

ELASTIC-PLASTIC ANALYSIS AROUND A CIRCULAR HOLE STRESS CONCENTRATION

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ABSTRACT

This paper takes a look at the ability of the 8 node solid (CHEXA) and the 4 node shell (CQUAD4) to calculate the plastic surface around a circular hole, when the plate is loaded bi-axially. The MSC calculated stresses will be compared to two analytical solutions; one by Galin, the other by Sokolov. The solution by Galin, which is exact, calculates the boundary of the plastic surface for a plane strain condition. The solution by Sokolov, which is approximate, calculates the boundary of the plastic surface for a plane stress condition. First, the CHEXA will be used in a plane strain condition with two load cases, and 3 different meshes in each load case. Second, the CHEXA and CQUAD4 will be used in a plane stress condition with the same two load cases, with 3 meshes for load case 1 and 1 mesh for load case 2. Load case 1 will produce an elliptical plastic surface, load case 2 will produce a circular plastic surface. The version of MSC/NASTRAN¹ is 67.7. The material property is elastic-perfectly plastic. Reading from the OUTPUT2 file, the nodes and their von Mises stress, along angles 0, 45 and 90 degrees, from the hole to the edge of the plate, will be determined. The von Mises stress for these nodes will be plotted, such that the nodes which are plastic and the nodes which are elastic are displayed. These nodal values will be compared with the analytical solutions of Galin and Sokolov. All of the models were run on a Cray Y-MP 8I, located at Marshall Space Flight Center.

INTRODUCTION

We have a flat plate with a 1 in radius hole, loaded biaxially as shown in Figure 1. The objective is to load the plate so as to achieve a plastic surface which surrounds the hole; either a circular plastic surface or an elliptical plastic surface. We will look at the following elements, conditions, and theories to see how well they can determine this plastic surface:

PART A: 8 node solid, CHEXA, plane strain, solutions by Galin and Sokolov.

PART B: 8 node solid, CHEXA, 4 node shell, CQUAD4, plane stress, solutions by Galin and Sokolov.

There will be two load cases. The first load case will produce an elliptical plastic surface, the second will produce a circular plastic surface.

PROBLEM DEFINITION

The geometry we will use is a 40 inch by 40 inch by 1 inch thick flat plate, with a 1 inch radius hole.

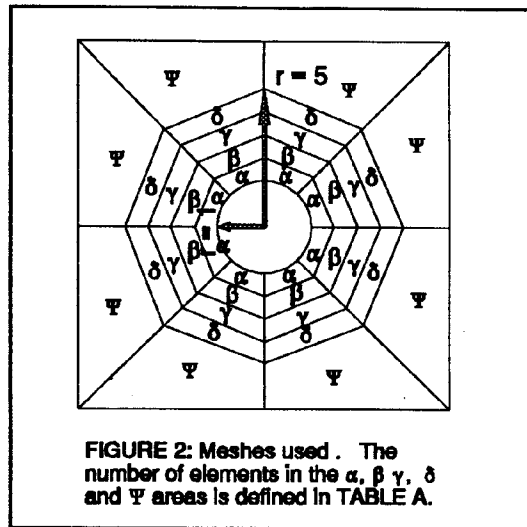
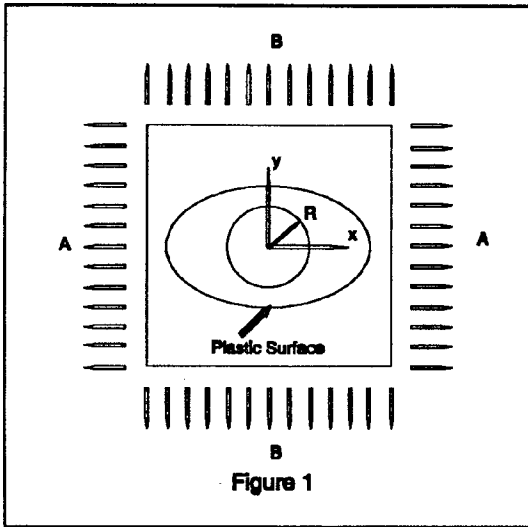


FIGURE 2: Meshes used. The number of elements in the α , β , γ , δ and Ψ areas is defined in TABLE A.

A: the pressure applied in the + x and - x direction.
 B: the pressure applied in the + y and - y direction.

TABLE A: NUMBER OF ELEMENTS IN EACH MESH

The number of elements in the angular, θ direction is constant with 4 elements every 45 degrees.

