Parameter Estimation Using Frequency Response Tests

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Abstract

Structural optimization techniques in MSC/NASTRAN may be adapted to improve the correlation between finite element calculations and dynamic test results. The goal of the system is to reduce the "errors" in the finite element results by predicting changes to selected structural properties. Modern methods, which minimize weighted differences between test and analytic results over many excitation frequencies, have been adapted to the MSC/NASTRAN structural design optimizer. Response amplitudes from forced sine-sweep excitations are used as the basic inputs and actual structural properties changes are the calculated results.

This approach bypasses many of the previous difficulties by using the following methods: 1) The error measures are defined directly from the solution vectors to avoid large complicated symbolic equation entries and manually transcribed data tables, 2) Frequency response solutions are used to avoid the difficult task of calculating eigenvector derivatives, and 3) Constraint equations are built into the solution to enforce test responses and produce faster convergence. A minor amount of automated preprocessing is the necessary extra effort to use the standard V68 system. Test results show the feasibility of the approach, and perhaps its practicality. Results will be shown for a classical example problem.

Background

Previous Modal Methods

The use of quantitative methods for assessing the differences between dynamic tests and finite element analysis has been an active field for many years. When finite element analysis was first introduced it was treated cautiously by the test-oriented engineers. If a difference existed between the different approaches the FE model was suspected and major modeling revisions were attempted to cure the discrepancy. With the gains in reliability with FE methods this attitude has changed rapidly over the years. Now the goal is to find the actual physical differences regardless of the source.

Modal correlation was the favorite method for dynamicists. The references contain only a small fraction of the hundreds of related publications issued since 1975. Methods like the MAC[1], COMAC[2], and many other variants have been used to match eigenvalues and eigenvectors for lightly damped structures. These methods provide direct information on the quality of the correlation but have difficulty in identifying specific physical properties. Also these methods had some difficulty when closely coupled modes and finite damping occurred in the structures. The problem was that the information available from a few measurements for a few mode shapes was only sufficient to identify a small number of structural changes.

Another step forward has been to adopt the design optimization techniques (Flanigan[3] and Blakely[4]) available in MSC/NASTRAN to minimize the differences between test and FE analysis results. The problem here has been the difficulty in obtaining sensitivity derivatives efficiently for the case of many eigenvectors or response solutions. This has led to a mixture of data comparisons, including combinations of modal and forced response samples.

Direct frequency response methods have been adapted to analyze larger, more complex structural systems when a large number of structural parameters may be modified and when the test frequency range includes many modes. The major proponent of these methods has been Lin /Ewins[5], with other recent developments by Ibrahim, et al[6] and Conti/Donley[7]. These methods try to match the actual test responses for many frequencies with sinusoidal steady state loads. The structural parameter changes are defined by real structural properties and some form of minimization technique is used to solve the underdetermined system. In particular, this paper uses the MSC/NASTRAN design sensitivity option to find linearized derivatives of the responses with respect to the property changes (AKA design variables) for multiple frequencies. Several other researchers are known to be pursuing this direction.

Structural optimization methods have also been improved in recent years. Current systems such as the MSC/NASTRAN code[8] are capable of dealing with thousands of design variables and design constraints (AKA output quantities) simultaneously with many frequencies. Several linear programming and nonlinear solution options are available as well as a general user-defined equation mode for defining constraints and objective functions. In essence, the optimizer modules will determine specific design changes that will result in a feasible solution that meets the constraint limits and minimizes an objective function. The result is a system that can be adapted to solve the Test/Analysis correlation problem for a variety of reasonable problems.

Current Difficulties

One difficulty with using design optimization methods for test/analysis correlation is that the size of the test data for frequency response can be large and awkward to process. The input for the definition of a single "constraint equation" for each response point at each frequency may require 2-3 lines of hand-transcribed numbers. A second obstacle is that the response quantities, especially near modal frequencies, are highly nonlinear and may cause trouble for the linear approximation methods used by the optimizer. An analogy is the example of a nonlinear structure with a solution in the post-buckled range. Attempts to use the linearized structural tangent matrix from the pre-buckled range predict an incorrect direction the search never gets past the discontinuity. Similarly, near a normal mode, the large slopes and discontinuities in the response curves may not allow the optimizer to move the natural frequencies across a measured frequency point. For this reason Ewins recommends that engineers avoid the modal frequencies in selecting test frequency samples.

The objective of the project described below was to: a) adapt the MSC/NASTRAN design optimization methods for determining the feasibility of parameter estimation, and b) expand the limitations of the system to allow for higher modal density, damping and larger parameter changes.

