MATCHING FREQUENCY RESPONSE TEST DATA WITH MSC/NASTRAN

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This paper describes the use of MSC/NASTRAN for matching frequency response test data. MSC/NASTRAN's design optimization capability (SOL 200) is used to minimize the difference between the MSC/NASTRAN results and test data. In the procedure, the model parameters are automatically updated until the analytical results match the test data. The procedure is enabled in MSC/NASTRAN via the use of a user-written equation (DEQATN Bulk data entry) that defines the difference between test and analysis. An example is shown to illustrate the method.

DESIGN OPTIMIZATION

In design optimization an objective function (weight, for example) is minimized or maximized. Design variables (element properties or geometry) are specified, as are side constraints (upper and lower bounds on the design variables) and performance constraints (minimum and maximum allowable response values, such as an eigenvalue limit). Multiple design variables can be linked together to form a single design variable, and an objective function can be created via a user-written equation.

Dynamic responses supported in MSC/NASTRAN's design optimization capability are eigenvalues for normal modes analysis and displacements, velocities, accelerations, and SPC forces for transient and frequency response analysis. Stress and force responses are also available for dynamic response analyses.

This paper presents a model refinement technique. The objective function to be minimized is not weight but rather the difference between test data and MSC/NASTRAN results. This is specified via a user-written equation defined with the DEQATN Bulk Data entry. Design constraints are specified to ensure that a realistic model is obtained.

MODEL REFINEMENT APPROACH

A model refinement approach called *Bayesian parameter estimation* [1,2] is used. This approach incorporates confidences in test data and model parameters. In this approach each "piece" of test data has a confidence assigned to it, each model parameter has a confidence associated with it, and the test data as a whole is weighted relative to the initial model as a whole. The Bayesian parameter estimation procedure is an iterative approach wherein an error is minimized. The goal is to find the set of final model parameters that minimize the following error:

$$E = wt*\Sigma wr*(r-r_0)^2 + wm*\Sigma wp*(p_f-p_0)^2 \qquad (1)$$

where
r = responses from the test (i.e., test data)
r_0 = responses from the original analysis
wr = weighting factors (confidences) for responses
p_f = parameters of the final model
p_0 = parameters of the original model
wp = weighting factors (confidences) for the parameters
wt = scalar weighting for the test data as a whole
wm = scalar weighting for the model parameters as a whole

In other words, the goal is to get a close match to the test data while making the smallest changes to the initial model parameters.

Equation 1 is minimized using MSC/NASTRAN's design optimization capability. The minimum of the error gives a model that matches the test data (making the experimentalist happy) while changing least from the initial model (making the analyst happy). Making both happy permits a balanced approach influenced by the "model updater's" choice of weightings wt and wm.

FREQUENCY RESPONSE MATCHING APPROACH

The need to match frequency response test data has arisen because people doing model updating have come to realize some of the limitations inherent in matching eigenvectors, not the least of which is computing accurate eigenvector derivatives in an efficient manner. Another problem is that the measured eigenvector is a derived quantity that is subjecty to error, as opposed to the frequency response which is measured directly and which is less susceptible to error. Also, eigenvectors do not take into account damping but frequency response does. With these problems and limitations in mind, people are starting to look at matching measured frequency response test data as opposed to the eigenvectors.

The ability to match measured frequency response data requires the following items:

Frequency response derivatives and optimization.

The ability to handle potentially large amounts of test data.

The ability to span multiple excitation frequencies in the user-defined equation for the error between test and analysis.

Frequency response derivatives have been available in MSC/NASTRAN starting with Version 67 and frequency response optimization has been available starting with Version 67.7. The ability to span multiple excitation frequencies using the DEQATN entry is available in Version 68. The size of the DEQATN entry has also been expanded in Version 68 to allow a longer equation (and therefore more terms) to be defined.

The matching of frequency response test data is a two-step process:

1. Match measured eigenvalues first, using the DEQATN entry to define the difference between the measured and computed eigenvalues.

2. With the design variables computed from step 1, use the DEQATN entry to define the difference between the measured and computed frequency response. In addition, use the DCONSTR entries to constrain the eigenvalues to be close to the test data. Set the lower and upper bounds on each DCONSTR entry to span the following frequency range of each eigenvalue according to the following formula:

   lower bound = min(computed or test eigenvalue)

   upper bound = max(computed or test eigenvalue)

   By defining eigenvalue bounds in the manner described above, the model will be feasible (i.e., there is no constraint violation) and the minimization process will proceed better than if the model is infeasible to begin with.

