VARIATION ANALYSIS BY MONTE CARLO RANDOMIZATION OF LOAD VARIATION SENSITIVITIES

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Abstract:

Most analysis considers only the nominal loads acting on a structure, but there may be significant impact due to the variation or error in the loads as well. When there are multiple load sources, the effect from the combination of these load variations is difficult, if not impossible, to predict. This paper describes the use of a Monte Carlo randomization method applied to the displacement results generated from MSC/NASTRAN analyses using sensitivity loads. A Monte Carlo process is used to efficiently obtain a statistical distribution of possible results from the random combination of load variations. Using the method presented minimizes the number of analyses which must be run in order to obtain a population of results from which accurate conclusions can be drawn. The model used represents the High Resolution Mirror Assembly (HRMA) for NASA's Advanced X-ray Astrophysics Facility-Imaging (AXAF-I). The variation analysis discussed considers the impact from support induced load variations during alignment and assembly of the AXAF-I mirrors to the mounting structure.

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Acronyms:

AXAF-I	Advanced X-Ray Astrophysics Facility-Imaging
HRMA	High Resolution Mirror Assembly
MAS	Mirror Alignment System
DOF	degree(s) of freedom
90% EE	90% encircled energy

Definitions: (italicized in text when first defined)

Monte Carlo	a numerical analysis method using random sampling
load variations	deviations in the loads and constraints from the nominal conditions
nominal analysis	analysis of the effects from nominal loads and constraints
variation analysis	analysis of the effects from load variations
nominal residual	strain resulting from perfect load and constraint conditions
variation residual	strain resulting from imperfect load and constraint conditions

1.0 Introduction

Typical finite element analyses involve the evaluation of the deformed shape or stress state of a structure for a given set of loads and boundary conditions. Many times the exact load magnitudes or support locations are known only within a specified tolerance, but nevertheless, analysis is performed using only the nominal load and constraint conditions. For many structures this type of analysis, a *nominal analysis*, may be all that is necessary, but for sensitive structures the deviations of the loads and constraints from nominal, collectively called *load variations*, can have an impact more significant than the nominal conditions themselves. Therefore, it may also be necessary to perform a *variation analysis* to determine the effects of the load and support variations from nominal. As an example, consider a structure uniformly supported level to gravity. Each support exerts a nominal load. However, if the structure is tilted with respect to gravity, then each support exerts the nominal load plus or minus a load variation--yielding a different solution.

To analyze the impact from the variations, one has to either assume a distribution of variations or consider many different cases with random variations. Although the latter method is more realistic, it has the potential of being too computer and manpower intensive. The first method, although efficient, does not reveal the probability, or confidence level, of obtaining the analyzed result.

This paper describes and provides an example of a technique that can be efficiently used to determine the probabilistic result of a structure subjected to random load and constraint variations within a specified tolerance. The technique uses a Monte Carlo method which is explained and demonstrated in its application to variation analyses (section 3). After a description of the analysis problem to be solved and an explanation of the variation analysis concept (section 2.2), the method will be explained (section 3) and then illustrated with a simple example (section 4). Finally, the technique will be applied to the analysis problem for which it was developed (section 5). The discussion in sections 3 and 4 is included as a teaching tool to demonstrate the useful application of this technique to virtually any process where variations from the nominal conditions may cause a significant result. The method has been implemented with a FORTRAN code [1], and all of the program input load variation sensitivities have been generated from MSC/NASTRAN analyses.

2.0 Analysis Problem Background

Although the Monte Carlo method as discussed herein is applicable to a vast range of situations, it was specifically developed for a class of analysis problems involved with the assembly of the Advanced X-ray Astrophysics Facility-I (AXAF-I) High Resolution Mirror Assembly (HRMA), a space telescope optical system. The performance of the optical system is dependent on a number of error sources including, but not limited to, mirror fabrication quality, system environmental conditions, and assembly residual strains. The tasks that use the Monte Carlo method discussed in this paper fall within the category of assembly residual strains. Residual strains can arise from both nominal or variation sources, and it is reasonable to find that the variation strain is larger than the nominal strain. The analysis problem that initiated the development of the Monte Carlo method for variation analysis was the evaluation of the performance impact from the mirror alignment system (MAS) load variations on the AXAF-I mirrors.

2.1 AXAF-I HRMA Structure

NASA's AXAF-I is one of four large orbital telescopes collectively known as the Great Observatories for Space Astrophysics. AXAF-I along with the Compton Gamma-Ray Observatory (GRO), Hubble Space Telescope (HST), and Space Infrared Telescope Facility (SIRTF) serve to gather information about the origin of the universe; fundamental laws of physics; and the birth of stars, planets, and life by imaging astronomical objects across the entire electromagnetic spectrum [2]. AXAF-I is a high-resolution telescope which doubly reflects x-rays off of the mirrors in the HRMA and focuses the image onto detectors in the science instrument module (Figure 1).

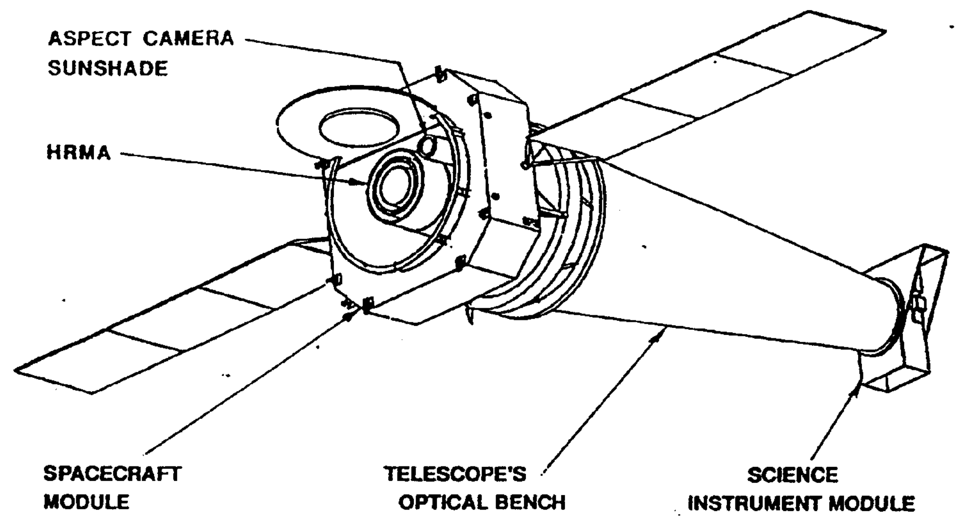


FIGURE 1. AXAF-I Telescope Configuration. Forward section of HRMA is exposed to x-rays.

The HRMA consists of four nested, confocal Wolter Type-I grazing incidence mirror pairs bonded to an elaborate assembly of structural as well as thermal components (Figure 2).

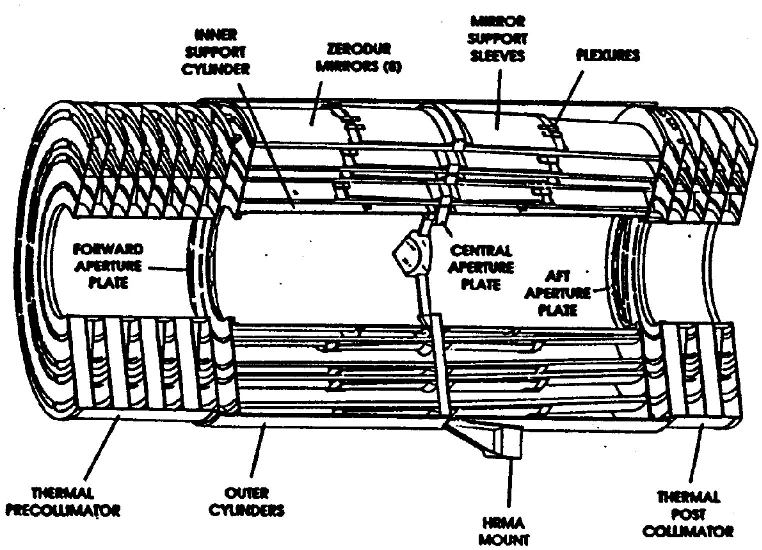


FIGURE 2. HRMA Configuration. Mirrors (8 total) are indicated.

Incoming x-rays at grazing incidence are reflected off the paraboloid mirrors and then reflected again off the hyperboloid mirrors (Figure 3). The focal plane is approximately 10 meters from the center of the HRMA and is located in the science instrument module.