Theoretical Development

Basic Matrix Methods

The information at the starting point for the process is the combination of a series of dynamic test results along with a finite element model (FEM) which produces similar but different results. Specifically a steady-state frequency response function, with displacements, $u_j(\omega_i)$, or accelerations at selected points j, and at selected frequencies, ω_i, from the test. An analytic model is defined in terms of linear matrices, At each frequency define the impedance matrix, [**Z**], as:

$$[Z^a(\omega_i)] = -[\omega^2 M^a + i\omega B^a + K^a] \qquad (1)$$

where $[M^a]$, $[B^a]$, *and* $[K^a]$ are the mass, damping, and stiffness matrices for the analytic model. Note that these matrices may be complex and unsymmetric in MSC/NASTRAN. The analytic solutions to the loads, $P_i = P(\omega_i)$, are:

$$[Z^a(\omega_i)]\{u_i^a\} = \{P_i\} \qquad (2)$$

The objective of this procedure is to find the modifications to the finite element properties which will produce a new set of impedance's, $[Z^a(\omega_i)]$, such that for each selected frequency:

$$[Z^x(\omega_i)]\{u_i^x\} \cong \{P_i\} \qquad (3)$$

where the vector, $\{u^x_i\}$ at each frequency contains results that match the test data. The change in impedance is defined as:

$$[\Delta Z(\omega_i)] = [Z^x(\omega_i)] - [Z^a(\omega_i)] \qquad (4)$$

The matrix changes in *M, B, and K* which form the matrix *Z* in turn may be defined by structural parameters such as cross sectional areas or nonstructural mass densities. Note that the potential number of parameters could grow to a large number if every element in the analytic system could change independently. Fortunately, the important parameters is usually a reasonable number. If a sufficient number of degrees of freedom are measured at enough frequencies the number of unknown quantities becomes larger than the unknowns it would appear that the approximation in Eq.(3) could be exact. However, the number of unknowns also includes all of the unmeasured degrees of freedom in the model! If this quantity is larger than the number of measured coordinates, and many unknown structural parameters have an effect on the whole structure, then an approximation is the best we can hope for, and a unique solution is impossible..

We can show the relationship to the Lin/Ewin's approach to solving Eq.(3) by subtracting Eq.(2) from Eq.(3) and substituting Eq.(5) to obtain:

$$[Z_i^a]\{u_i^x - u_i^a\} \cong -[\Delta Z_i]\{u_i^x\} \qquad (6)$$

and by inverting $[Z^a]$ at each frequency we obtain a basis for iteration :

$$\{u_i^x - u_i^a\} \cong -[Z_i^a]^{-1}[\Delta Z_i]\{u_i^x\} \qquad (7)$$

In other words the changes in the responses are restricted by the constant analytic structural properties in $[Z^a]$. The advantage to this method is that the rows corresponding to the measured points may be solved simultaneously with the unmeasured points. The unmeasured points in the

$\{u^x_i\}$ vectors are updated with each estimate of $[\Delta Z_i]$. The errors in the tested degrees of freedom become the measure of convergence in the iterations. The problem is that the whole right side vector, $\{u^x_i\}$ needs to be approximated on the first step. If the vectors, $\{u^a_i\}$ are used for the first estimates, large differences in the incremental impedance may cause local stresses. Another difficulty is the practical aspect of automating the iteration procedure. The test displacements or accelerations become difficult to bring into the solution process. An alternate method, developed below, uses a constrained initial solution to initiate the search and adopts the MSC/NASTRAN optimization system for the parameter updates and search process.

The MSC Optimizer

Many of the processes necessary for dynamic test parameter updates are available as automatic procedures in the design sensitivity and optimization solutions in MSC/NASTRAN[8]. These include processing of structural parameter changes as "design variables" with matrix updates and sensitivity calculations, automated search iterations, and controls over the errors using the "design constraints". Although the system was designed primarily for changing the performance of the structural design it is general enough to accommodate other definitions of an "improved design". However, in order to adapt the test update process to the optimization requirements we need to modify the equations above.

The Constrained Test Point Method

Define the partitions of the estimated solution vector for each frequency, ω_i, and each iteration, n, as:

$$\{u^{xn}(\omega_i)\} = \begin{Bmatrix} u^n_{oi} \\ u_{ti} \end{Bmatrix} \qquad (8)$$

where the "t" partition contains the known displacements of the tested points and the "o" partition contains the untested degrees of freedom. From Eq.(3) define a set of errors to be minimized as:

$$\{\delta^i_n\} = [Z^{xn}_i]\{u^{xn}_i\} - \{P_i\} \Rightarrow 0 \qquad (9)$$

Expand Eq.(9) into partitions:

$$\begin{Bmatrix} 0 \\ \delta^n_{ti} \end{Bmatrix} = \begin{bmatrix} Z_{oo} & Z_{ot} \\ Z_{to} & Z_{tt} \end{bmatrix} \begin{Bmatrix} u^n_{oi} \\ u_{ti} \end{Bmatrix} - \begin{Bmatrix} P_{oi} \\ P_{ti} \end{Bmatrix} \qquad (10)$$

We could now solve the upper half of Eq.(10) for $\{u^n_{oi}\}$ and substitute the result into the lower half to calculate the error. In effect this is analogous to an enforced displacement solution technique. For each iteration the matrices are updated, and Eq.(10) is evaluated at all frequencies, and the errors are collected and evaluated for use in predicting the next set of changes.