This two-step process--match eigenvalues first and then match frequency response--is illustrated for matching the measured frequency response of a disk drive enclosure.

STEP ONE: MATCHING EIGENVALUES

Consider the disk drive enclosure shown in Figure 1. The drive was modeled with 1404 grid points and 1345 plate elements. Fifteen different plate thicknesses were used. Bar elements were used in the seams of the joints (where plates met at a 90 degree angle) in order to stiffen the joints, as shown in Figure 2. Table 1 shows the measured and computed frequencies for the lowest nine flexible resonant frequencies.

Eigenvalues are first matched in the updating procedure and then this revised model is used as a starting point to match the frequency response results.

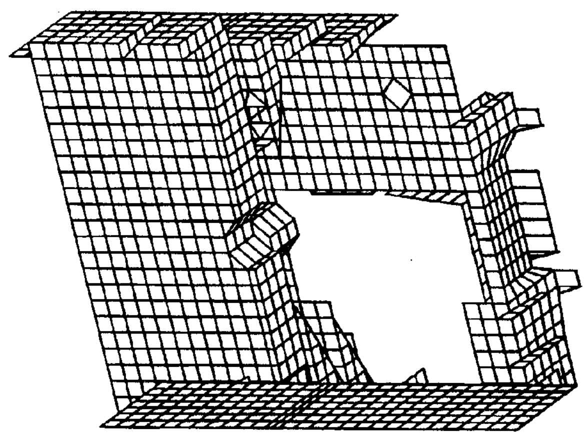

Figure 1: MSC/NASTRAN Model of the Disk Drive Enclosure

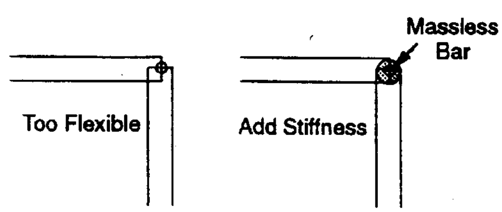

Figure 2: Technique to Stiffen Corners

Table 1: Measured and Computed Frequencies for Enclosure

Mode	Frequencies (Hz) Measured	Computed
1	373	301
2	1374	1154
3	1570	1390
4	1773	1565
5	1980	1871
6	2120	2029
7	2224	2234
8	2672	2328
9	2864	2466

The model refinement process was first run to update the model until its computed resonant frequencies matched those from the test. There were uncertainties in the thicknesses because the plates had significant taper. Therefore, each of the thicknesses had an initial value (the average plate thickness) and some variation (represented by lower and upper bounds on the values). Three additional design variables were the two bending and torsional moments of inertia of the bars that stiffen the 90 degree intersection of the plates. Initial values of I and J were computed based upon the cross-section of the intersection.

An equation was written to specify the error to be minimized. This equation was essentially the same as Eq. 1 except that the thicknesses and resonant frequencies were divided by their initial values to produce fractional changes. Test data as a whole (wt) was weighted at 100.0 and the initial model as a whole (wm) was weighted at 0.1, or one-thousandth as much. The lowest four resonant frequencies were used. The bar properties were considered as design variables but were not included in the equation due to their large uncertainty. Equation (2) shows the error that was minimized:

$$E = wt*\Sigma((Ti-Ai)/Ai)^2 + wm*\Sigma((Pj-Oj)/Oj)^2 \quad (2)$$

where
wt = confidence of the test data (100)
Ti = test eigenvalue
Ai = analysis eigenvalue
wm = confidence of model parameters (0.1)
Pj = updated model parameters
Oj = original model parameters

The intent is to find the set of updated parameters Pj that minimize the error. Note the scaling used to derive fractional changes.

MSC/NASTRAN's optimizer uses response sensitivities to select new parameters at each iteration in the process. This iterative cycle continues automatically until convergence is achieved by a lack of change in the parameters or the objective function between consecutive iterations.

Results of the model updating process are summarized in Table 2. The overall process required 10 analyses to converge (the initial analysis plus nine optimization iterations).

Table 2: Frequencies After Eigenvalue Matching

	Frequencies (Hz)		
Mode	Measured	Initial	Revised
1	373	302	374
2	1374	1154	1364
3	1570	1390	1497
4	1770	1565	1758

There was another piece of test data that was matched: the weight of the structure. This weight, about 0.95 lb, was input as a performance constraint (not as part of the objective function). Thus, the optimized model matched not only the lowest four resonant frequencies but also the weight.

