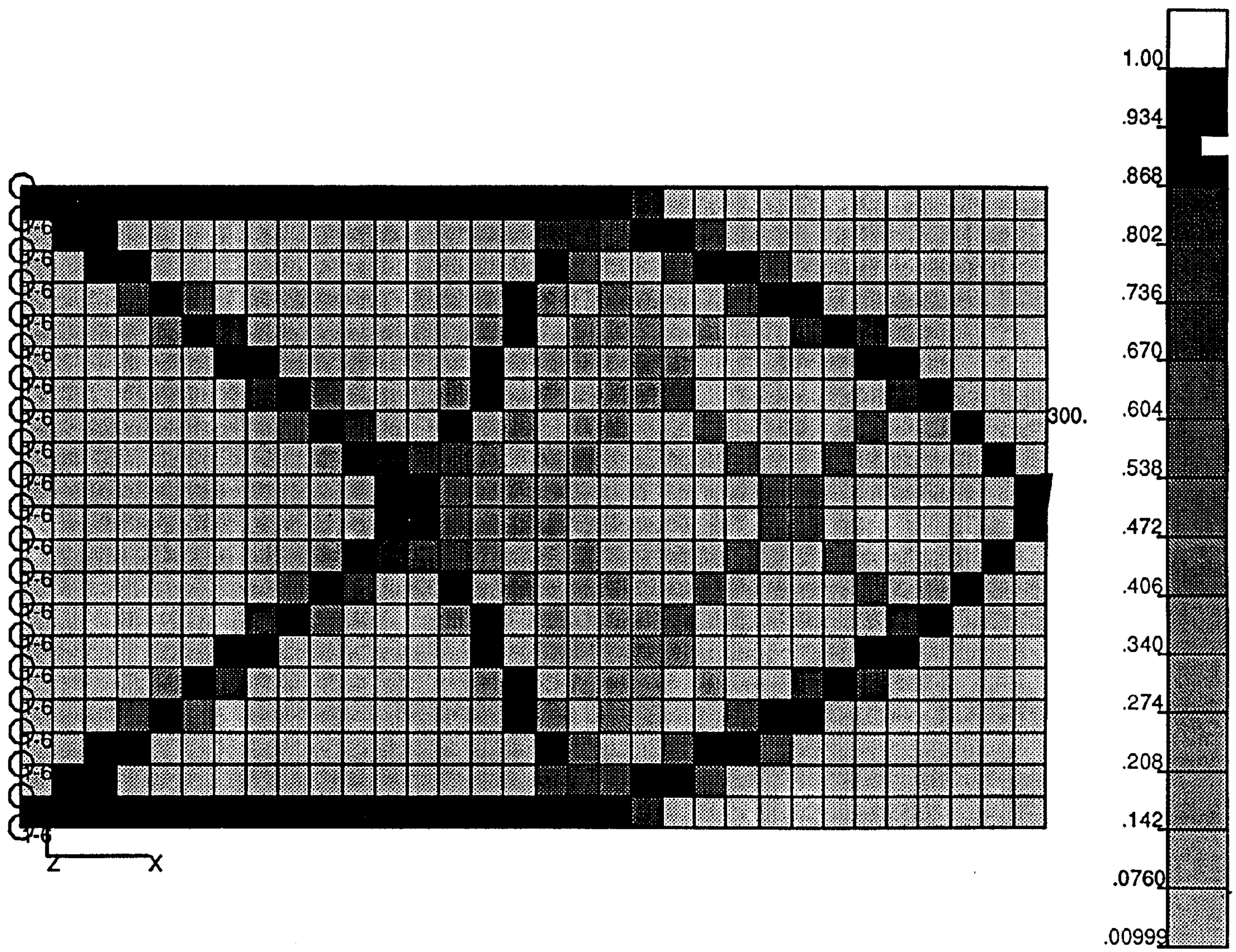

Figure 2. Optimal Material Distribution of Example 1.