This table defines the number of elements in each radius out from the hole.

Note: the MESH DESCRIPTIONS A, B, and F are from a previous paper presented by the author⁴

MESH DESCRIPTION	FIRST RADIUS	SECOND RADIUS	THIRD RADIUS	FOURTH RADIUS	TO EDGE OF PLATE
	α	β	γ	δ	Ψ
A	5	5	5	5	5
B	10	10	10	10	10
F	20	15	10	10	10

PART A - Plane Strain

We will model this geometry with 8 node solid (CHEXA) elements. There will be two load cases. In the first load case we will use three mesh densities (A, B and F, see Figure A). In the second load case we will again use meshes A, B, and F. The plane strain condition is accomplished by restraining all of the nodes in the z direction, by use of the GRDSET card.

The full geometry is used in each case, that is, symmetry is not exploited. The number of elements in the angular, θ direction is constant with 4 elements every 45 degrees. The number of elements in the radial direction is defined in FIGURE 2, and TABLE A.

PART B - Plane Stress

We will model this geometry first with 8 node solid (CHEXA) elements. There will be two load cases. In the first load case we will use three mesh densities (A, B and F, see Figure A). In the second load case we will use mesh F.

Second we will model this geometry with 4 node shell (CQUAD4) elements. In load case 1 we will use meshes A, B, and F. In load case 2 we will only use mesh F.

The full geometry is used in each case, that is, symmetry is not exploited. The number of elements in the angular, θ direction is constant with 4 elements every 45 degrees. The number of elements in the radial direction is defined in FIGURE 2, and TABLE A.

In both PART A and PART B, the following material properties will be used.

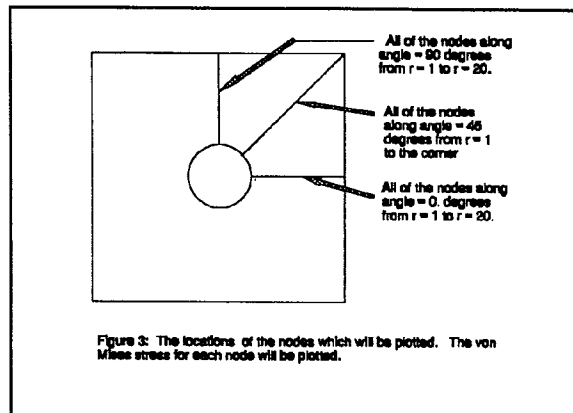
The material properties are:

$$\nu = .3$$

$$E = 30,000,000 \text{ psi.}$$

The material is elastic perfectly plastic. The yield, σ_T is set at 1,500. psi. The NLPARM card for all cases is in APPENDIX A.

All of the data presented is read from the OUTPUT2 file, using a program written by the author. The procedure is to capture the nodes along angles 0, 45 and 90 degrees, from the hole to the edge of the plate. See Figure 3. For the solid element, the nodes on the top will be used. The von Mises Stress for each of these nodes will be plotted. The nodes which are plastic will be marked with a triangle, the nodes which are still elastic will be marked with a circle. The position where the theories of Galin and Sokolov indicate the plastic surface should be will be marked with vertical lines; a dotted line for Galin and a dashed line for Sokolov. Each plot will have the nodes along 0, 45 and 90 degrees.



THEORY

Galini²: The Elasto-Plastic Problem for an Infinite Plane with a Circular Hole (plane strain)

This is an exact solution first derived by Galin. The development of the theory will not be developed and presented here. What will be presented are the equations which define the contour of the plastic surface around the hole. As stated earlier, the plastic surface is either a circle, or an ellipse, depending on the loading. That is, if A and B are equal (see Figure 1), the plastic surface is a circle, if they are not, the plastic surface is an ellipse.

The equations presented here are from Savin⁵, pages 205-213.

$$c = R e^{\frac{1}{2k} \left(\frac{A+B}{2} - p + k \right)} \quad (4.27)$$

Where: R = the radius of the hole, in this case $R = 1$.

A = the pressure applied at ∞ : σ_x B = the pressure applied at ∞ : σ_y

p = the pressure applied at the contour of the hole, in this case it is 0.

k = is a material constant such that:

a) according to the theory of maximum shear stress: $k = \frac{\sigma_T}{2}$

b) according to the theory of the octahedral shearing stress: $k = \frac{\sigma_T}{\sqrt{3}}$

where σ_T is the yield point of the material in the case of uniaxial tension.