STEP TWO: MATCHING MEASURED FREQUENCY RESPONSE

Frequency response measurements were made at 49 locations and at numerous frequencies. Based on high signal-to-noise ratios at the resonant frequencies, it was decided to match the measured frequency response at the lowest four resonant frequencies (373, 1374, 1570, and 1773 Hz) and for a subset of points (16 out of the 49 were used). The design variables and design constraints were the same as those used for matching the resonant frequencies. The initial model is the optimized model that resulted after the eigenvalues were matched. Equation (3) shows the error to be minimized:

$$E = \Sigma(1-(Ti/T)/(Ai/A)^2 \qquad (3)$$

where Ti = test acceleration magnitude
T = test reference acceleration
Ai = analysis acceleration
A = analysis reference acceleration

The intent is to find the set of updated parameters Pj that minimize the error between the measured and analytical response shape at each of the four lowest resonant frequencies.

Note that because we got a very close match to the eigenvalues it was decided not to take into account the difference between the initial and final model parameters in the error equation. In other words, we have relatively high confidence in the updated model derived from step 1 and at this point want to get the best match to the frequency response test data, regardless of the model changes. The measured lowest four eigenvalues were input as performance constraints, to force the eigenvalues to remain close to the eigenvalues derived in step 1. The weight was also used as a performance constraint.

A reference accelerometer was chosen in order to normalize the results and test data. This ensures that the relative response shape is measured and not the absolute value of response. Matching the shape minimizes the dependence on the dynamic amplification near resonance, as illustrated in Figure 3.

The appendix shows the Executive Control, Case Control, and a portion of the Bulk Data for the input file for matching frequency response test data.

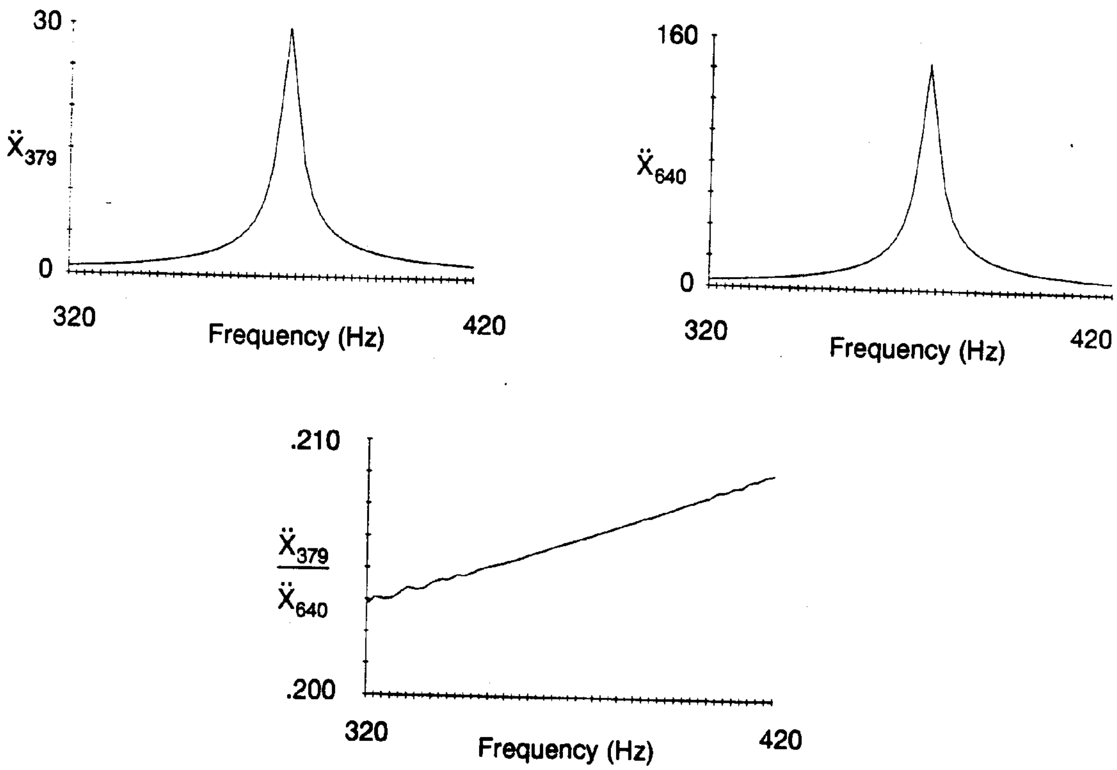

Figure 3. Absolute Values of Response (top) vs. Response Shape (bottom).