Another method to evaluate Eq.(10) without the extra partitioning steps is to use a hybrid technique as shown below. Starting with Eq.(10), we add another set of degrees of freedom to represent the force errors at the test points, $\{\delta^n_i\}$ and merge the constraint equations into the matrix. At each frequency the modified system is:

$$[Z^c_i]\{u^{cn}_i\} = \{P^c_i\} \tag{11}$$

where:

$$[Z^c_i] = \begin{bmatrix} Z_{oo} & Z_{ot} & 0 \\ Z_{to} & Z_{tt} & c \\ 0 & c & \frac{-c^2}{k} \end{bmatrix}, \tag{12}$$

$$\{u^{cn}_i\} = \begin{Bmatrix} u^n_{oi} \\ u^n_{ti} \\ \delta^n_i \end{Bmatrix}, \tag{13}$$

and

$$\{P^c_i\} = \begin{Bmatrix} P_{oi} \\ P_{ti} \\ cu_{ti} \end{Bmatrix} \tag{14}$$

In this "constrained" system the form is identical to the standard frequency response solution with the error vector, $\{\delta^n_i\}$ conveniently occurring as a partition of the solution vector. The actual meaning of $\{\delta^n_i\}$ depends on the coefficients c and k. Some observations are: -

The terms in the diagonal coefficient matrix, [c], act as scale factors to provide scaling on δ^n_i to supply proper magnitudes for the optimizer.

The diagonal terms, 1/k act like local flexibilities attached between the structure and the test points to attenuate local measurement errors. These are chosen to absorb the effects of random noise and local nonlinearities. Note that if the matrix terms are zero, the values, δ^n_i correspond to Lagrange Multipliers, and the test points are constrained exactly.

With this method the known test values are input as load vectors and also act as direct constraints on the solution. This is more natural and direct than using an iteration procedure to solve for a response quantity which is already known. Any differences between the test specimen and the analytic model are now measured as force errors, which relate directly to finite element properties. The drawback to this approach is that a certain amount of random noise may cause large force errors. By using a loose error criteria the solution will be able to tolerate the noise as allowable error.

Optimization Approach

The theoretical approach for the MSC/NASTRAN Optimization code is given in Ref. 7. Applying Eqs. (11-14) from above to the optimization procedure we can define the following interfaces for V68:

- Design Variables are the structural parameters to be varied. Options include finite element property data, scalar element coefficients, and grid point locations. They define the effects of the matrices $[\Delta K]$, $[\Delta B]$ and $[\Delta M]$ indirectly through the finite element sensitivity matrices..
- Optimization constraints are simply upper limits of the magnitude of the residual loads, δ, for selected points and selected frequencies. Indirect constraints are also available to limit the changes in the design variables for each iteration.
- The objective function may include a variety of parameters, including RMS. for the response errors, limits on weight, and modal properties. Current tests have used a minimization of a simple sum over key frequencies of the squared errors at important points.
- Normal modes for the constrained system will be different than the original analytic model. These natural frequencies will indicate the dynamic response of the unmeasured parts of the structure. With a proper selection of measured degrees of freedom these modal frequencies should be higher than the range of the test.

An important aspect of the optimization procedure is the linearized iteration method using "sensitivity matrices". The actual calculation relates changes in design variables, x , -to matrix changes, by calculating the "pseudo load" vectors, $\{F_{ix}\}$, for all displacements and at all frequencies, where:

$$\{F_{ix}(\omega)\} = \left[\frac{\partial Z^c}{\partial x}\right]\{u_i^c\} \qquad (15)$$

In order to predict the derivatives of the error with respect to the design variables we must find the derivatives of the vector, $\{u_i\}$. First take derivatives of Eq.(11), and obtain the following equation that may be solved for the derivatives of the response, u, with respect to each design change, x:

$$[Z_i^c]\left\{\frac{\partial u_{ix}^c}{\partial x}\right\} = \left\{\frac{\partial P_j}{\partial x}\right\} - \left[\frac{\partial Z_i^c}{\partial x}\right]\{u_i^c\} = \{F_{ix}\} \qquad (16)$$

In MSC/NASTRAN this equation is the basis for design sensitivity calculations in a frequency response system. The vectors $\{F_{ix}\}$ are calculated at the finite element level for all frequencies and design variables using the most recent estimates of displacements, velocities, and accelerations. They is treated as a right-hand-side load vector in Eq.(16) in a second frequency response calculation with results of sensitivity derivatives instead of displacements. The most important derivatives are the partitions corresponding to the force errors, $\{\partial\delta/\partial x\}$,which are used to direct the optimizer in the proper direction. In cases where the analytic solution is close to the test speciman these terms are nearly linear and the convergence is very quick. However for cases with large differences in structural properties and response due to changing modes, these terms become nonlinear and can easily cause divergence in the optimizer search process.