In this paper $k = \frac{\sigma_T}{\sqrt{3}}$

The equation for the boundary of the plastic surface will be an ellipse with semi-axes:

$$a = c (1 + \beta) \quad b = c (1 - \beta)$$

$$\text{where } \beta = \frac{B - A}{2}$$

If β is 0., the plastic surface is a circle.

Sokolov³: Plastic Zone Around a Circular Hole in the Case of Bi-axial Tension (plane stress)

These equation are taken from Savin², pages 225-230.

$$A = \lambda_1 \sigma_T \quad B = \lambda_2 \sigma_T$$

where λ_1 and λ_2 are dimensionless parameters such that

$$0 \leq \lambda_1 \leq 1 \quad \text{and} \quad 0 \leq \lambda_2 \leq 1$$

The equation for the plastic contour is:

$$r(\theta) = \frac{R}{2 - \lambda_1 - \lambda_2} - \frac{2 R (\lambda_1 - \lambda_2)}{(2 - \lambda_1 - \lambda_2)^2} \cos 2\theta$$

where: R = the radius of the hole. (in this case $R = 1$.)

σ_T is the yield point of the material in the case of uniaxial tension

LOAD CASE 1

Load case 1 will use the bi-axial loading of : $A = .7125 \sigma_T = .7125 * 1500. = 1068.75$

$B = .7375 \sigma_T = .7375 * 1500. = 1106.25$

These values were taken from Savin, page 229.

The Plastic Contour - Load Case 1

Galin -

$$c = R e^{\frac{1}{1732.060} \left(\frac{1106.25 + 1068.75}{2} + 0 - 800.025 \right)} = 1.13640$$

$$\beta = \frac{1106.25 - 1068.75}{2k} = .02165$$

$$a = 1.13640 (1. + .02165) = 1.16100 \quad b = 1.13640 (1. - .02165) = 1.11179$$

The equation for the plastic surface is:

$$x = 1.16100 R \cos \theta \quad y = 1.11179 R \sin \theta$$

Where R is the radius of the hole: R = 1.

Therefore, the radial distance from the hole to the contour of the plastic surface:

$$\text{angle, } \theta = 0 = 1.16100 \quad \text{angle, } \theta = 45 = 1.13666 \quad \text{angle, } \theta = 90 = 1.11179$$

Sokolov -

$$\lambda_1 = .7125 \quad \lambda_2 = .7375 \quad R = 1.$$

$$r(\theta) = \frac{R}{2 - .7125 - .7375} - \frac{2 R (.7125 - .7375)}{(2 - .7125 - .7375)^2} \cos 2\theta = r(\theta) = R \frac{11. + \cos 2\theta}{6.05}$$

Therefore, the radial distance from the hole to the contour of the plastic surface is:

$$\text{angle, } \theta = 0 = 1.98347 \quad \text{angle, } \theta = 45 = 1.81818 \quad \text{angle, } \theta = 90 = 1.65289$$

LOAD CASE 2

Load case 2 will use the bi-axial loading of :
 A = .8500 σ_T = .8500 * 1500. = 1275.
 B = .8500 σ_T = .8500 * 1500. = 1275.

The Plastic Contour - Load Case 2

Galin -

$$c = 1. e^{\frac{1}{1732.06} \left(\frac{1275. + 1275.}{2} + 0 - 800.025 \right)} = 1.26632 \quad \beta = \frac{1275. - 1275.}{2k} = 0.0$$

$$a = 1.26632 (1. + 0.) = 1.26632 \quad b = 1.26632 (1. - 0.) = 1.26632$$

The equation for the plastic surface is a circle with a radius of 1.26632

Sokolov -

$$\lambda_1 = .8500 \quad \lambda_2 = .8500$$

$$r(\theta) = \frac{1.}{2 - .8500 - .8500} - \frac{2 R (.8500 - .8500)}{(2 - .8500 - .8500)^2} \cos 2\theta = r(\theta) = 3.33333$$

Therefore, the plastic surface is a circle with a radius of 3.33333

THE PLOTS

Plots 1-3 are load case 1, plane strain, CHEXA with meshes A, B and F. Plots 4-6 are load case 2, plane strain, CHEXA, with meshes A, B and F. Plots 7-9 are load case 1, plane stress, CHEXA, with meshes A, B and F. Plots 10-12 are load case 2, plane stress, CQUAD4, with meshes A, B and F. Plot 13 is load case 2, plane stress, CHEXA, with mesh F. Plot 14 is load case 2, plane stress, CQUAD4, with mesh F. The plots are at the end of the paper.