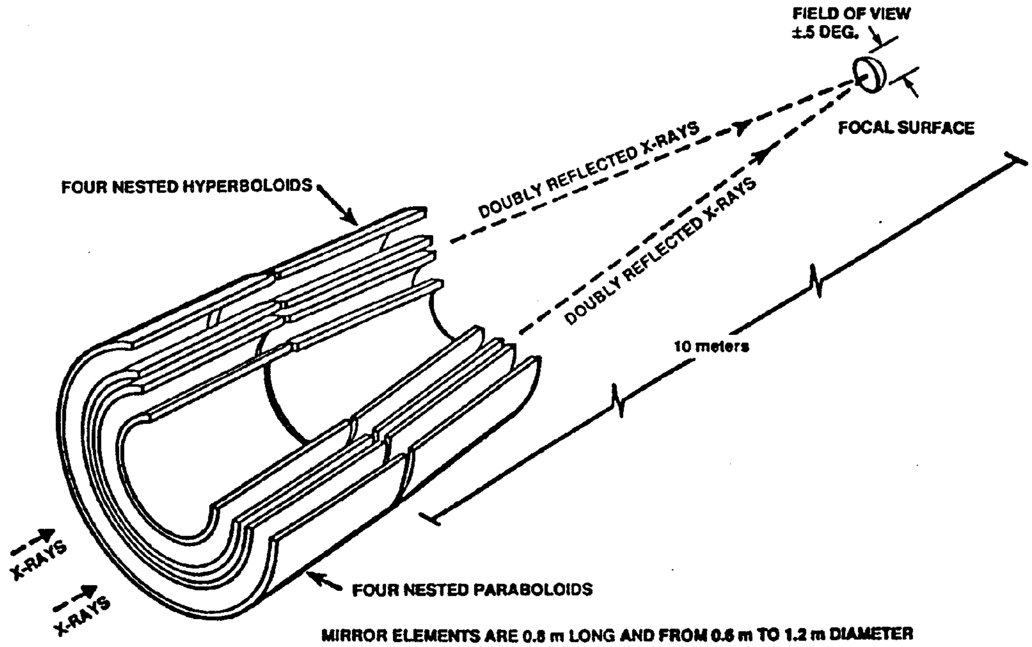


FIGURE 3. HRMA Optical Form. Mirrors shown without support structure.

NASA awarded the AXAF-I contract to the TRW Space and Electronics Group in August 1988. As prime contractor, TRW is responsible for systems engineering and the integration of the entire AXAF-I Observatory, including the design and development of the AXAF-I Spacecraft. The TRW team includes the Eastman Kodak Company as the telescope system subcontractor. Kodak will design and integrate the x-ray telescope, including assembly and alignment of the HRMA--the x-ray imaging portion of the telescope. Other TRW subcontractors include Hughes Danbury Optical Systems, who will grind and polish the mirror elements, and Ball Electronics and Space Division, who will build the housing and components of the science instrument module. The AXAF-I program is managed by the Marshall Space Flight Center with technical support from the Harvard College Smithsonian Astrophysical Observatory [3].

2.2 Mirror Alignment System Variations

One of the major areas of concern during the assembly process of the HRMA at Kodak is the residual impact of the mirror supports during the alignment and bonding of the mirrors to the rest of the HRMA. The mirror residual strains that are added from this assembly process can be described by the combination of the nominal and variation residuals. The *nominal residual* is the strain resulting from a perfect, flawless assembly process in a gravity environment. Even though perfect assembly conditions are assumed, the difference of the 1g environment during assembly and the 0g environment on-orbit causes a residual to be built into the system. The nominal residual from this assembly process and the method used to perform the analysis of an assembly process in general (where loads and boundary conditions change within the solution and each assembly step adds deformation to the previous deformed shape) are described in a MSC 1993 Users' Conference paper [4]. The *variation residual* is the strain resulting from all the imperfections and tolerances that lead to net forces and moments on the mirrors at the support points. These load variations (deviations from nominal) will occur at unknown random magnitudes within the tolerances determined from requirements and will each contribute some strain to the mirror. The net mirror strain is the sum of all the strain contributions. The example analysis used to illustrate the method discussed in this paper deals with these load variations that are present during the assembly operation.

For the purposes of illustrating the Monte Carlo method, a simplified version of the Mirror Alignment System (MAS) variation analysis will be used. Only the first stage of the analysis will be discussed as it applies to the Monte Carlo method. The complete analysis uses the solution method explained in the referenced paper [4] with the same application of the Monte Carlo method. Therefore, the illustration of the Monte Carlo method itself will remain unchanged.

The simplified MAS variation analysis can be reduced to just a mirror with random load variations applied to it at the support points. At each support point, loads can be applied in any of the unconstrained degrees of freedom (DOF). All the mirrors are supported one at a time by the MAS which contacts the mirror at 12 independent, equidistant points. Figure 4 illustrates one mirror (of the eight total) as it is typically supported by the MAS. Three of the 12 points are 'hard points' (spaced 120° apart) which provide a kinematic support. The hard points provide axial and circumferential constraint on the structure at each of the three locations and apply load errors in the other four DOF. The remaining nine points are 'offloaders,' or intricate levers, which provide axial support (force, not constraint) by means of a fixed counter weight at a known distance from the pivot point. The offloaders can apply load errors in all six DOF.

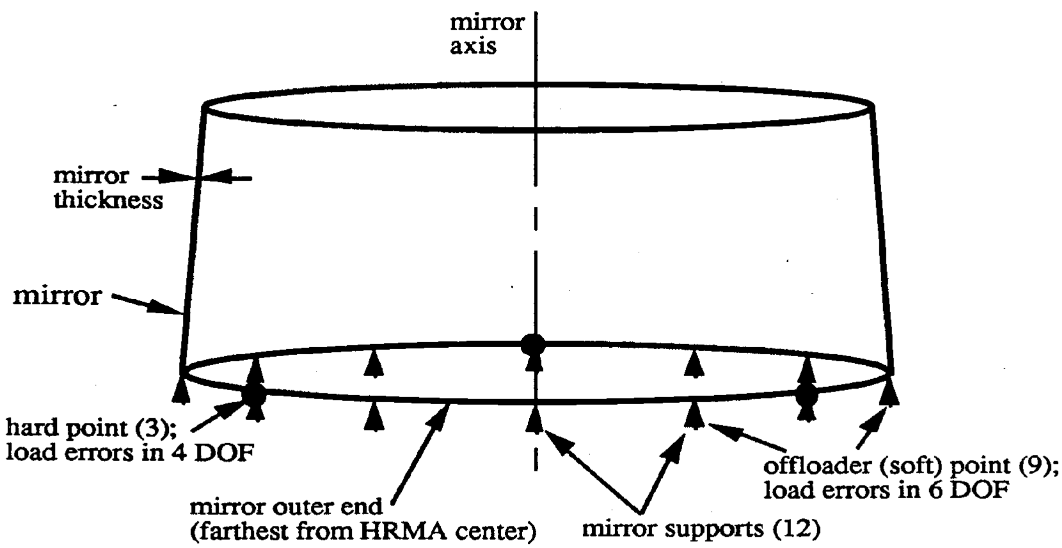


FIGURE 4. Mirror supported by the Mirror Alignment System with support errors in unconstrained degrees of freedom at pickup points.

The load errors in each of the degrees of freedom are caused by the geometry and load tolerances of the system. For instance, the offloader counter weight and counter weight radial location are only known to within required tolerances. The actual weight and location on the lever could be anywhere within the allowable tolerances. The error from these tolerances is an axial load (force) variation from the nominal, exact value. Similarly, there is a tolerance on the nominal mirror contact location in the radial direction. The actual contact location will be within the allowable tolerance, and the resulting theta moment (moment about tangent line) is caused by the radial offset times the axial force applied to the mirror by the device. Likewise, the other tolerances in the system cause load variations at the support points.

The exact values of the variables within the tolerances are random in nature. Biases, known variations from nominal, are analyzed independently from the unknown variations and are therefore not included in these calculations. Totaling up all of the load variation contributors for a particular DOF provides the net maximum load variation for that DOF. The method used to total up the contributors can be simple addition or RSS (root sum of squares) depending on the behavior of the errors. In general, adding the contributors is more conservative than using an RSS, since a RSS value includes the unlikelihood of multiple contributors occurring simultaneously at their extreme magnitudes.

The goal of the MAS variation analysis is to determine the 3σ performance impact from assembling the HRMA using the MAS with load errors in tolerance. The displacement results from the unit variation sensitivity loads are generated from MSC/NASTRAN (v67). The discussion of the finite element analysis and Monte Carlo use is included in section 5 after the explanation of the Monte Carlo method.

3.0 Monte Carlo Method

The *Monte Carlo* method, in general, is a numerical analysis technique which uses random sampling to construct the solution of a physical problem [5] whose evolution is determined by random events [6]. Conclusions are drawn by statistical means from the population of possible results. This method is extremely useful for problems with many variables of a random nature such that the probable net results are difficult, if not impossible, to determine by conventional analysis methods.

3.1 Load Variations

Any analysis of random errors occurring on a structure is appropriate for the Monte Carlo method as long as the individual error sources are modeled so they can be linearly combined with each other. Typically, forces and moments are applied at discrete points on a structure to simulate the load variations. Enforced displacements (constraint variations) at discrete points on the structure are also acceptable since they are actually loads as well. In fact, there is great flexibility to the type of loads allowed as long as all the error sources can be described as specific loads on the model and the loads can be combined linearly with the other loads in the system.