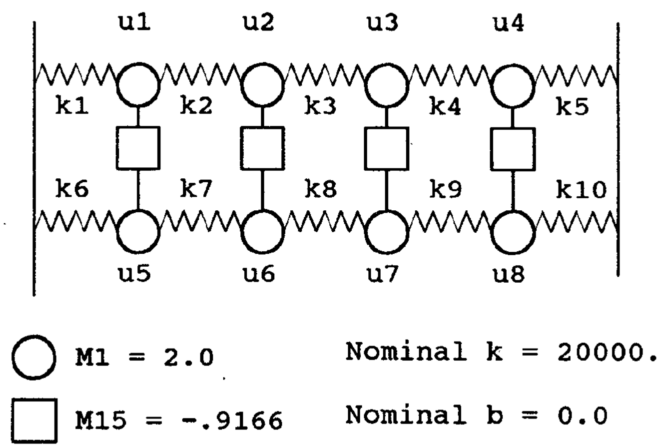
A Discussion of Unique Solutions

If the solution were linear with respect to the design variables we could always expect unique solutions. The number of design variables N_d is the number of unknown variables. The potential number of known coefficients N_u is equal to the number of test degrees of freedom times the number of sample frequencies times two (complex numbers). We recommend using an overdetermined system to allow for possible redundant data. Of course, a large number of frequencies in a small range with only a few active modes will not provide a great deal of independent information. However the use of many sample test points on the structure and the use of multiple load cases will always help. If the design changes are large the nonlinearities may produce several different reasonable solutions.

Examples and Tests

Example Problem: Eight dof Model with Damping

A simple example problem has been used by Ewins to test concepts and feasibility. The properties of the analytic model are given on the sketch below. The displacements are scalar points and the model represents four masses with inertia suspended by cables with fixed ends. The masses are assumed to be fixed and excited by a unit load on point 1. The response results will be, in effect, the transfer functions.



Eight Dof Model, Nominal Properties

The " Test model" is simply the "Analytic model" with the springs, k1 and k9 increased by 30% and two dampers of b= 9.31 introduced at the same connections. The accelerations were sampled at 10 evenly-spaced intervals between 10Hz. and 145 Hz. No attempt was made to sample the natural frequencies. Note that Ewins only used a range covering two or three modes. In our example the test range covers five modes. Results of four points (1,3,6,and 8) from the test were saved for use in the subsequent updating procedure. The results of the experimental model and the finite element properties of the analytic model were combined in the optimization analysis to produce estimates of property changes. In the optimization analysis all 10 stiffness properties were allowed to vary.

The resulting displacements and accelerations in the optimization run are nearly exact, but th somewhat misleading since we are forcing these results with our constraints. The errors are measured by the residual forces of constraint as indicated by the hybrid force output, λ. Thes values of the four measured points, over all 10 frequencies were the functions chosen to be minimized by the optimizer. Some of the many variations tested were:

1 Reduced the stiffness property difference in the test run from 30 % to 5%. This run conve almost immediately with less than 0.1% error. (The optimizer had a much more difficult ti with the larger deviation in the remainder of the tests).

2 Used a constant damping factor on all models instead of varying the damper elements. It necessary at one stage because design sensitivity for damping properties was unavailable i V67.5 and this is a typical application.

3 Tried different constraints combinations of using real or imaginary components of the co response errors as well as requested magnitudes and phase angles. Both options worked equally well.

4 Objective functions to be minimized included combinations and summations of errors-squ and design variables.

5 Discovered an effective method (to be shown in the V68 Design Sensitivity and Optimiza User Guide) which uses the maximum error as the objective function to be minimized. W not sure who originated this method.

In all cases the optimizer was limited to a reasonable number of iterations to simulate the expected results from a larger problem. Some example results are shown in the plots in the attached figures. Three curves are shown in each plot, Analytic, Test, and Modified. The "Modified" results are obtained from another frequency response run using the properties predicted by the optimizer.

Figures 1a and 1b show the results for a case with known damping, infinite stiffness for the enforced motions, and magnitudes of the force error used as both constraints and objective function. (dof8a). The response at 30 frequencies is shown for the analytic model, the test m and the automatically modified model. Figure 2 shows the different properties used for the 1 springs in the three models. Note that the calculated properties of the modified system show relatively high stiffness for elements 2 and 9, while the stiffness of the other elements was reduced.

Figures 3a and 3b show the results at two points using Method 5, above, and using dampers in the test run with both stiffness and damping properties as design variables in the optimizer run. Again the results show an improvement. However, the overall stiffness estimated by the optimizer decreased again.

Figures 4a and 4b show the results with the same inputs as the last case above, except with added flexibility for the enforced motions (The k term in Eq. 12). The optimizer produced much better stiffness properties but now had difficulty with the damping design variables. However note that the peak frequencies corresponding to the modes were very close. An experienced test engineer will say it is impossible to get five normal modes from four accelerometers.

Conclusions

A few large steps have been made for finding easier optimization methods and extending the range for parameter estimation. Problems with high modal density, discrete damping, and large property changes appear to be feasible. DMAP coding has been avoided and the interfaces use simple available formats. Most of the effort has been spent in determining the most effective options in the MSC/NASTRAN optimizer and choosing the best physical variables to include in the optimization criteria. Future work should include larger and more complex realistic test cases and examination of methods for choosing an optimum set of test frequencies.