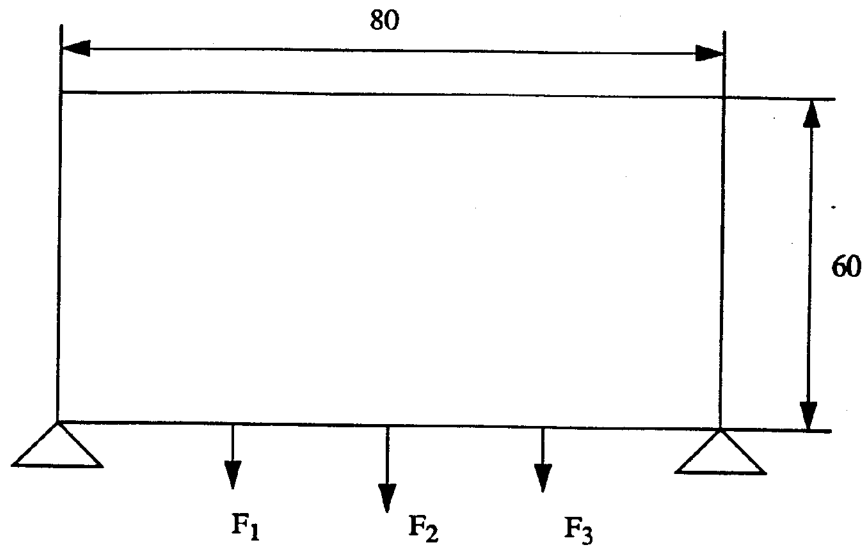

Figure 3. Example 2. Beam with Simple Supports.

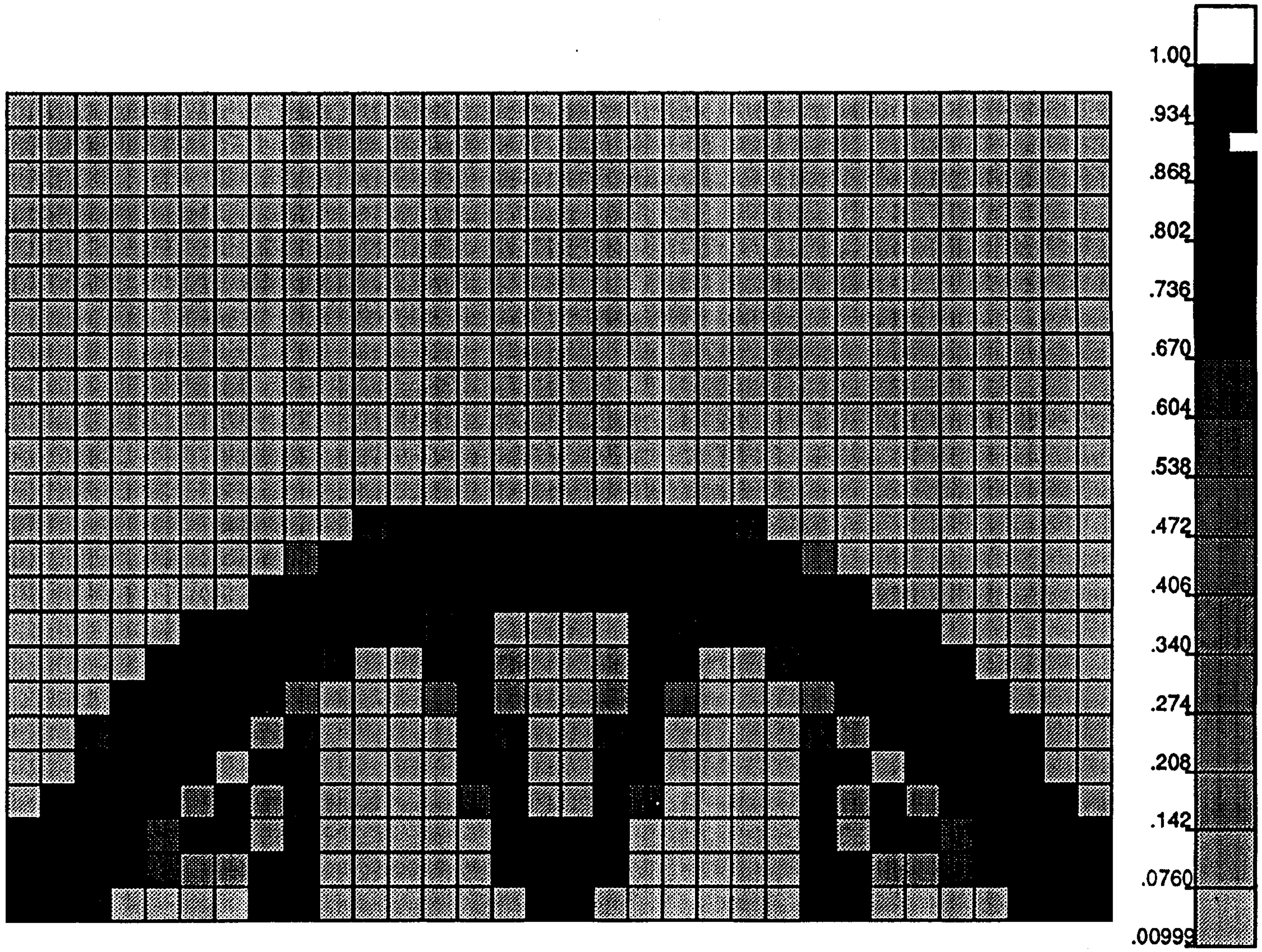

Figure 4. Optimal Material Distribution of Example 2.