DISCUSSIONS

The nonlinear analysis was performed without difficulty. Initially, some of the models were run using a various number of load steps. All of them ran to completion, even though some had many iterations and bi-sections to perform. It appeared to the author that the number of load steps may be a variable, so it was set to 30 for all models.

CONCLUSIONS

From looking at Plots 1-6, it appears that the Galin theory, when applied to plane strain cases, is right on. The plane stress theory of Sokolov is very conservative. For plane stress situations, it appears that an average of Galin and Sokolov may give a good answer.

The CHEXA and the CQUAD4 gave identical answers for the plane stress situation.

ACKNOWLEDGEMENT

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2. Galin, L. A., *PLANE ELASTICO-PLASTIC PROBLEM PLASTIC ZONES IN THE VICINITY OF CIRCULAR APERTURES*, Prikladnaia Matematika i Mekhanika, Volume 10, 1946.
3. Sokolov, A. P., *THE ELASTIC-PLASTIC STATE OF A PLATE*, Dokl. Akad. Nauk SSSR, X, No. 1, 1948.
4. Thacker, R. P. Jr., *MSC/NASTRAN: SHELL AND SOLID ELEMENT MESH REQUIREMENTS IN THE VICINITY OF A CIRCULAR HOLE STRESS CONCENTRATION*, Presented at the 1993 MSC WORLD USERS' CONFERENCE.

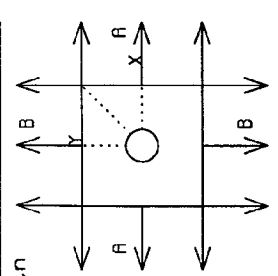
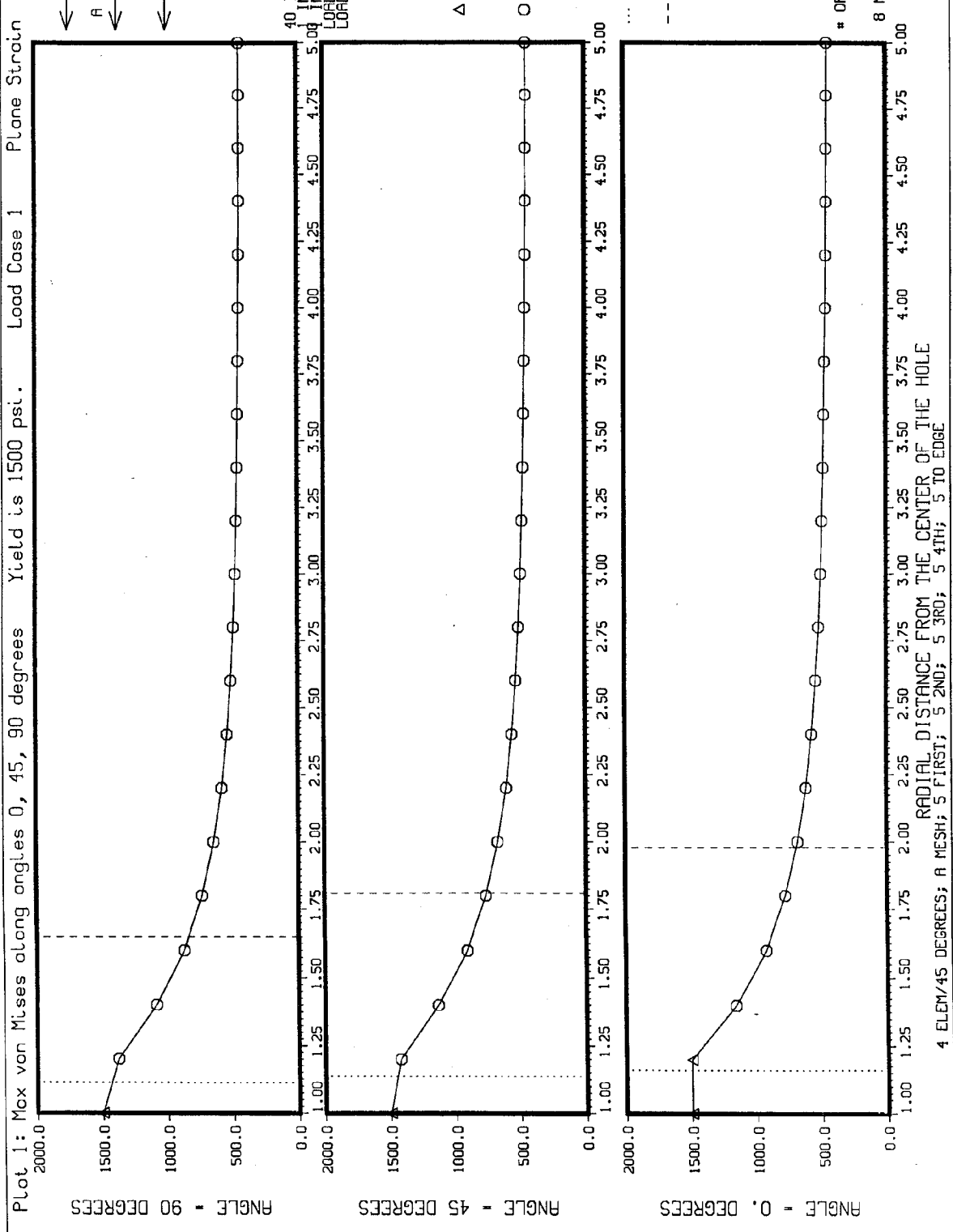
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5. Savin, G. N., *STRESS CONCENTRATION AROUND HOLES*, Pergamon Press, 1961