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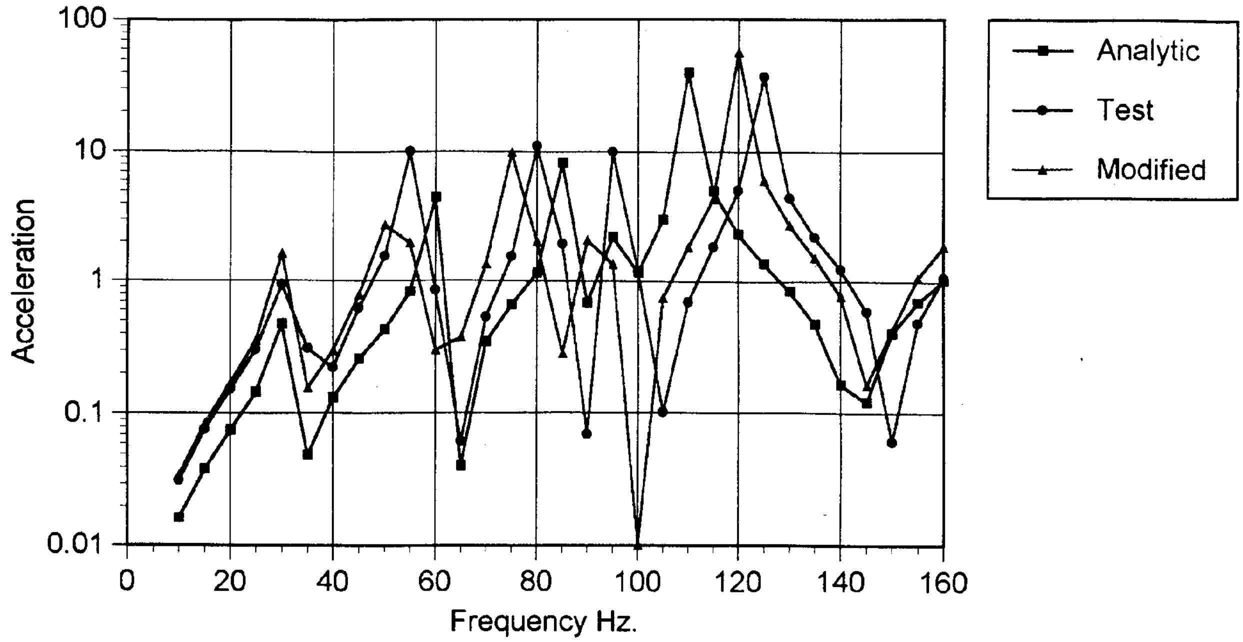