The linear combination of load variations is required so that the resultant displacement from any random load set can be calculated from just load sensitivities (unit loads). In this way, each unique load need only be analyzed once for a given structure. Consider the following set of fundamental equations [7]:

$$\{F\} = [K]\{d\} \tag{1}$$

$$\{d\} = [K]^{-1}\{F\} \tag{2}$$

where the applied forces, $\{F\}$, for the structure of stiffness, $[K]$, cause the displacements, $\{d\}$. Since the displacement is linear with the force, any scalar multiple of the force set, $\{F\}$, results in the same multiple of the displacement set, $\{d\}$. Now consider two different load sets, $\{F_1\}$ and $\{F_2\}$, which cause displacement sets, $\{d_1\}$ and $\{d_2\}$, respectively, or the total displacement, $\{d_t\}$.

$$\{d_1\} = [K]^{-1}\{F_1\} \tag{3a}$$

$$\{d_2\} = [K]^{-1}\{F_2\} \tag{3b}$$

$$\{d_t\} = \{d_1\} + \{d_2\} = [K]^{-1}(\{F_1\} + \{F_2\}) \tag{4}$$

$$\{d_t'\} = (R_1\{d_1\} + R_2\{d_2\}) = [K]^{-1}(R_1\{F_1\} + R_2\{F_2\}) \tag{5}$$

Variables R_1 and R_2 are random numbers (scalar values between -1 and +1), and $\{d_t'\}$ is the particular displacement solution using the random numbers. As a result of Equation 5, if the deformed shapes, $\{d_1\}$ and $\{d_2\}$, caused by the loads, $\{F_1\}$ and $\{F_2\}$, respectively, are known from Equations 3, then the particular solution, $\{d_t'\}$, using random numbers, R_1 and R_2, is known simply by scaling the original displacement sets by the random number magnitudes.

In the above discussion, the force sets, $\{F_1\}$ and $\{F_2\}$, could correspond to the maximum load variation at two different points and/or DOF on the model. The displacement solution, $\{d_t'\}$, would therefore be the resulting deformed shape from one random case of load variations (within tolerance) acting on the structure.

3.2 Monte Carlo Simulation

The power of the monte carlo method is the ability to determine the results of many cases (1,000's) of random load variations on a structure in a timely, efficient way. MSC/NASTRAN is efficiently used to generate sensitivities (output displacements from known loads), and the actual randomization occurs outside the MSC/NASTRAN solution sequence thereby eliminating the need to apply many (1,000's) of predetermined load vectors on the structure.

The analysis problem must be generalized by the number of points that have load variations on them and the number of DOF that will be allowed to vary. For each point and each DOF, one sensitivity (unit load) analysis must be run to determine the structure response to one unit of load at that point in that particular DOF. Each sensitivity can be one subcase in a MSC/NASTRAN solution (i.e., SOL 101).

The Monte Carlo routine must read in the input sensitivity data (nodal displacement sets) and store the information in a matrix. Each set of displacements must be scaled by the allowable tolerance (maximum/minimum load variation) and a random number. Summation of all the displacement sets gives a net deformed shape for one random case. By repeating the random number scaling and summation, more deformed cases can be generated. Figure 5 shows a simple flow diagram for a generic Monte Carlo program. The seven main steps are shown and then summarized into four general headings as labeled to the right of the figure. Note that in step 4 the original matrix scaled by the tolerances is saved for use when the process is repeated from step 7.

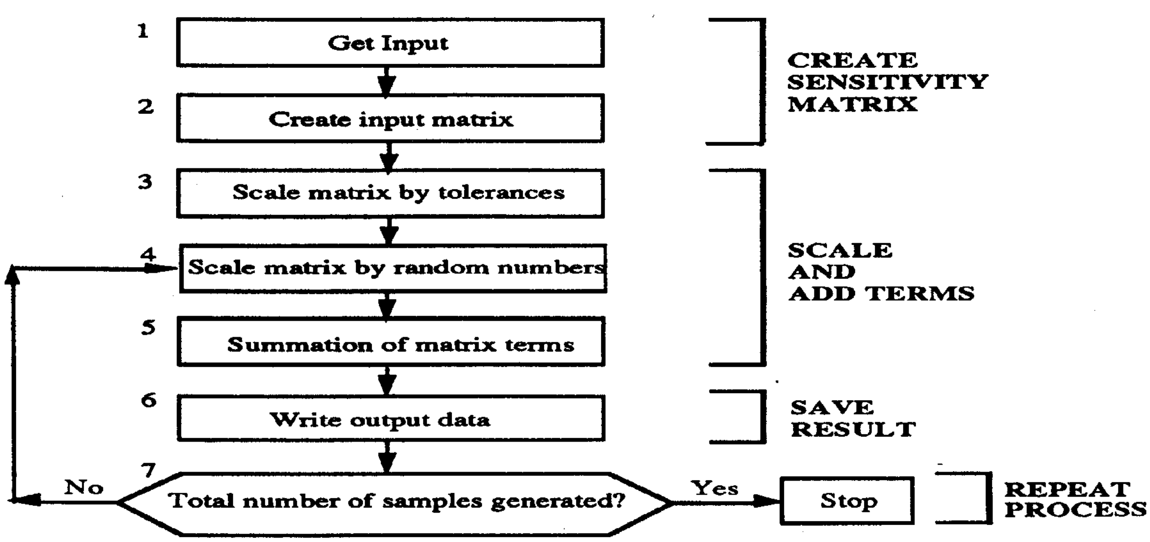


FIGURE 5. Monte Carlo Flow Diagram. Generic program outline.

3.3 Sensitivity Matrix

At the heart of the method is the sensitivity matrix. This matrix contains all of the behavior of the structure based on the specified load points and structural constraints. With this matrix any possible solution using the specified load points can be generated by combining the sets of data using different scale factors. The same Monte Carlo program can be used for different analyses simply by reading in different sensitivity data and creating sensitivity matrices unique to each analysis problem. An illustration of a simple sensitivity matrix is shown in Figure 6.

DOF

$$\begin{bmatrix} C(i,j) \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix} \quad \text{POINTS}$$

$$\begin{bmatrix} C'(i,j) \end{bmatrix} = \begin{bmatrix} d_1(T)(R_1) \\ d_2(T)(R_2) \end{bmatrix}$$

$$D = d_1(T)(R_1) + d_2(T)(R_2)$$

FIGURE 6. Sensitivity Matrix. The sensitivity matrix, C, is scaled to obtain a net displacement, D, for one random case.

The example sensitivity matrix in Figure 6 represents the problem of two varying points with one DOF of error as introduced in section 3.1. The displacement results from the MSC/NASTRAN unit load sensitivity analyses are the quantities d_1 and d_2. These quantities could be the displacement sets for the whole structure or the displacement of a single grid whose net displacement result is desired. The sensitivity matrix, C, is scaled by the tolerance for DOF 1, T, and random numbers, R_1 and R_2, to form the particular solution, C'. The net displacement, D, is found by summation within C'. The tolerance, T, could have been specified by point instead of DOF in which case there would have been two tolerances, one for each point (row of the matrix).

3.4 Symmetry

It may be advantageous to make use of structural symmetry to reduce the quantity of data that needs to be generated for Monte Carlo input. Since a typical system may have at least 10 load points with 6 DOF of possible load variations at each point, there may be more than 60 analysis subcases with 60 sets of output required to complete the input generation. There is the potential of having a cumbersome quantity of data to generate. It may be easier and more efficient to reduce the problem with symmetry and then generate the remaining data within the Monte Carlo code. If the 10 load points of this example are all symmetric with respect to the model and constraint conditions, then only one point needs to be analyzed in all DOF; the other points can be derived from the first. Symmetry is used in the Monte Carlo analysis of the MAS variations to reduce the initial 12 points with variations to only three unique points.

3.5 Results Evaluation

The results from a Monte Carlo analysis is a set (population) of possible results (samples). Naturally, the population of samples can be statistically summarized by a mean and deviation. Consider the problem that has one point randomly varying in one DOF. Assume that the load varies randomly in a Gaussian manner and some output quantity is under study. The results from the Monte Carlo randomization will show the distribution in Figure 7 which reflects ~8000 random samples. The output has been sorted into bins (x-axis) for output magnitudes from -1 to +1 units, and the corresponding frequencies are shown (y-axis). The mean of the distribution in Figure 7 is zero.