APPENDIX A

NLPARM CARD USED - IN ALL CASES

1	2	3	4	5	6	7	8	9
1234567812345678123456781234567812345678123456781234567812345678								
NLPARM	1	30		AUTO	5	25	UPW	NO



40 INCH BY 40 INCH BY 40 INCH FLAT PLATE WITH 1 INCH RADIUS HOLE. LOADING (A) IS 1068.75. LOADING (B) IS 1106.25. MSC 67.7.

CHEXFB
DEFAULT
△ Nodes which are plastic
○ Nodes which are linear elastic

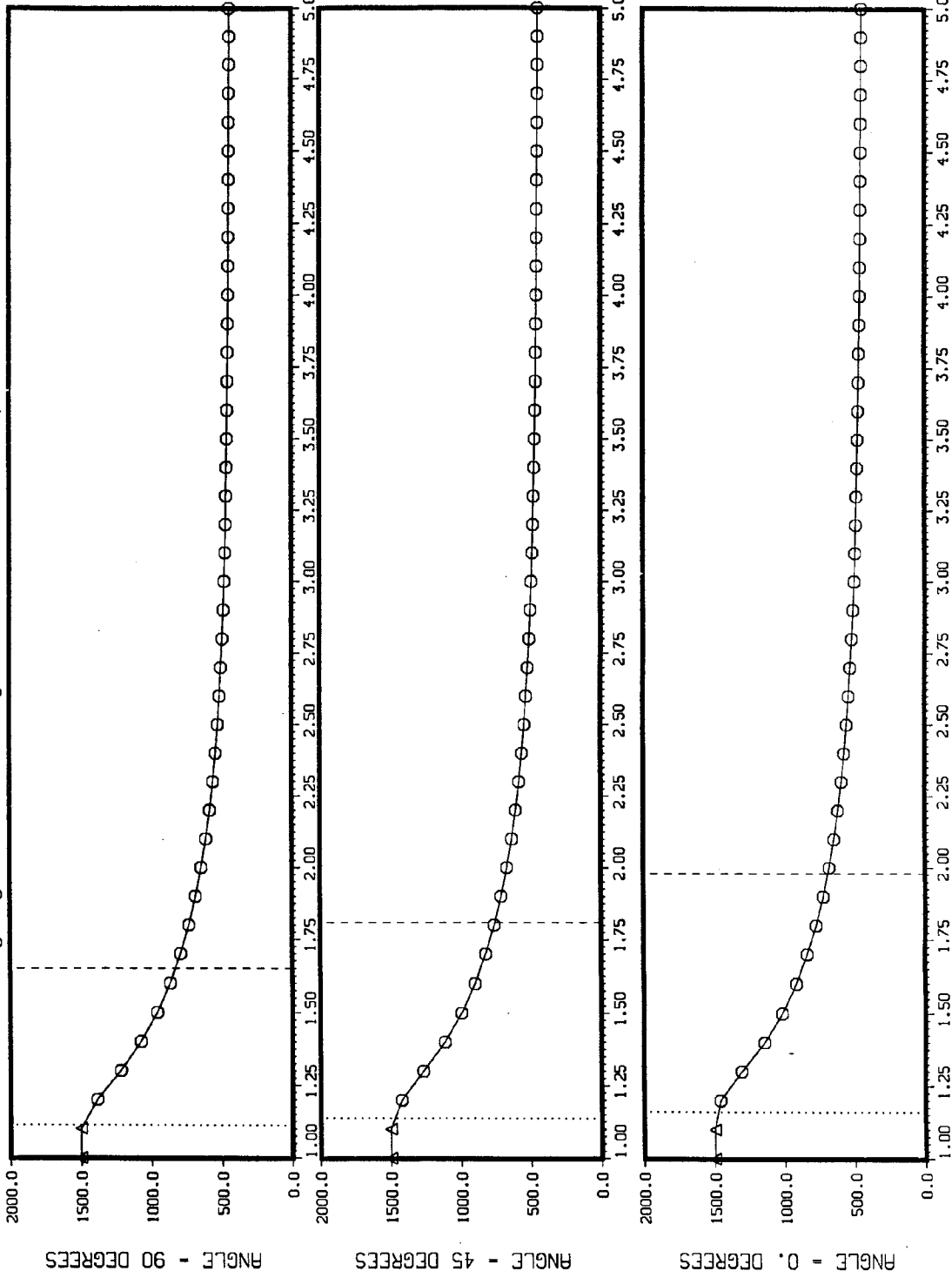
Theory:
... Galin (Plane Strain)
--- Sokolov Stress