Figure 1a. Response, Point 1, No Damping Elements

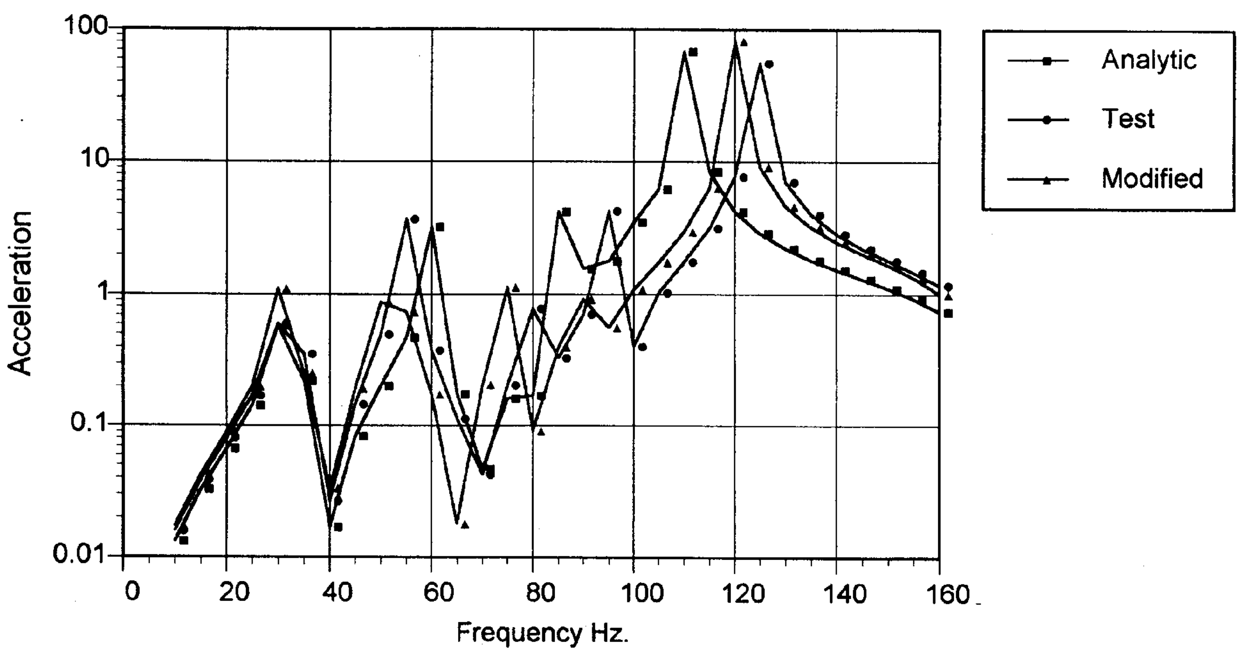

Figure 1b. Response, Point 2, No Damping Elements

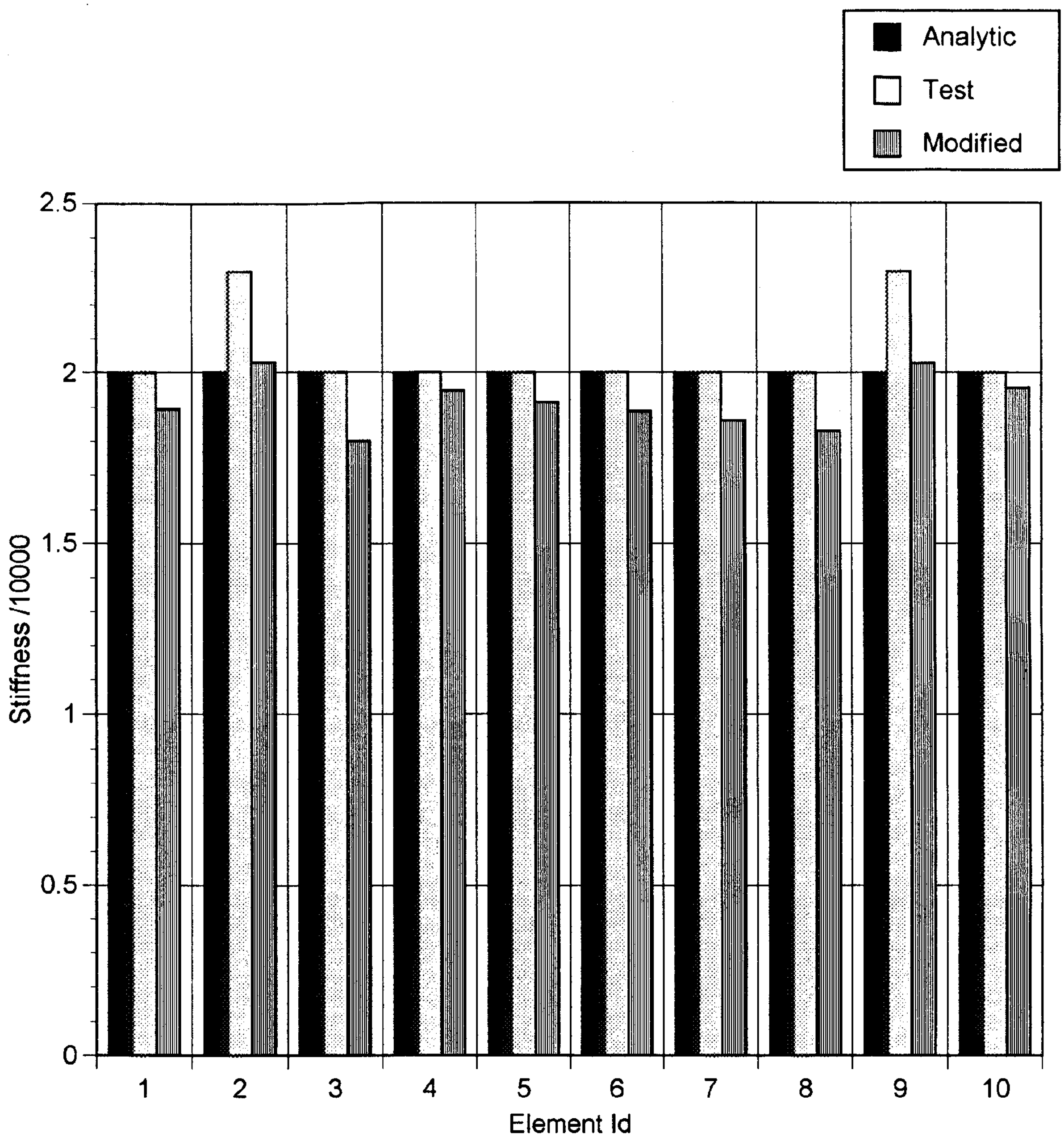