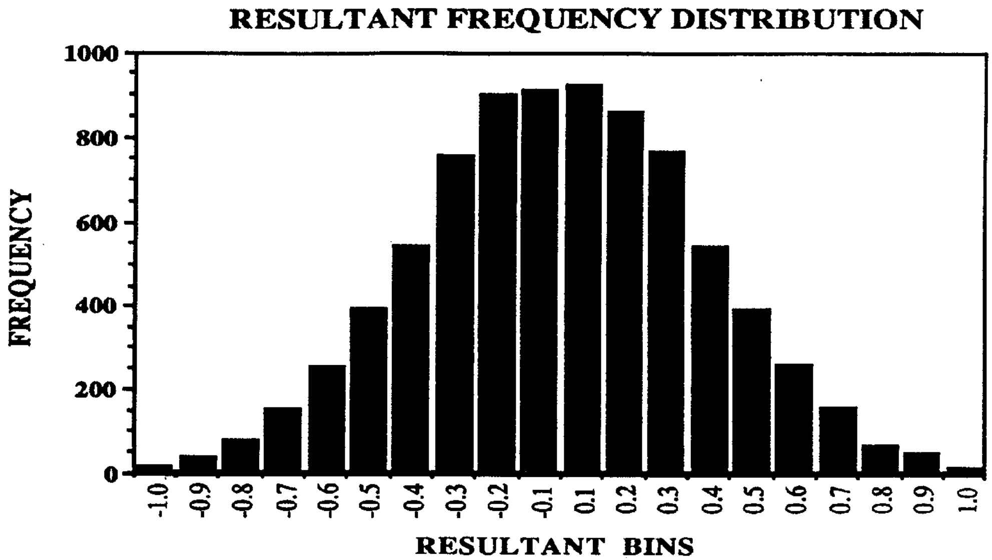
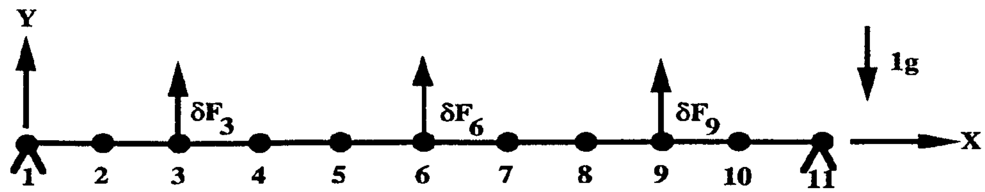


FIGURE 7. Monte Carlo Results Distribution Example.

For this particular example, the distribution is normal, and the standard deviation can therefore be calculated from theory (~0.33). The 3σ value for this problem would therefore be ~0.99. However, many distributions for problems are not normal, and it may not be convenient to directly compute the standard deviation value. However, if the output quantity must be kept within a certain range, then the probability of obtaining a value outside the range can be calculated. For example, if the output quantity values are needed that correspond to the values between which 99.74% (3σ) of the other values occur [8], then the output can be sorted by absolute magnitude and values discarded until 99.74% of the data remains. The next value in the list is the resultant magnitude with a positive or negative sign that bounds 99.74% of the data. In a similar manner, if the output quantity is always a positive value, such as the case of von Mises stress, then the quantity below which 99.87% (3σ) of the other values occur [8] can be determined by sorting the results by magnitude and discarding values until 99.87% remain. A variety of other statistical conclusions can also be drawn as appropriate for the case analyzed.

4.0 Example Variation Analysis Solution

The previous example involved a problem with one point and one DOF of load variation. This example gives the trivial solution illustrated in Figure 7 which is simply a reflection of the random Gaussian numbers used. A more realistic problem has more than one load point and possibly multiple DOF of load variations. In this section, consider the simply supported beam example illustrated in Figure 8, in which there are one, two, or three load variation points with one DOF at each point in every case. This simple example not only illustrates the use of the Monte Carlo process, but also the dangers of overlooking or underestimating the variation analysis problem.



LOAD CASE:

1. **One load variation: point 6**
2. **Two load variations: points 3 and 9**
3. **Three load variations: points 3, 6, and 9**

Load in DOF 2 only.
δF_i is load variation within (±) tolerance.

FIGURE 8. Simply supported beam example with 10 identical elements. Three different load cases considered.

Assume that this example represents a critical component in a structure that is deformed from some body load, such as gravity. It is required to have the beam as undeformed as possible at the center point of the span (point 6). Three additional support points are available: points 3, 6, and/or 9 as indicated in Figure 8. At any of these three locations, a force of a specified magnitude can be applied to the beam. Assume the devices that are to apply the load are only accurate to within 1 unit of load, ±1 pound for this example. The device is to be calibrated so that it will most likely apply the desired force and will least likely apply the maximum load variation, ±1 pound from the desired load; the device has a random Gaussian distribution of error about the set point. Consider the three different options of supporting, or offloading, the beam as listed in Figure 8:

Design #1: One support device total: at point 6
Design #2: Two support devices total: at points 3 and 9
Design #3: Three support devices total: at points 3, 6, and 9

The nominal analysis considers the beam shape from the structure loaded by gravity while supported by the end constraints plus the additional nominal load applied at each of the points as indicated in the three different designs. The nominal loads at points 3, 6, and/or 9 are those required to keep the center of the beam at its initial undeformed position. Therefore, each design is equally acceptable for nominal performance since each of the three results in zero displacement at point 6--a perfect result. Based only on the nominal analysis, all three support designs are equally acceptable; although design #1 is simplest.

The variation analysis considers the beam shape from the structure supported by the end constraints while subjected to variation loads at each of the points as indicated in the three different designs. At each contact point, there is a random (Gaussian) load that is applied to the beam within the specified tolerance. Design #1 has the statistical result already shown in Figure 7. In this section, the magnitude of the extreme displacements (maximum and minimum value on the x-axis of Figure 7) will be determined as well. Designs #2 and #3 involve more than one load variation with different sensitivities at each load point (point 3 or 9 versus point 6). Therefore, it is not immediately obvious what results will be obtained. On one hand, the additional load points may add additional deflection to the beam. On the other hand, the additional load points may allow some loads to cancel each other thus reducing the net deflection of the beam.

Figure 9 illustrates some of the possible beam shapes for different combinations of load variations at the two points of Design #2. The beam shapes illustrated only reflect the cases where the load variations are at the maximum or minimum values (+1 or -1 pound exactly). The two curves that show the greatest deflection at point 6 bound the problem for Design #2; the displacements at point 6 for random cases must lie between these two displacement values.

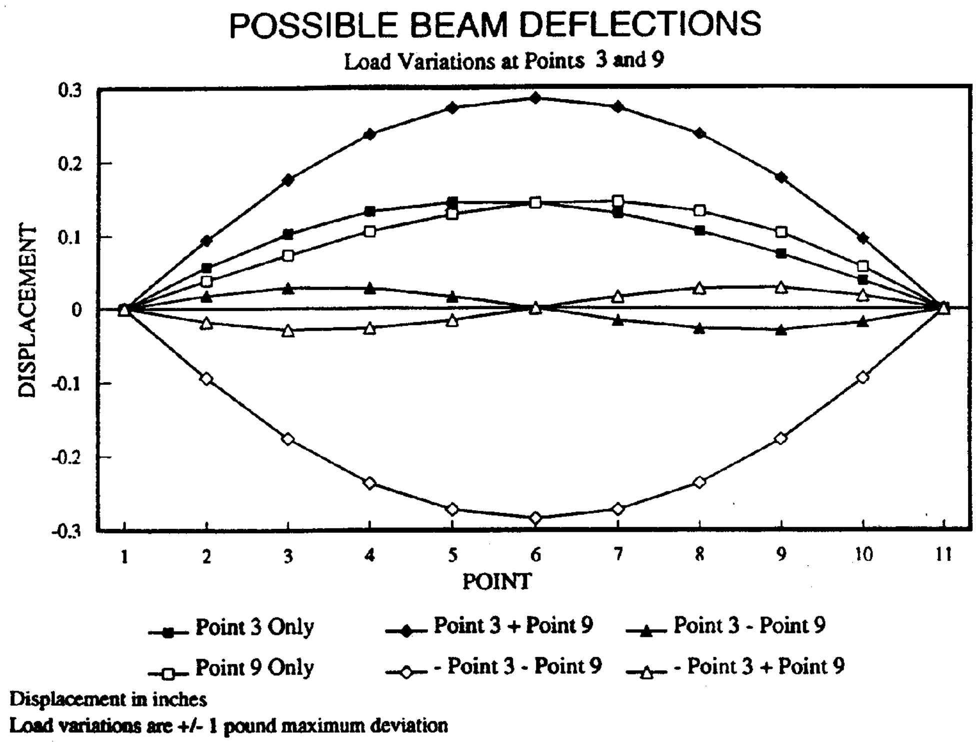


FIGURE 9. Deformed Beam Shapes. Possible deformed beam shapes from variation loads at two points (Design #2).

In order to evaluate the impact from the load variations in each of the three designs, many random samples of possible net displacements at point 6 must be generated for each design. The random samples for this example problem of a Monte Carlo application are generated with the FORTRAN code, Monte-D (Monte Carlo-Displacement), supplied in Appendix A. This code is a simple example of implementing the Monte Carlo technique with MSC/NASTRAN displacement output. Figure 5 already showed the flow diagram which is the basis for the code. Note that the sensitivity matrix, [C], for the analysis of Design #2 is the same matrix illustrated in Figure 6.

The Monte Carlo process can be summarized in three basic steps outlined below.