OF ELEMENTS = 800

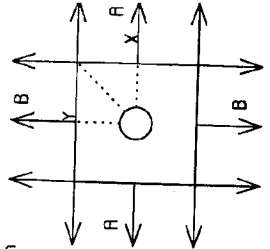
8 NODE SOLID

RADIAL DISTANCE FROM THE CENTER OF THE HOLE
4 ELEM/45 DEGREES; A MESH; 5 FIRST; 5 2ND; 5 3RD; 5 4TH; 5 TO EDGE

Plot 2: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 1 Plane Strain



Plot 3: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 1 Plane Strain

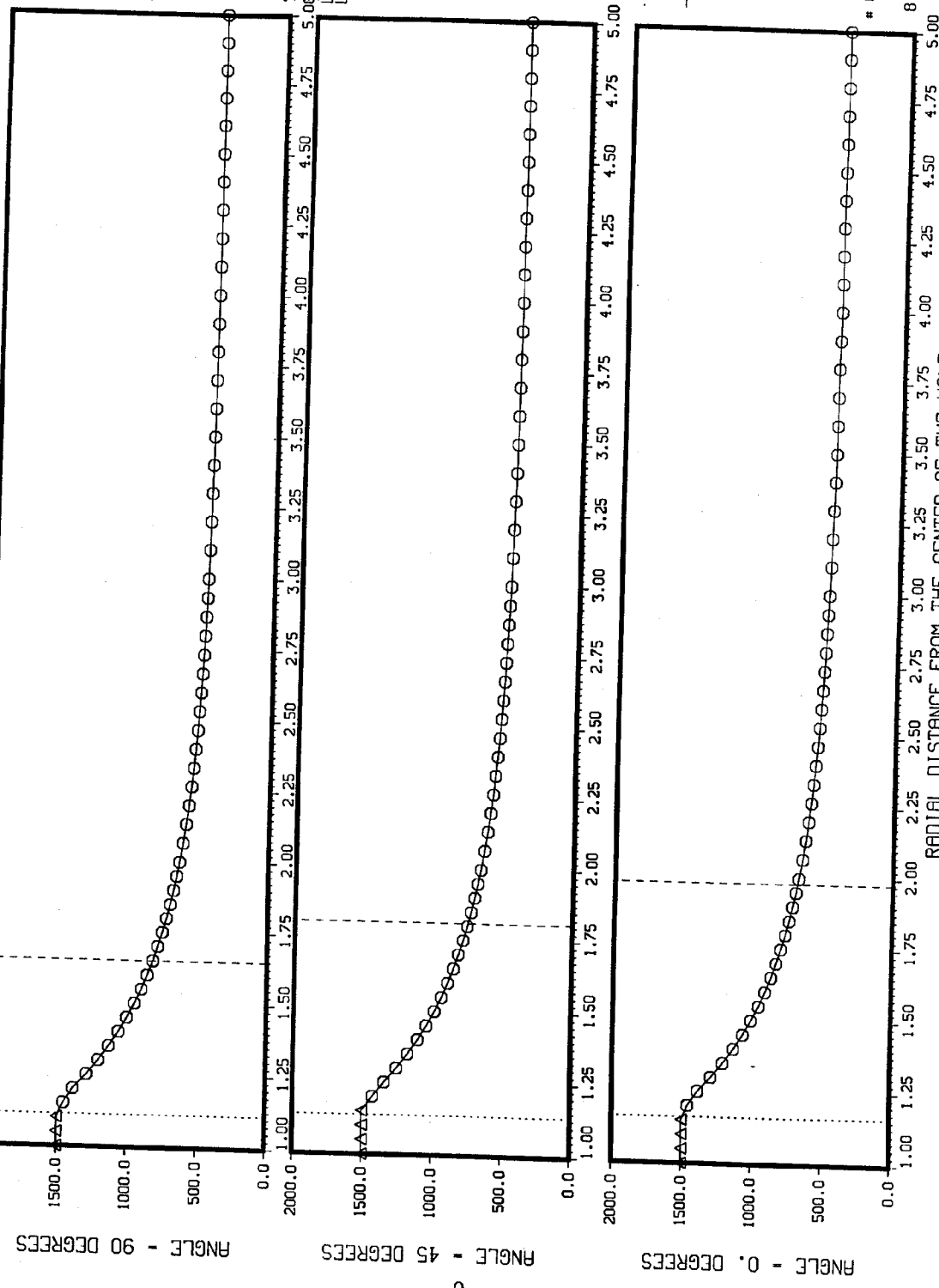


40 INCH BY 40 INCH BY
 1/4 INCH FLAT PLATE WITH
 1/4 INCH RADIUS HOLE WITH
 LOADING (A) IS 1068.75
 LOADING (B) IS 1106.25
 MSC 67.7
 CHEXAB
 DEFAULT

△ Nodes which are plastic
 ○ Nodes which are linear elastic

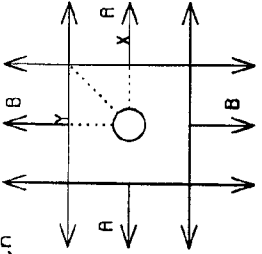
Theory:
 ... von Mises (Plane Strain)
 --- Sokolov (Plane Stress)