Figure 2. Element Properties, dof8a

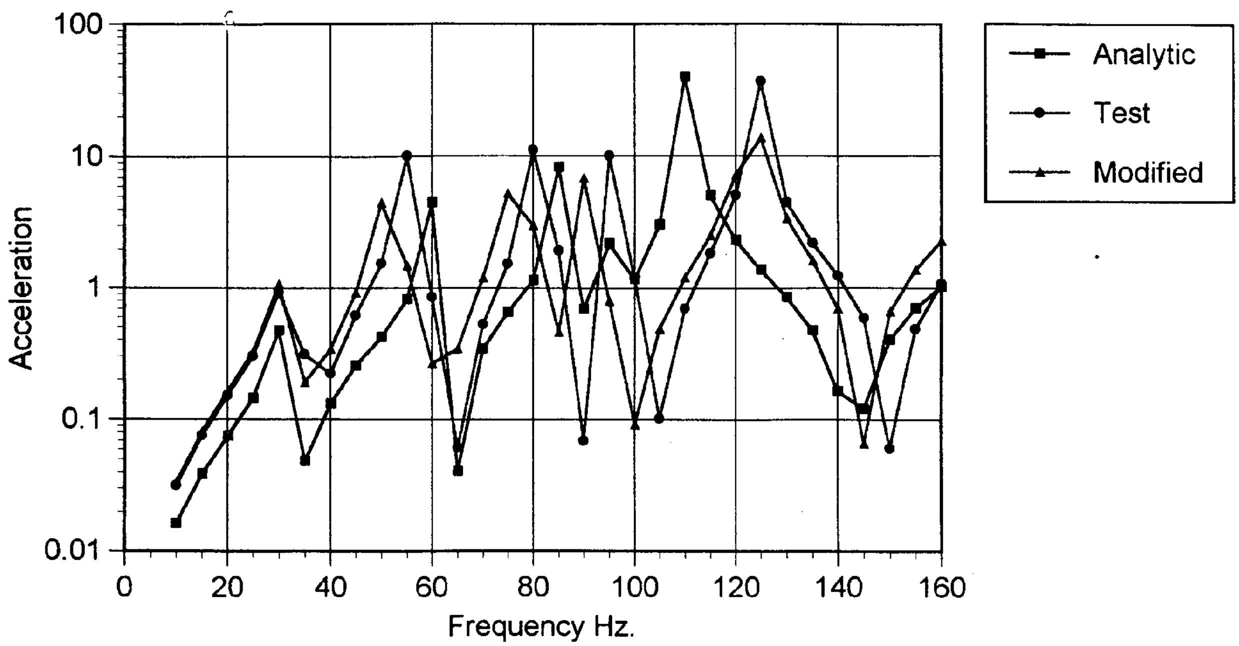

Figure 3a. Response Point 1, Damper Elements

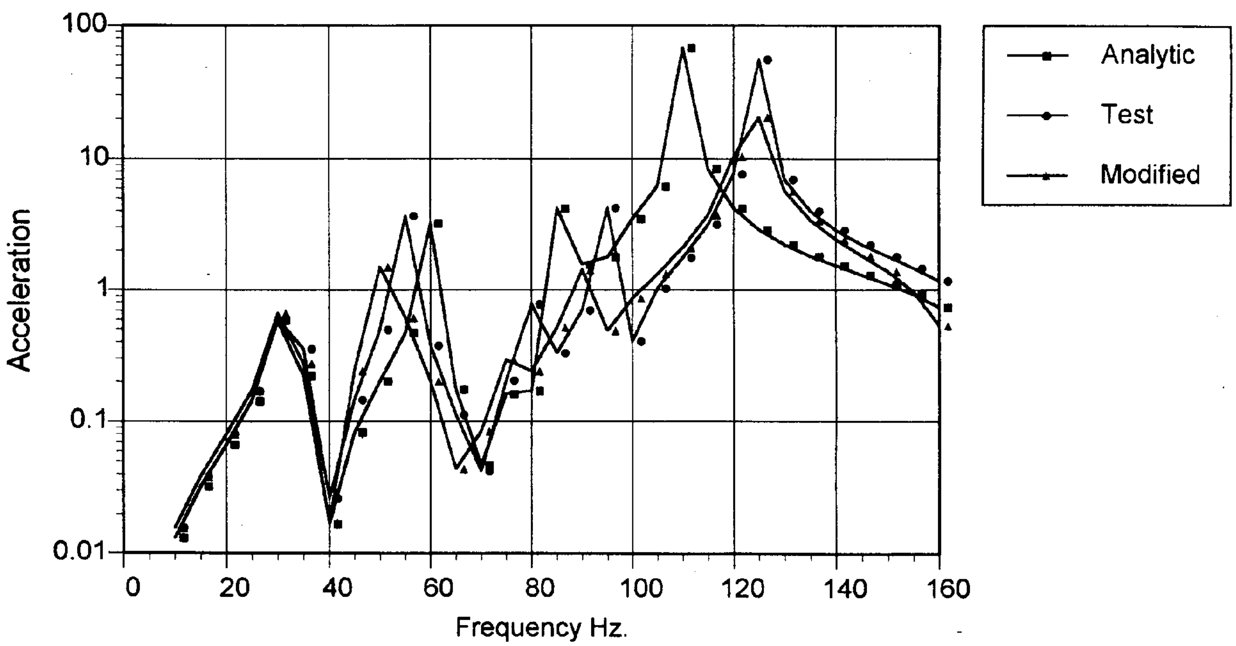

Figure 3b. Response Point 2, Damper Elements