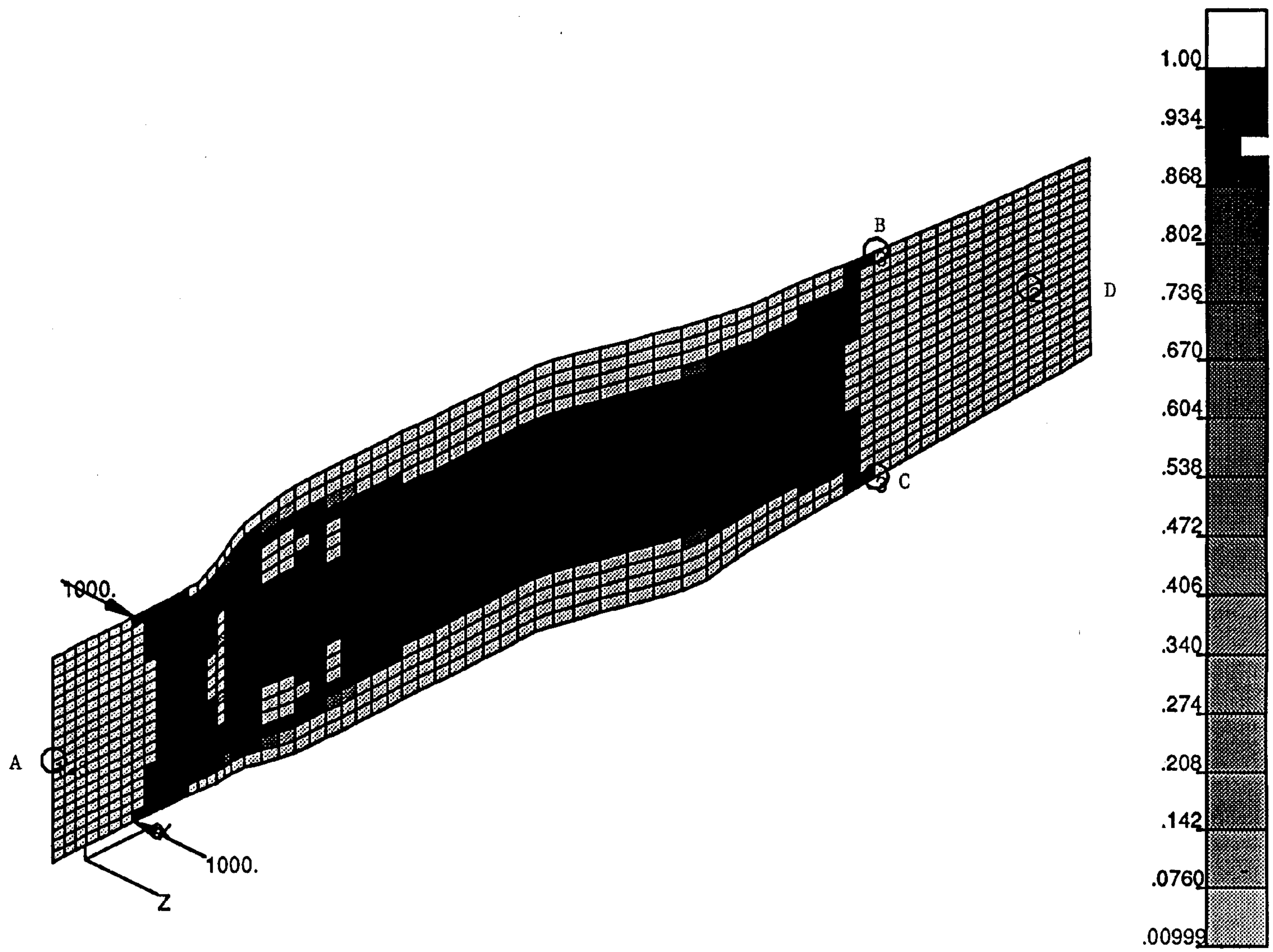

Figure 5. Example 3. Truck Frame for Twisiting and Optimal Material Distribution