The objective function (the error in Eq. (3)) was decreased from 101 to 25. The decrease would probably have been greater (thereby leading to a better match of the response shape) if there were no performance and design constraints in the model, but the constraints have to be used to ensure realism.

It was discovered during this second step that the approximation method used in the optimizer made a difference in how the optimization proceeded. (The approximation method is set by the APRCOD field on the DOPTPRM Bulk Data entry.) The default method, the Mixed Method, could not "get started" in minimizing Eq. (3). Direct Linearization, on the other hand, performed the minimization and is the recommended method when dynamic excitations are near or above the fundamental resonant frequency.

MAC values [3] for the lowest four analytical modes were high (0.90 or greater) in the initial model, and they did not improve in the revised model. Had the MAC values of the initial model been lower, the frequency response matching would have yielded a substantial improvement in their values.

Table 3 shows the resonant frequencies after matching the frequency response. The "initial" model frequencies in the table are the frequencies after the eigenvalue matching process (step 1).

Table 3: Frequencies After Matching Frequency Response

	Frequencies (Hz)		
Mode	Measured	Initial	Revised
1	373	374	375
2	1374	1361	1361
3	1570	1571	1570
4	1773	1782	1779

DISCUSSION AND SUMMARY

There are several advantages to using design optimization for model refinement to match dynamic test data. Model updating using design optimization requires no user interaction since all iterations take place within MSC/NASTRAN. Also, factors can be applied to weight the test data versus computed responses, reflecting the relative confidence of the experimentalist and analyst. Factors can also be used to weight test data and model parameters relative to each other. In addition, upper and lower bounds can be applied to each of the design variables to ensure that a realistic model is obtained.

When matching frequency response results, it is recommended that a match first be made to the measured eigenvalues, and then constrain these eigenfrequencies while trying to match the frequency response test data. It is also recommended that measurements be taken at numerous locations and to then match *ratios* of measurements instead of the actual measurements (this greatly lessens the effect of the dynamic amplification near resonance).

When matching test data it is recommended to use as much data as possible. For example, an earlier model revision process for the enclosure [4] did not take into account the weight and as a consequence the final model obtained was unrealistic.

The matching of frequency response results is in its infancy. Investigators are still proposing numerous methods for doing the matching. The matching is complicated by the fact that there is potentially a lot of data, there is not a good way of handling the large amount of data, the effect of noise is significant, the measurements may not have been made exactly at the model's grid points, and the tools available for matching have only been recently available.

Further investigations by MSC will continue, primarily to determine better (i.e., less nonlinear) equations that define the error. Also, data handling methods will be explored in order to determine the feasibility of automatically finding a minimum subset of measurements that still yields an accurate model after updating. The author would greatly appreciate hearing about any ideas and experiences with model updating.

REFERENCES

1. Collins, J.D., Hart, G.C., Hasselman, T.K., and Kennedy, B., "Statistical Identification of Structures," *AIAA Journal*, Vol. 12, No. 2, 1974.

2. Isenberg, J., "Progressing from Least Squares to Bayesian Identification," *Proceedings Winter Annual Meeting*, ASME, Dec. 1979.

3. Blakely, K. and Rose, T., "Cross-Orthogonality Calculations for Pre-Test Planning and Model Verification," *Proceedings MSC World User's Conference*, The MacNeal-Schwendler Corporation, 1993. (Also reprinted in the *Proceedings MSC European User's Conference*, 1993.)

4. Blakely, K., "Updating MSC/NASTRAN Models to Match Test Data," *Proceedings MSC World User's Conference*, The MacNeal-Schwendler Corporation, 1991.