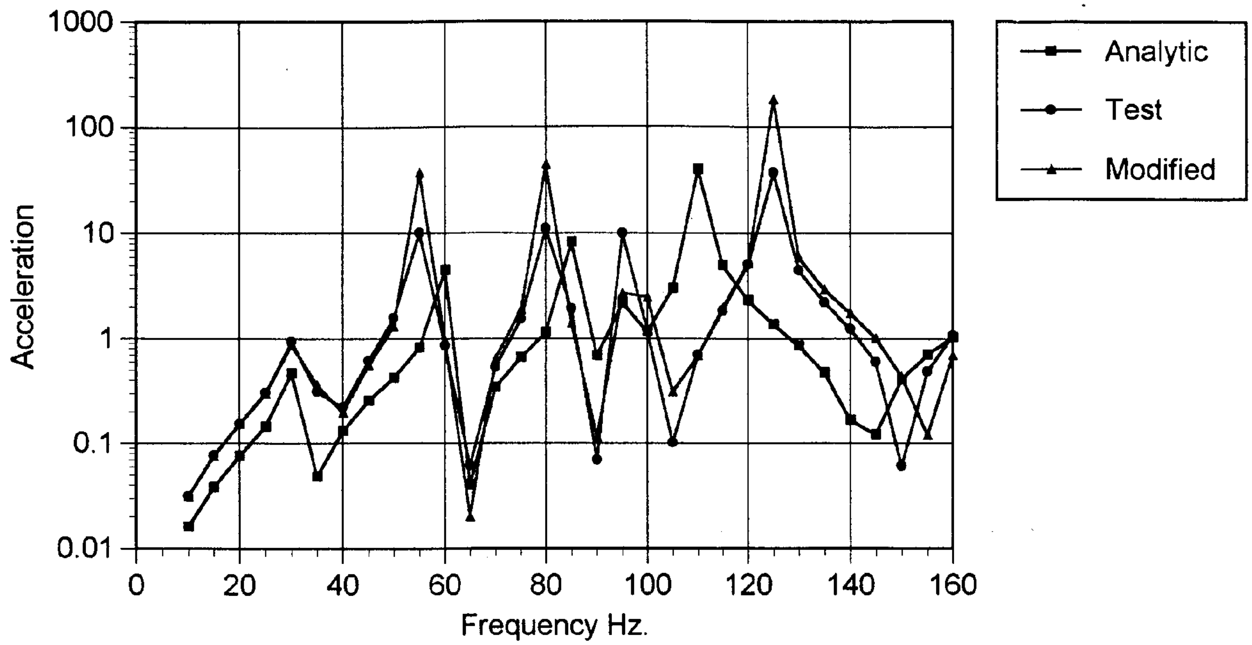

Figure 4a. Response Point 1,Hybrid Method

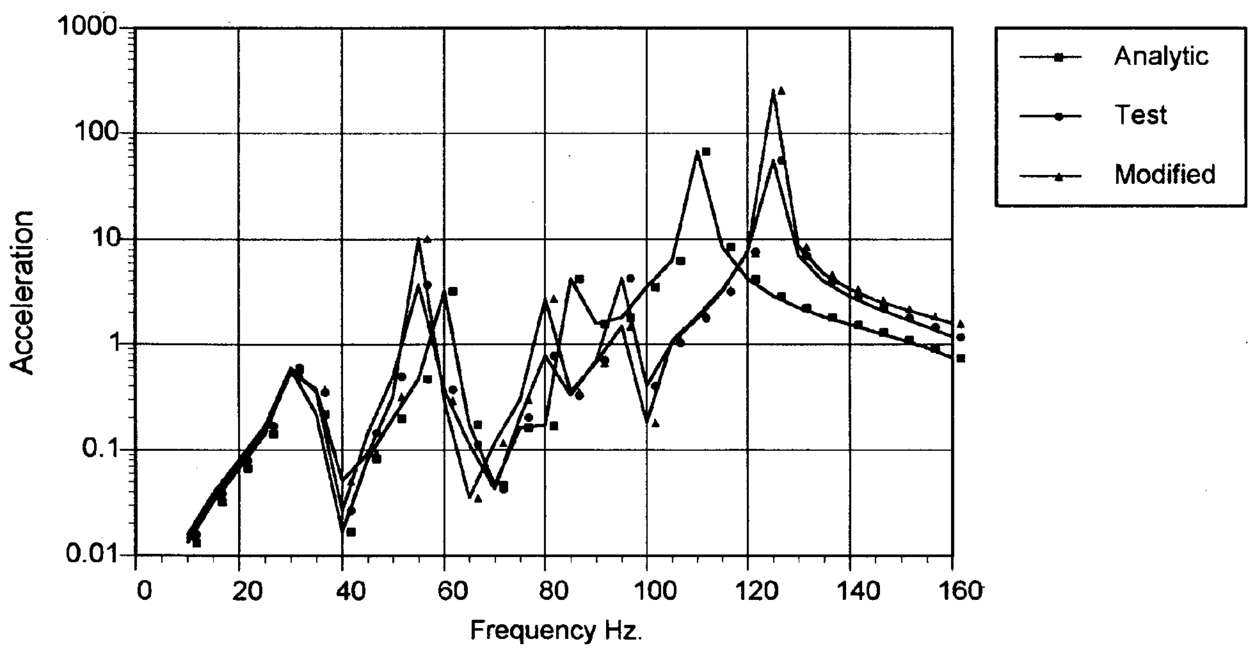

Figure 4b. Response Point 2,Hybrid Method