1. Generate unit load sensitivity output (displacements) using MSC/NASTRAN
2. Run Monte Carlo (FORTRAN) with parameters for the particular design analyzed
3. Statistically summarize random Monte Carlo results

The first step requires that the displacement sensitivities to unit load variations be determined. A unit load is applied in each DOF at each point one at a time in unique subcases. The output file (.f06) with displacements for each subcase is then used as input for the Monte Carlo code. The Monte Carlo program is executed in step 2 with the appropriate parameters for the particular design. For instance, in Design #1, one point would be specified with one DOF varying, and the tolerance would be input as a scale factor applied to the unit load displacement results. Random value parameters are also assigned, such as the number of random samples to generate and the random seed value to start the random number sequence. Finally, the results, a list of output displacements based on random load variations, can be summarized using any valid statistical means.

Based on the three designs outlined for this example problem, the displacements at grid 6 are required for load variations occurring at any of three points (3, 6, or 9) in one DOF (force in ydirection) at each point. All three designs were analyzed in one MSC/NASTRAN (v67, SOL 101) solution (Appendix B). For each of the designs the Monte Carlo code was utilized once, and the output from the designs was postprocessed in a spreadsheet/graphing package (Lotus® 1-2-3®).

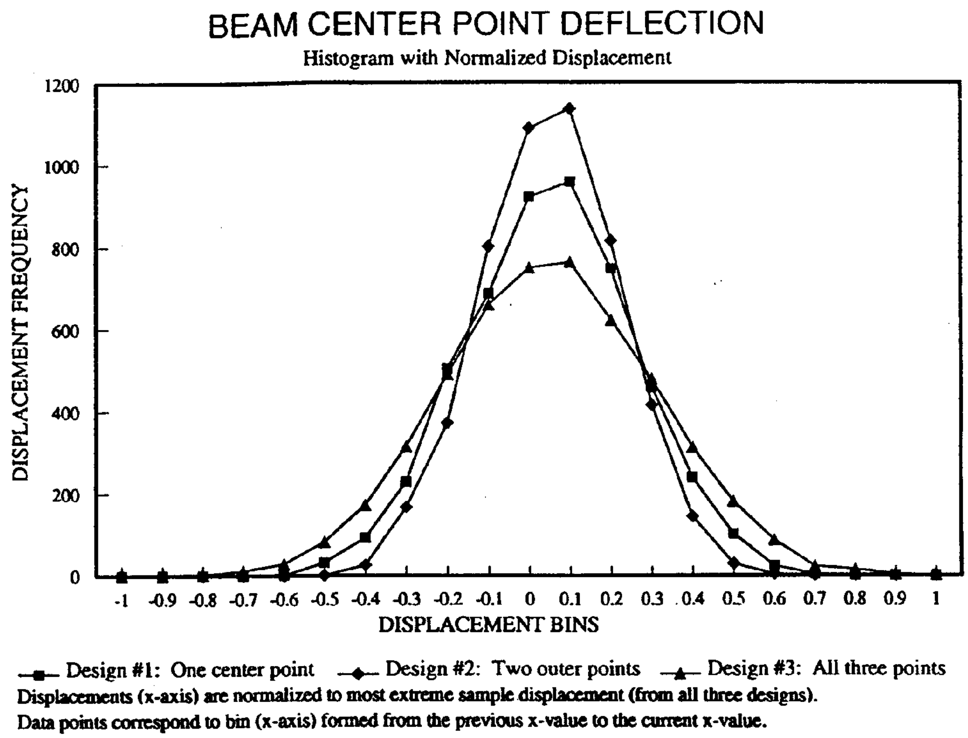


FIGURE 10. Example Problem Monte Carlo Results. Histogram of random sample results for three different designs.

The results are summarized in the histogram shown in Figure 10. The histogram shows all three designs and reflects a population of 5,000 random samples for each design. The y-axis is the frequency of occurrences, and the x-axis is the displacement magnitudes normalized by the maximum (absolute value) displacement. For the random cases generated, this maximum displacement was 0.40". This maximum magnitude occurred in Design #3 which has a maximum possible displacement of about 0.53" corresponding to all three points at their maximum (all plus or all minus) load variation (Appendix B).

The three curves in Figure 10 are the outlines of the three different bar graphs--one for each design. By comparing the different curves, the expectation values of the designs can be compared. The data points correspond to bins that span from the previous x-value to the x-value with the data point. With this note in mind, the curves are symmetric about zero displacement within random error. Most notable is the fact that the peaks (which occur at 0.") are clearly different for the three designs. Likewise, the three designs have very different maximum deviations from zero. More specifically, the largest and smallest maximum deviation occur in Design #3 and Design #2 respectively. In other words, the design with the largest probability of obtaining zero displacement at the beam center point and also the smallest maximum deviation from zero is Design #2--two points varying (points 3 and 9). In quantitative terms, the results can be sorted by the absolute magnitude, and the 99.74% (3σ) probable displacement can be determined as explained in section 3.5. The following table (Figure 11) shows the 99.74% values for each of the three designs as determined from the sorted list of Monte Carlo results (not included).

BEAM CENTER POINT DISPLACEMENT

DESIGN	99.74% (3σ) DISPLACEMENT
#1	0.23"
#2	0.18"
#3	0.31"

FIGURE 11. Resultant 99.74% (3σ) Probable Center Point Displacement.

The results from the variation analysis indicate that there is a 99.74% probability that the center point displacement will be within 0.18" for Design #2, while as much as 0.31" for Design #3. Clearly, Design #2 is better than either of Designs #1 and #3 based on the design criterion. Recall that based on the nominal analysis alone, all three designs were deemed equal. The 28% difference between Design #1 and #2 could have a significant impact on the performance of the beam structure used in this example.

As this example indicates, ignoring or incorrectly estimating the impact of the load variations can have significant effect on the performance of a design. It is interesting to note that if load variations were considered in this example by simply comparing the results for each design using the worst-case load variations (maximum variation at every point), the incorrect conclusion would be drawn. The maximum possible center deflections for the three designs are 0.25", 0.28", and 0.53" respectively (Appendix B). Therefore, the 'worst-case' analysis would result in the incorrect conclusion that Design #1 is best.

5.0 Mirror Alignment System Variation Analysis

The MAS analysis background was introduced in section 2.2. Now with the Monte Carlo method explained in section 3, the MAS variation impact on mirror performance can be determined. The discussion considers just one mirror (labeled P1) of the eight total, but the process for the other seven mirrors (P3, P4, P6, H1, H3, H4, and H6) is identical.

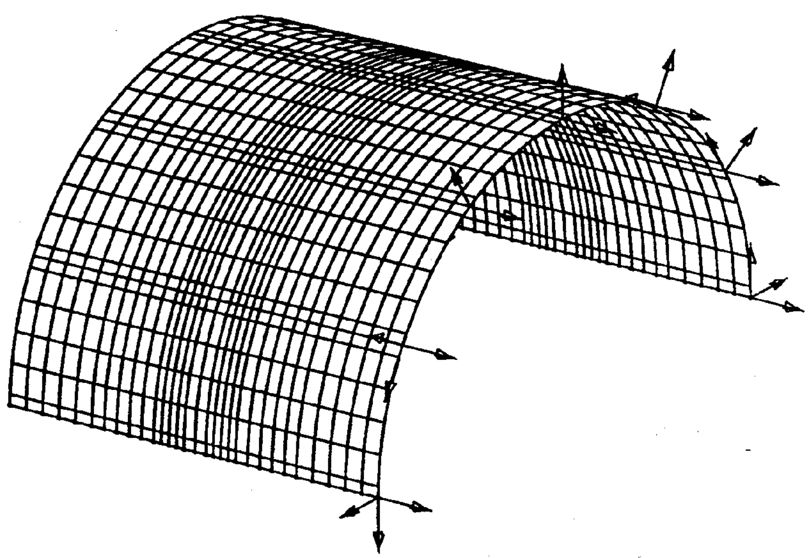


FIGURE 12. Mirror Finite Element Model with Load Variations.

5.1 Finite Element Model

Recall that the simplified analysis which is considered in this discussion involves only the mirror and applied load variations at each of the 12 support points. The half model used is taken from a much larger model of the complete HRMA with 180° symmetry. As a result, symmetric and antisymmetric analyses are completed, and the results are combined appropriately to yield the full 360° asymmetric structural response. The P1 mirror model with loads is shown in Figure 12.

Additional symmetry is used by realizing that only three of the 12 support points are unique due to the symmetry of the model and constraints. Figure 13 indicates the unique points used (A, B, and C) and how the other points are generated from them within the Monte Carlo code.