Appendix

This appendix lists the Executive Control, Case Control, and a portion of the Bulk Data for frequency response matching. Note that the input file is for Version 68.

```
ID KB, MSC    $ Version 68
$
TIME 9999
SOL 200
$
CEND
TITLE = DISK DRIVE FREQUENCY RESPONSE ANALYSIS
SUBTITLE =    MATCH TEST FREQUENCY RESPONSE--USE UPDATED PROPS.
LABEL =       FORCE NEAR FIRST FOUR FREQUENCIES
SET 9998 = 640,379,31,383,387,44,392,653,542,
 666,57,1221,1207,1194,1338,1357
$
METHOD = 10
$
DESGLB = 9999   $ "Global" constraint--mass of structure
$
LOADSET = 2010
$
$ EIGENVALUE ANALYSIS
SUBCASE 1
DESSUB = 1
ANALYSIS = MODES
DISPLACEMENT(PLOT) = 9998
$
$ FREQUENCY RESPONSE ANALYSIS
SUBCASE 2
ANALYSIS = MFREQ
DESOBJ = 700    $ Reference DRESP2
ACCELERATION(PHASE)=9998
DLOAD=2020
FREQUENCY=2005
SDAMP=2007
$
BEGIN BULK   $ Optimization entries (abridged) only
$
$ Change optimization method to Direct Linearization
DOPTPRM APRCOD  1       DESMAX  20
$
$ Multiply BAR props. by 1000; divide by 1000 on DVPREL1
PBAR    999     999     0.      6.6E-4  4.3E-4  7.446E-4
DESVAR  991     I1      6.6E-1  1.0E-7  1.0
DESVAR  992     I2      4.3E-1  1.0E-7  1.0
DESVAR  993     I3      7.446E-11.0E-7  1.0
DVPREL1 991     PBAR    999     5       1.0E-7
        991     1.0E-3
DVPREL1 992     PBAR    999     6       1.0E-7
        992     1.0E-3
DVPREL1 993     PBAR    999     7       1.0E-7
        993     1.0E-3
$
```

```
PSHELL  1       1       .1104   1
PSHELL  2       1       .1600   1
... etc. ...
PSHELL  300     1       0.1045  1
DESVAR  1       PS1     .1104   .07     .130
DESVAR  2       PS2     .1600   .10     .20
... etc. ...
DESVAR  300     PS300   .1045   .07     .130
DVPREL1 1       PSHELL  1       4
        1       1.
DVPREL1 2       PSHELL  2       4
        2       1.
... etc. ...
DVPREL1 300     PSHELL  300     4
        300     1.
$
$ Weight response
DRESP1  9999    WT      WEIGHT                                          ALL
$
$ Mass constraint
DCONSTR 9999    9999    0.0023  0.0025
$
$ Eigenvalue responses
DRESP1  101     FREQ1   EIGN                    1
DRESP1  102     FREQ2   EIGN                    2
DRESP1  103     FREQ3   EIGN                    3
DRESP1  104     FREQ4   EIGN                    4
$
$ Eigenvalue constraints
DCONSTR 1       101     5.47E6  5.53E6
DCONSTR 1       102     72.6E6  74.8E6
DCONSTR 1       103     97.2E6  97.6E6
DCONSTR 1       104     124.0E6 125.4E6
$
$ Synthesized response based on equation (100),
$  test data (DTABLE), and computed frequency response (DRESP1)
DRESP2  700     FREQ1   100
        DTABLE  A640T3  A379T3  A31T3   A383T3  A387T3  A44T3   A392T3
                ... etc. ...
                D1357T2
        DRESP1  1001    1003    1006    1010    1015    1019    1022
                ... etc. ...
                4049
$
$ Objective function; 640T3 is test reference and 640 is analysis ref.
DEQATN  100     F(A640T3,A379T3,A31T3,A383T3,A387T3,
        A44T3,A392T3,A653T3,A542T3,A666T3,
        ... etc. ...
        D1194,D1338,D1357) =
        (1.-(A640T3/A640T3)/(A640/A640))**2 +
        (1.-(A379T3/A640T3)/(A379/A640))**2 +
        ... etc. ...
        (1.-(D1357T2/D640T3)/(D1357/D640))**2
$
$ Frequency response accelerations
DRESP1  1001    FA640   FRACCL                  3       374.    640
DRESP1  1003    FA379   FRACCL                  3       374.    379
... etc. ...
DRESP1  4049    FD1357  FRACCL                  2       1782.   1357
$
$ Table of test data (measured frequency response accelerations)
DTABLE  A640T3  11.88   A379T3  3.03    A31T3   9.55    A383T3  2.24
        ... etc. ...
        D1207T2 1.27    D1194T2 0.59    D1338T2 0.99    D1357T2 0.04
```