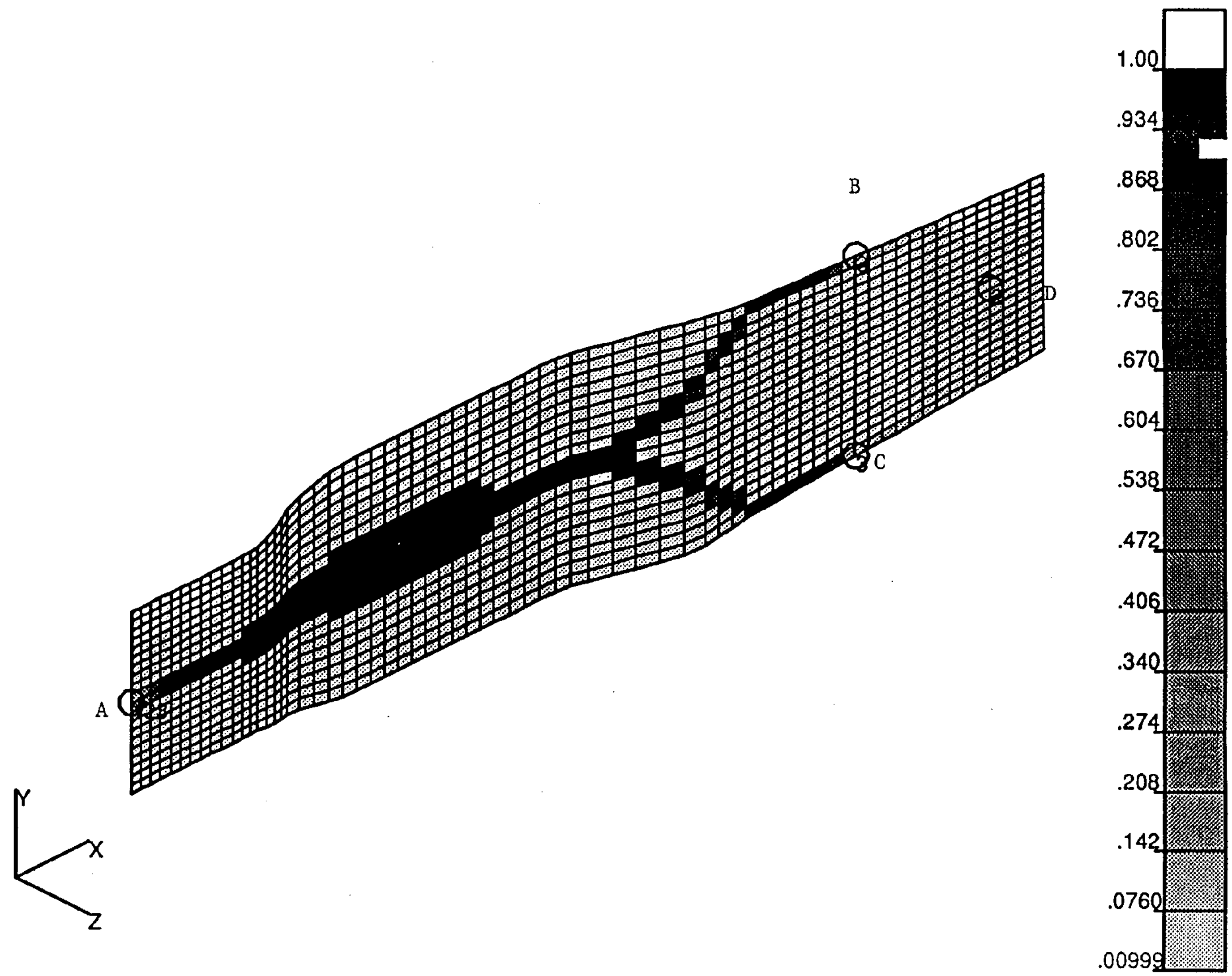

Figure 6. Example 4. Truck Frame for Maximum Eigenvalue
and Optimal Material Distribution