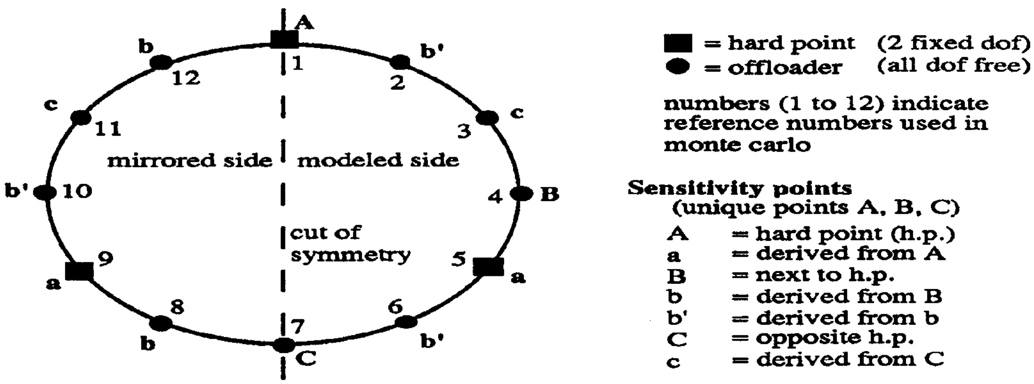


FIGURE 13. Symmetry of Load Variation Points on a Mirror.

5.2 Load Variations

At each of the three unique support points, random load variations are applied in all of the unconstrained degrees of freedom. Figure 12 shows the three force components at each support point. In addition there are three moments (not shown) about the same components. Figure 4 also showed a mirror with load variations. The hard points shown in Figure 4 have only four free DOF, so two of the DOF of load applied to the mirror model are loads into a constraint (zero mirror displacement). These two DOF are not actually analyzed due to the obvious result, so a prepared, generic zero mirror displacement file is used instead.

It is necessary to determine the actual tolerances in each DOF that can be maintained. The manufacturing tolerances, gauge resolution, and additional capabilities are all itemized in a list and then combined appropriately (RSS or add) depending on the nature of the errors. For the P1 mirror these capabilities result in the net tolerances listed in Figure 14. The moments are based on a P1 mirror weight of about 430 pounds.

P1 Mirror Load Variation Tolerances

DOF	Component	Magnitude	Primary Cause
1	Radial Force	0.05 lb	Bearing friction
2	Theta Force	0.05 lb	Bearing friction
3	Axial Force	0.05 lb	Calibration accuracy/bearing friction
4	Radial Moment	1.07 inch-lb	Theta position accuracy
5	Theta Moment	0.28 inch-lb	Radial position accuracy
6	Axial Moment	0.05 inch-lb	Theta constraint misalignment

FIGURE 14. P1 Net Mirror Tolerances in All 6 DOF.

5.3 Output Evaluation Criterion

Up until now only displacements at a single output point have been illustrated. For the MAS variation analysis solution it is necessary to consider the entire mirror shape and its performance impact or image quality. The performance of the AXAF-I Telescope can be quantified by the angle measured along the focal length and subtending the diameter encompassing 90% of the energy incident on the collective eight mirrors. This quantity is called 90% encircled energy (90% EE) and is measured in arcseconds (1 arcsec ≈ 5 μm diameter at a 1 meter distance). The 90% EE is determined by ray tracing the deformed set of mirrors in a ray tracing code, CYGNUS, developed for the AXAF-I Program. In addition to using raw mirror displacement data, the ray tracing algorithm can use a set of Fourier-Legendre polynomials which when summed reproduce the net mirror displacements. This latter ray tracing option is used for the MAS variation analysis.

Fourier-Legendre coefficients are orthonormal polynomials that collectively describe the net deformed shape of cylindrical optics in the same manner that Zernike polynomials describe the deformation of flat optics. The AXAF-I optics are slightly conic, but fit well with Fourier-Legendre polynomials. Figure 15 shows a few mirror shapes corresponding to common polynomials with reference names as indicated. The polynomials fit the mirror deformed shape in the same manner that a set of sine functions could describe the different beam shapes from the example illustrated in Figure 9. Using a set of polynomials is more efficient than using the complete set of raw displacements since only two values, A and B, per coefficient are needed. The AXAF-I optics are usually fit with a series of 154 coefficients total with less than 1% truncation error. Each set of displacements for the load variation sensitivities is therefore fit with polynomials before Monte Carlo is performed. Since the Monte Carlo method linearly combines scaled displacements, the polynomials can be likewise scaled and linearly added. The Monte Carlo code, Monte-I (Monte Carlo-Imaging), used for the MAS variation analysis manipulates the polynomial input sensitivities instead of the actual displacements and then calls the ray tracing program.

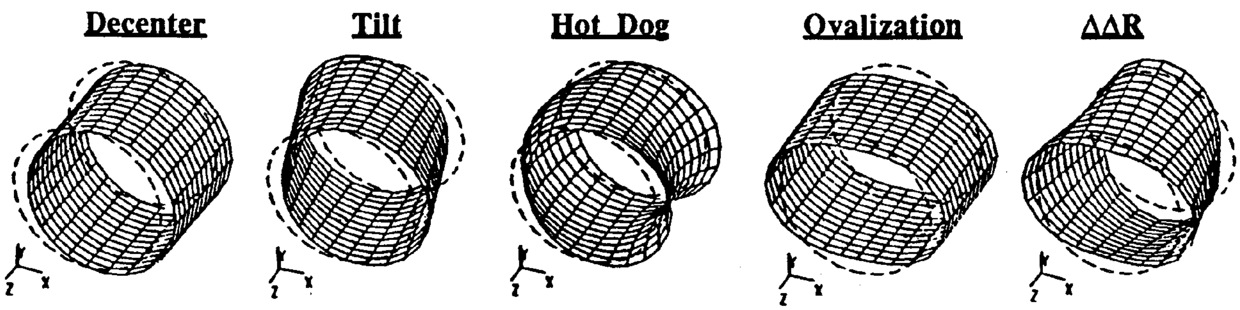


FIGURE 15. Example Fourier-Legendre Coefficient Shapes on a Mirror. Net mirror displacement can be fit by sum of coefficients.

5.4 Sensitivity Matrix

The sensitivity matrix, [C], for the MAS variation analysis has a similar structure to the matrix shown in Figure 6 except that each cell in the matrix contains a submatrix of coefficients instead of a single displacement value. The matrix is shown in Figure 16 and has 12 rows for the different load variation points and 6 columns for the different DOF that can vary. The third dimension to the matrix is for the 8 different mirrors of the HRMA. Each submatrix of coefficients contains the 154 coefficient pairs used to fit a net deformed mirror. The tolerances, (T), and the random numbers, (R), are sets of scale factors applied to the terms in the matrix, [C], to form the sample solution, [C']. Each column of [C] is multiplied by one value from (T) and each submatrix, [c(i,j,k)], is multiplied by one value from (R); matrix multiplication is not performed.

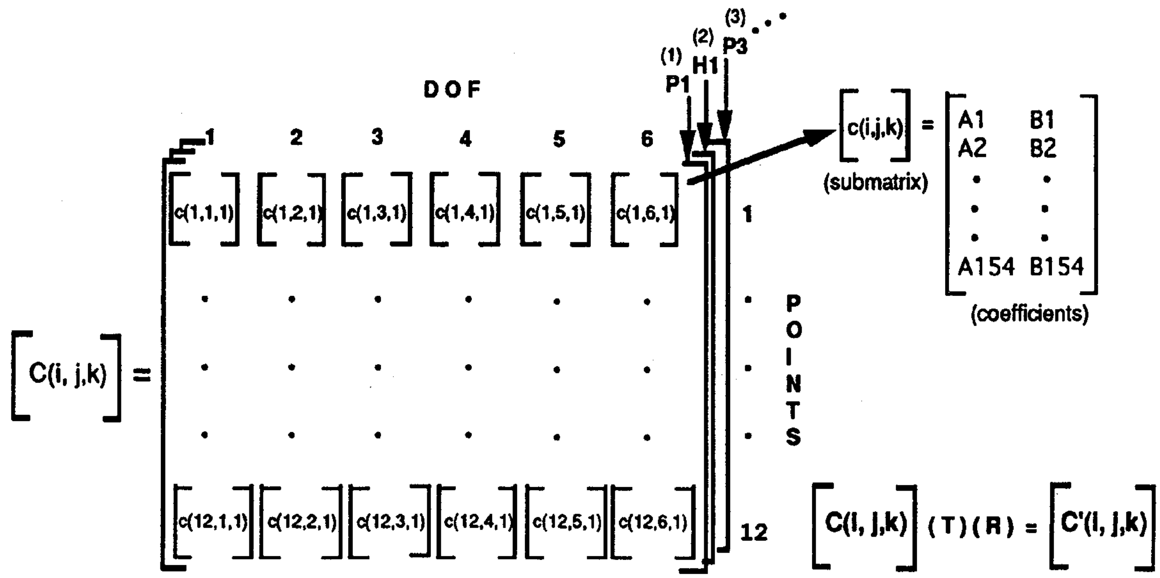


FIGURE 16. Sensitivity Matrix for the MAS Variation Analysis.

5.5 Analysis Results

The requirements for the HRMA are based on 3σ performance estimation. Therefore, the Monte Carlo results, 90% EE values, are sorted by magnitude; and since the quantities are always positive, the 99.87% value is selected as the answer. The value selected depends on the number of samples available. Obviously the larger the sample size the more statistical confidence there is in the answer. Figure 17 indicates the convergence rate of the 3σ 90% EE values for the analysis.