OF ELEMENTS = 2080
 8 NODE SOLID

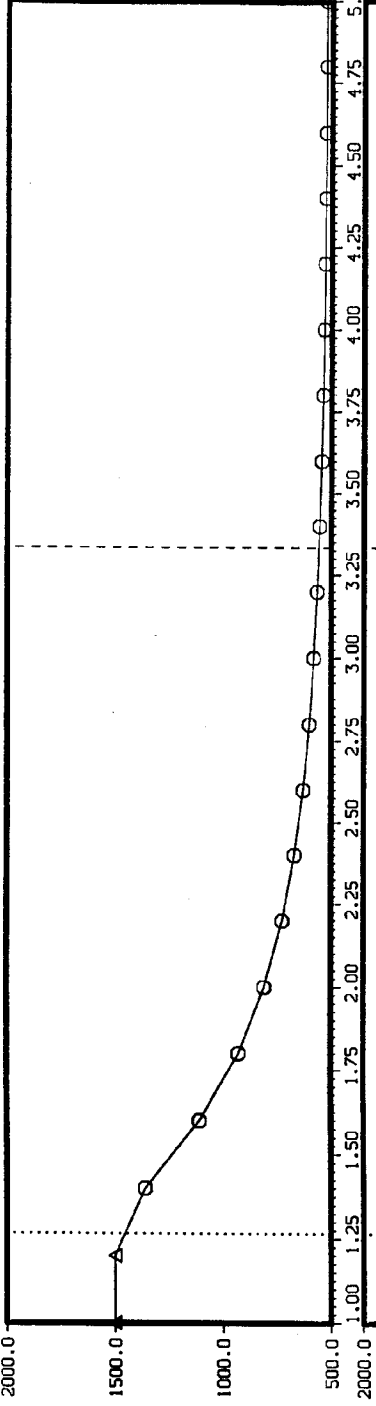


4 ELEM/45 DEGREES; 6 MESH; 20 FIRST; 15 2ND; 10 3RD; 10 4TH; 10 TO E

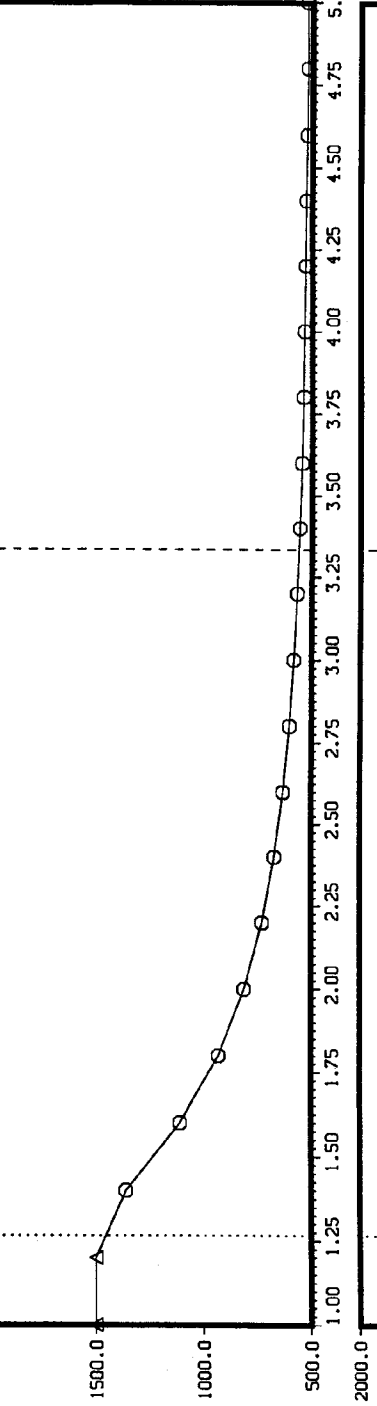
Plot 4: Max von Mises along angles 0, 45, 90 degrees Yield is 1500 psi. Load Case 2 Plane Strain