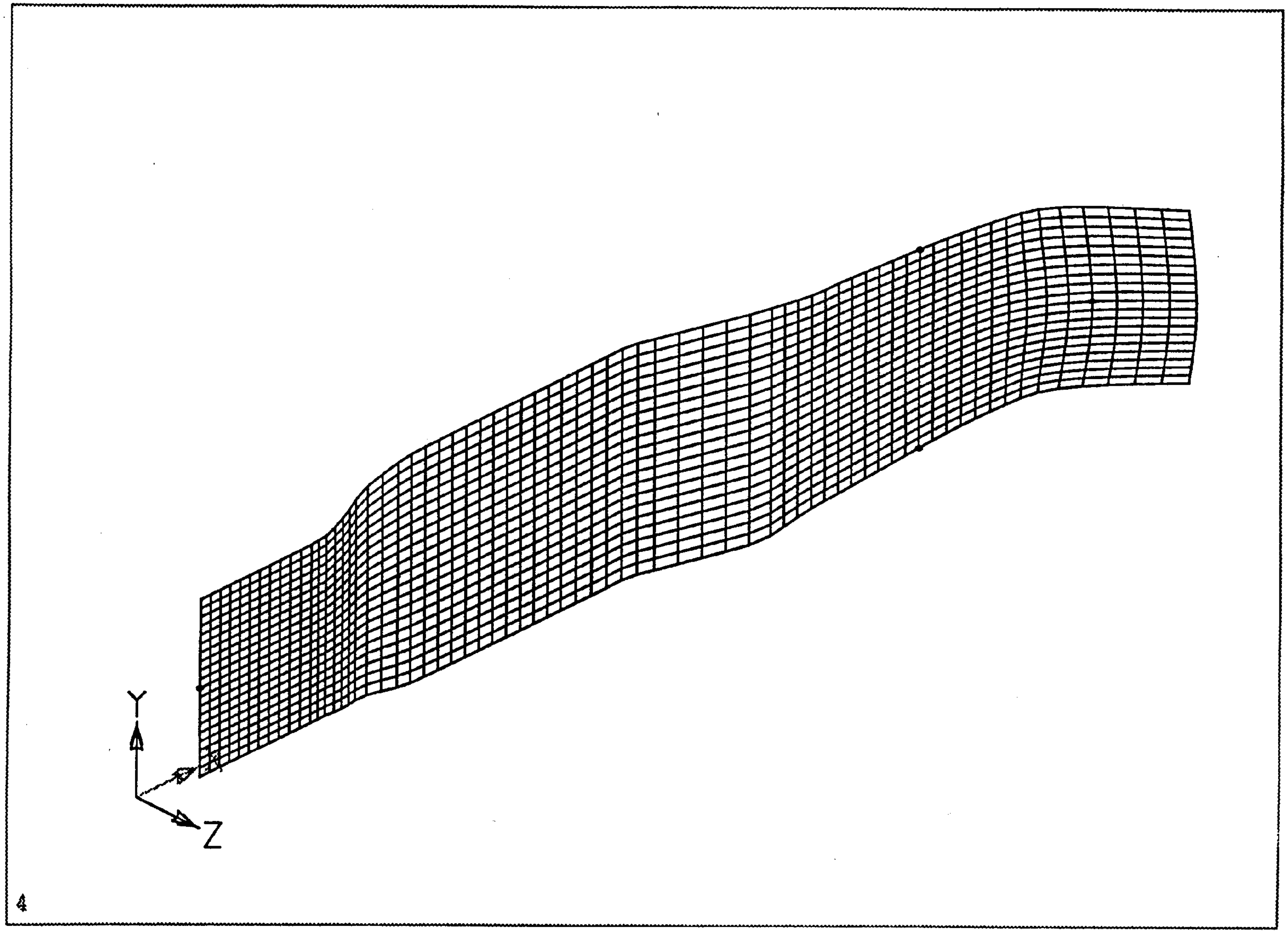

Figure 7. Second Mode Shape of Example 4.

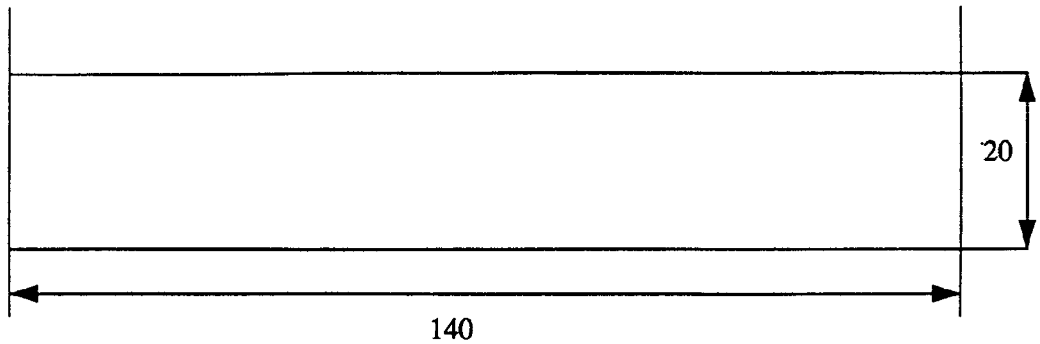

Figure 8. Example 5. Clamped-Clamped Beam.