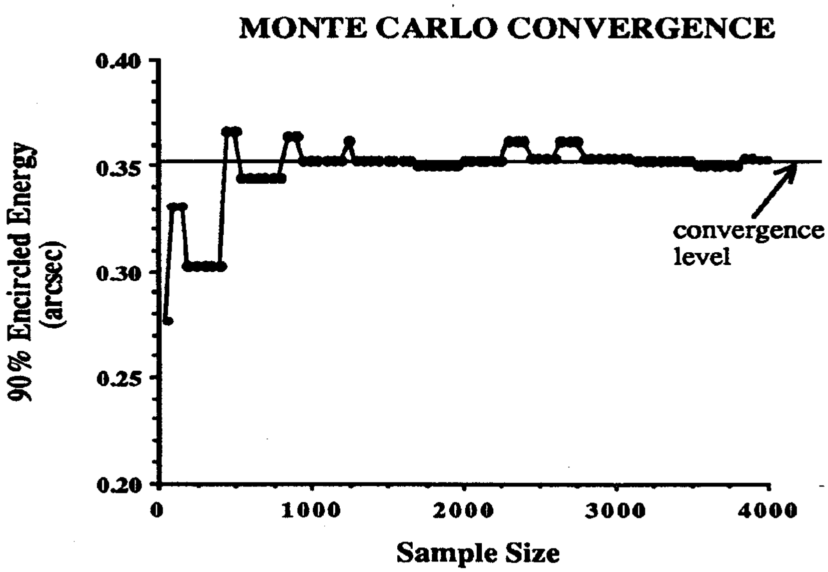


FIGURE 17. Monte Carlo Convergence of 3σ Results.

Based on Figure 17 it appears that the solution converges with about 1,000 samples. However, it is also important to see how much variation there is between different populations of the same number of samples. The difference between populations provides insight into the magnitude of the error bars on the data points in Figure 17. Figure 18 shows how repeatable the results are for different populations of a few sample sizes. As expected, the larger sample sizes have much less variation than the smaller samples.

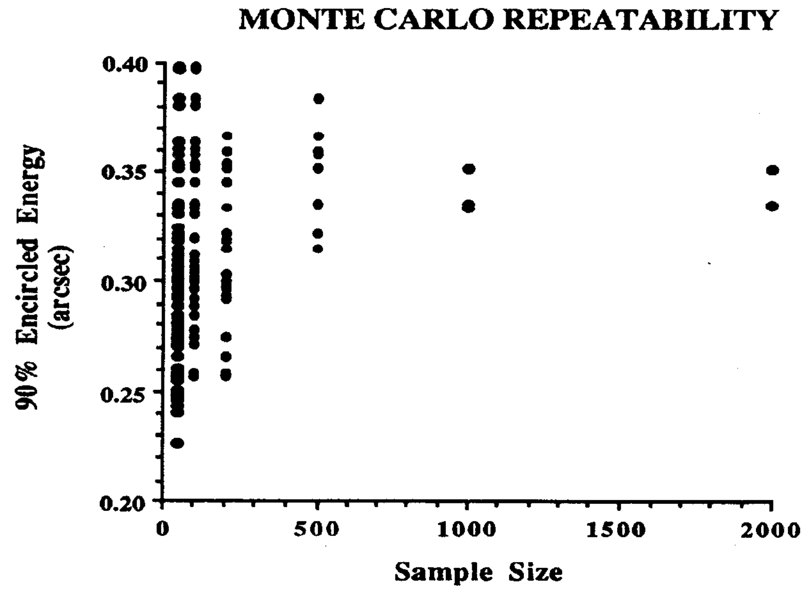


FIGURE 18. Monte Carlo Repeatability of 3σ Results.

The 3σ 90% EE for the MAS variation analysis discussed is 0.35 arcsecond (Figure 17 or 18). By tightening manufacturing and assembly tolerances and reevaluating the answer with the Monte Carlo method, the result has decreased to 0.19 arcsec. This result compares to a total error budget of 1.0 arcsecond for all HRMA errors. This variation analysis turns out to be one of the large error contributors in the HRMA assembly. The nominal analysis of the same assembly process yields a negligible result of 0.0006 arcsecond [4]. Obviously, ignoring the load variation effects would be a serious error.

6.0 Conclusions

The load variation effects can have a very significant impact on the performance of a structure. Therefore, in addition to a nominal analysis of the structure, the impact due to variations from nominal should also be considered. Using a 'worst-case' analysis instead of Monte Carlo may be difficult and incorrect when there are several different load variations involved.

The Monte Carlo method as presented can be used in many different applications. In addition to randomizing nodal forces and moments; grid point temperatures, gravity load vectors, pressure loads, and many other loads can all be used for the load variation. Similarly, in addition to the displacement (or displacement polynomial) output quantity presented; stress, strain, internal force, velocity, acceleration, or any other quantity that is linearly dependent on the applied loads can be used as an output quantity for the Monte Carlo results.

With the Monte Carlo method, the analyst can not only predict the result of the variation analysis, but can also determine the probability for a result to occur.

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Ed Baraban
Systems Engineer -- Eastman Kodak Company
Use of the Monte Carlo code with the optical ray tracing code to generate the encircled energy samples shown in Figures 17 and 18.

References:

[1] FORTRAN 77, DEC Fortran X3.2(u2) for ULTRIX/RISC Systems

[2] "The Great Observatories for Space Astrophysics," National Aeronautics and Space Administration, Astrophysics Division, NP-128.

[3] Casey, T. M., Matthews, G. W., Waldman, M., "After Einstein: The AXAF Program," *Space Technology International*, Cornhill Publications Limited, London, England, 1991.

[4] Genberg, V., Stone, M. J., "Nonlinear Superelement Analysis to Model Assembly Processes," *MSC 1993 World Users' Conference Proceedings*, Session 5A, The MacNeal-Schwendler Corporation, 1993.

[5] Carter, L. L., Cashwell, E. D., Particle-Transport Simulation with the Monte Carlo Method, Energy Research and Development Administration, Oak Ridge, Tennessee, 1975.