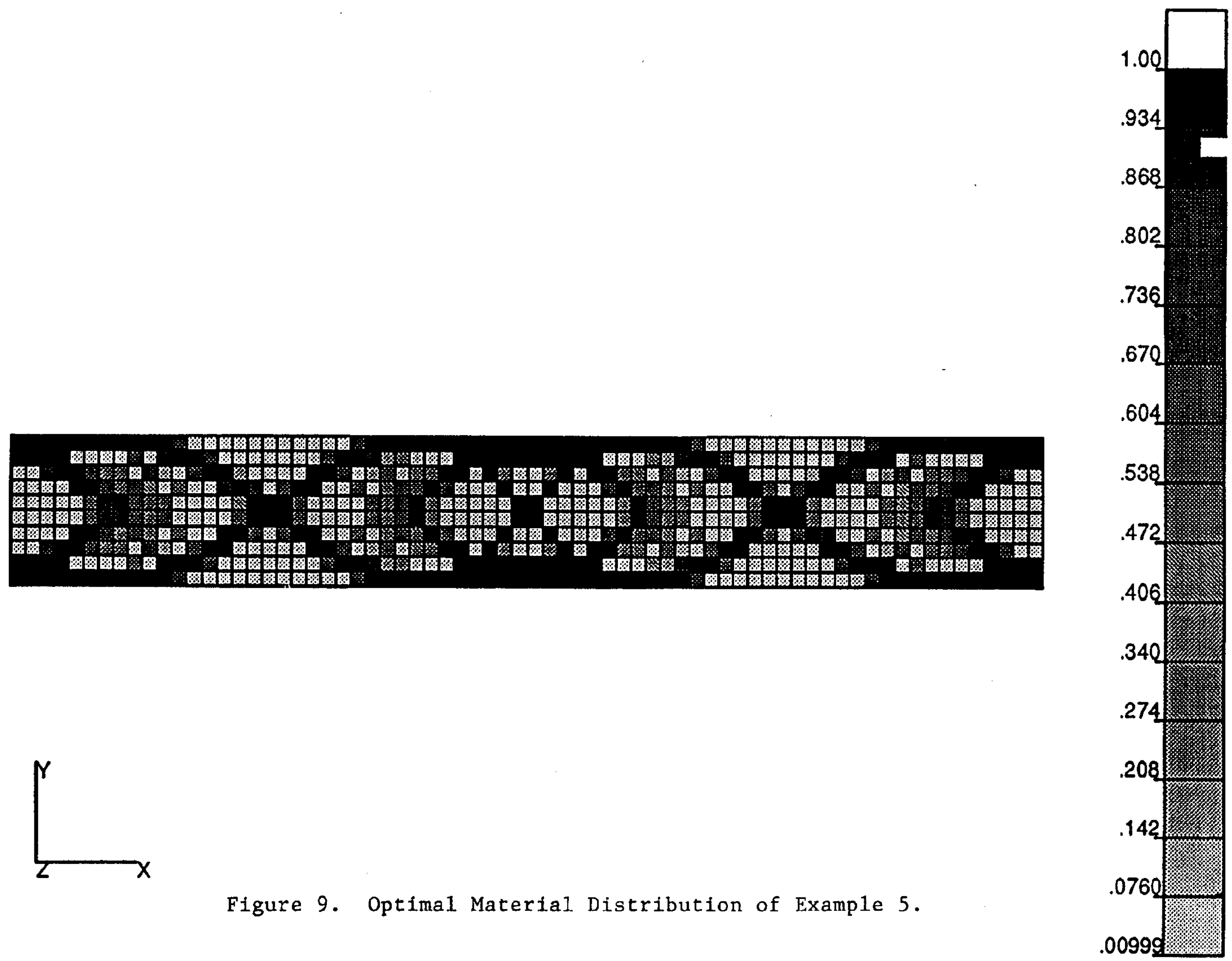

Figure 9. Optimal Material Distribution of Example 5.