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APPENDIX A. Monte-D FORTRAN Code (for Section 4.0)

```fortran
C**********************************************************************
C       PROGRAM MONTE-D (MONTE CARLO for DISPLACEMENT Input)
C**********************************************************************
C       MONTE CARLO RANDOMIZATION APPLIED TO LOAD VARIATION SENSITIVITY
C       OUTPUT —> NASTRAN DISPLACEMENTS
C
C       WRITTEN BY:  MARK STONE 1/03/94
C       BASED ON CONCEPTS FROM PROGRAM:  'MONTE-I' (Imaging)
C——/——————————————————————————————————————————————————————————————————/
C*******Parameters
        INTEGER IN,OUT,POINT,DOF
        PARAMETER (OUT=11,IN=12,POINT=10,DOF=6)
C
C*******Declared variables
        INTEGER PTS,DF,DFR,GID,SUB(POINT,DOF),NTC,SD,SC,SCI,SCJ
        INTEGER GIDTMP,I,J,Q
        REAL*8 TOL(DOF),C(POINT,DOF),CP(POINT,DOF),DISP,RNDM
        CHARACTER*132 CARD
        CHARACTER*60 TITLE,LABEL
        CHARACTER*30 FILEIN,FILEOUT,FN(10)
C
C*******User input section
C*******File information
        WRITE(6,*) ' Enter filename of the MSC NASTRAN displacements'
        READ(5,'(A)') FILEIN
        WRITE(6,*) ' Enter filename for the random case output file'
        READ(5,'(A)') FILEOUT
        WRITE(6,*) ' Enter the output file TITLE'
        READ(5,'(A)') TITLE
        WRITE(6,*) ' Enter the output file LABEL'
        READ(5,'(A)') LABEL
C*******Variation Analysis parameters
        WRITE(6,*) ' Enter the number of load points in the problem'
        READ(5,*) PTS
        WRITE(6,*) ' Enter the number of load DOF per point to use'
        READ(5,*) DF
C*******Model information
        WRITE(6,*) ' Enter the grid point number (GID) to investigate'
        READ(5,*) GID
        WRITE(6,*) ' Enter MSC NASTRAN output DOF for randomization'
        READ(5,*) DFR
        DO I = 1,PTS
           DO J = 1,DF
              WRITE(6,*) ' Enter subcase number for load at point:',I
              WRITE(6,*) '                              in DOF labeled:',J
              READ(5,*) SUB(I,J)
           ENDDO
        ENDDO
C*******Tolerances — can be by DOF or alternately, by POINT
C*******tolerance by DOF shown here
        WRITE(6,*) ' Tolerances are scale factors applied to input.'
        WRITE(6,*) ' For unit input sensitivities, tolerances are '
        WRITE(6,*) ' actual max/min values of load variations.'
        DO J = 1,DF
           WRITE(6,*) ' Enter the tolerance to apply to DOF:',J
           READ(5,*) TOL(J)
        ENDDO
C*******Random parameters
        WRITE(6,*) ' Enter the number of random test cases'
        READ(5,*) NTC
        WRITE(6,*) ' Enter random seed value (large odd integer)'
        READ(5,*) SD
C
C*******File Administration
        OPEN(UNIT=OUT,NAME=FILEOUT,TYPE='NEW')
        OPEN(UNIT=IN,NAME=FILEIN,TYPE='OLD')
C
C*******Write output file header
        WRITE(OUT,11) TITLE,LABEL
C
C*******Find displacement section of output (.f06) file; sort by subcase
100     READ(IN,1,END=300) CARD
        IF (CARD(113:116).EQ.'CASE') READ(CARD(118:121),2) SC
        IF (CARD(46:68).EQ.'D I S P L A C E M E N T') THEN
           DO I = 1,PTS
              DO J = 1,DF
                 IF (SC.EQ.SUB(I,J)) THEN
                    SCI = I
                    SCJ = J
                    GO TO 200
                 ENDIF
              ENDDO
           ENDDO
        ENDIF
        GO TO 100
C
C*******Find displacement value for desired grid
200     READ(IN,1,END=300) CARD
        IF (CARD(113:116).EQ.'CASE') THEN
           READ(CARD(118:121),2) SC
           IF (SC.NE.SUB(SCI,SCJ)) GO TO 100
        ENDIF
        IF (CARD(20:22).EQ.' G ') THEN
           READ(CARD(9:14),3) GIDTMP
           IF (GIDTMP.EQ.GID) THEN
              READ(CARD((12+(15*DFR)):(24+(15*DFR))),4) C(SCI,SCJ)
           ENDIF
        ENDIF
        GO TO 200
C
C*******Input data reading completed
300     CONTINUE
C
C*******Scale matrix by tolerances
        DO I = 1,PTS
           DO J = 1,DF
              C(I,J) = C(I,J)*TOL(J)
           ENDDO
        ENDDO
C
C*******Perform Monte Carlo randomization on matrix:  C[POINT,DOF]
C*******Loop for random cases
        DO Q = 1,NTC
C*******Scale by random numbers
           DO I = 1,PTS
              DO J = 1,DF
                 CALL RANGAU(RNDM,SD)
                 CP(I,J) = C(I,J)*RNDM
              ENDDO
           ENDDO
C*******Add terms for random case
           DISP = 0.D0
           DO I = 1,PTS
              DO J = 1,DF
                 DISP = DISP + CP(I,J)
              ENDDO
           ENDDO
C*******Write output data
           WRITE(OUT,12) Q,DISP
C*******End random cases
        ENDDO
C
C*******Format statements
1       FORMAT(A132)
2       FORMAT(I4)
3       FORMAT(I6)
4       FORMAT(E13.6)
11      FORMAT(A60,/,A60,//,T4,'CASE',T11,'DISPLACEMENT ',/,
     1         T4,'——',T11,'————————')
12      FORMAT(T4,I4,T11,1PE13.6)
C
        STOP
        END
C
C———————————————————————————————————————————————————————————————————————
        SUBROUTINE RANGAU(RNDM,SD)
C———————————————————————————————————————————————————————————————————————
C       Subroutine RANGAU generates a Gaussian random number between
C       the ranges -1. to +1.
C———————————————————————————————————————————————————————————————————————
C*******Passed variable
        INTEGER SD
        REAL*8 RNDM
C*******Local variables
        REAL*8 RN1,RN2,RN3,EPS,MN,SIG,RPD
C
C*******Initialize variables
        EPS = 1.0D-10
        MN = 0.D0
        SIG = 1.D0/3.0D0
        RPD = (DACOS(-1.D0))/180.D0
C
C*******Pick random numbers (square) and form Gaussian
100     RN1 = RAN(SD)
        RN2 = RAN(SD)
        RN3 = RAN(SD)
        SGN = (-1.D0)**(INT(10.D0*RN1))
        IF (RN3.EQ.0.0) RN3 = EPS
        RNDM = MN + SGN*SIG*DCOS(360.D0*RPD*RN2)*
     1          DSQRT(-2.D0*DLOG(RN3))
        IF (DABS(RNDM).GT.1.0D0) GO TO 100
C
        RETURN
        END
```

APPENDIX B. MSC/NASTRAN Analysis (for Section 4.0)

```
NASTRAN MESH
$
ID STONE,MARK
TIME 100
SOL 101
CEND
$
TITLE = MONTE CARLO LOAD VARIATION SENSITIVITY
SUBTITLE = EXAMPLE BEAM PROBLEM — SECTION 4.0
LABEL = DISPLACEMENT OUTPUT FOR A UNIT APPLIED LOAD
$
SPC = 1
$
SUBCASE 1
   LABEL = LOAD AT POINT 3
   DISP = ALL
   LOAD = 1
$
SUBCASE 2
   LABEL = LOAD AT POINT 6
   DISP = ALL
   LOAD = 2
$
SUBCASE 3
   LABEL = LOAD AT POINT 9
   DISP = ALL
   LOAD = 3
$
```

```
BEGIN BULK
$
PARAM,POST,0
$
EGRID,1,,0.,0.
EGRID,2,,10.,0.
$
GRIDG,1,,,10,-1,-2
GRIDU,1,1
$
CGEN,BEAM,1,1,1,L
,0.,1.,0.
$
FORCE,1,3,,1.,0.,1.,0.
FORCE,2,6,,1.,0.,1.,0.
FORCE,3,9,,1.,0.,1.,0.
$
SPCG,1,1,2,A
SPCG,1,1,2,B
GRDSET,,,,,,,1345
$
PBEAM,1,1,0.01,8.33-6,8.33-6
MAT1,1,1.+7,,0.3
$
ENDDATA
```

LOAD AT POINT 3 — SUBCASE 1

D I S P L A C E M E N T V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	0.0	0.0	0.0	0.0	0.0	5.762305E-02
2	G	0.0	5.604321E-02	0.0	0.0	0.0	5.282113E-02
3	G	0.0	1.024826E-01	0.0	0.0	0.0	3.841536E-02
4	G	0.0	1.316891E-01	0.0	0.0	0.0	2.040816E-02
5	G	0.0	1.440888E-01	0.0	0.0	0.0	4.801921E-03
6	G	0.0	1.420828E-01	0.0	0.0	0.0	-8.403361E-03
7	G	0.0	1.280720E-01	0.0	0.0	0.0	-1.920768E-02
8	G	0.0	1.044574E-01	0.0	0.0	0.0	-2.761104E-02
9	G	0.0	7.363985E-02	0.0	0.0	0.0	-3.361344E-02
10	G	0.0	3.802041E-02	0.0	0.0	0.0	-3.721489E-02
11	G	0.0	0.0	0.0	0.0	0.0	-3.841536E-02

LOAD AT POINT 6 — SUBCASE 2

D I S P L A C E M E N T V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	0.0	0.0	0.0	0.0	0.0	7.503001E-02
2	G	0.0	7.404261E-02	0.0	0.0	0.0	7.202881E-02
3	G	0.0	1.420828E-01	0.0	0.0	0.0	6.302521E-02
4	G	0.0	1.981182E-01	0.0	0.0	0.0	4.801921E-02
5	G	0.0	2.361464E-01	0.0	0.0	0.0	2.701080E-02
6	G	0.0	2.501650E-01	0.0	0.0	0.0	4.427882E-16
7	G	0.0	2.361464E-01	0.0	0.0	0.0	-2.701080E-02
8	G	0.0	1.981182E-01	0.0	0.0	0.0	-4.801921E-02
9	G	0.0	1.420828E-01	0.0	0.0	0.0	-6.302521E-02
10	G	0.0	7.404261E-02	0.0	0.0	0.0	-7.202881E-02
11	G	0.0	0.0	0.0	0.0	0.0	-7.503001E-02

LOAD AT POINT 9 — SUBCASE 3

D I S P L A C E M E N T V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	0.0	0.0	0.0	0.0	0.0	3.841536E-02
2	G	0.0	3.802041E-02	0.0	0.0	0.0	3.721489E-02
3	G	0.0	7.363985E-02	0.0	0.0	0.0	3.361344E-02
4	G	0.0	1.044574E-01	0.0	0.0	0.0	2.761104E-02
5	G	0.0	1.280720E-01	0.0	0.0	0.0	1.920768E-02
6	G	0.0	1.420828E-01	0.0	0.0	0.0	8.403361E-03
7	G	0.0	1.440888E-01	0.0	0.0	0.0	-4.801921E-03
8	G	0.0	1.316891E-01	0.0	0.0	0.0	-2.040816E-02
9	G	0.0	1.024826E-01	0.0	0.0	0.0	-3.841536E-02
10	G	0.0	5.604321E-02	0.0	0.0	0.0	-5.282113E-02
11	G	0.0	0.0	0.0	0.0	0.0	-5.762305E-02