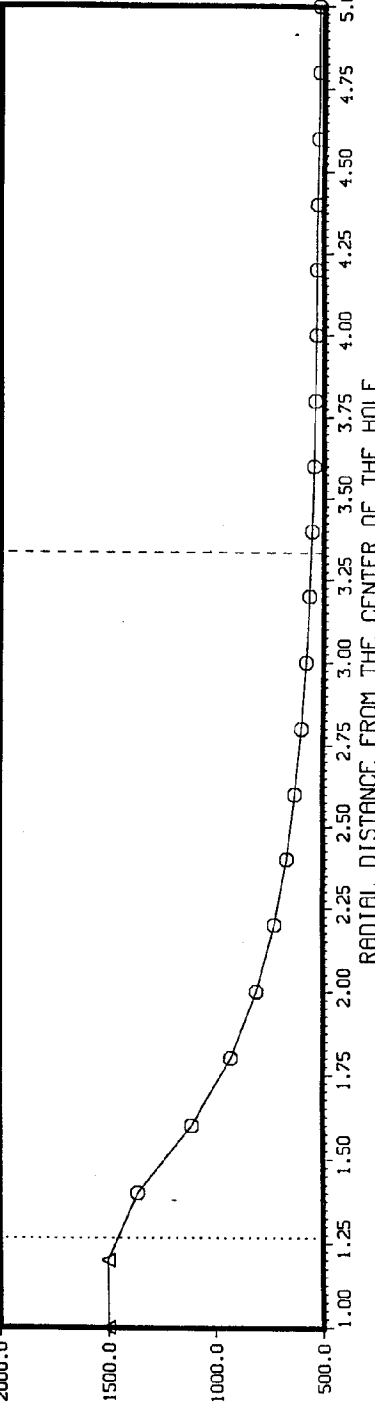
ANGLE = 90 DEGREES



ANGLE = 45 DEGREES



ANGLE = 0. DEGREES



CHEXRB
DEFAULT
△ Nodes which are plastic
○ Nodes which are Linear elastic

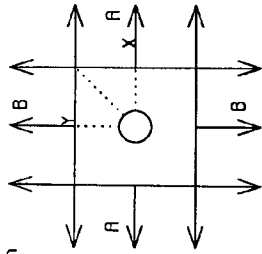
Theory:
--- Galin (Plane Strain)
--- Sokolov (Plane Stress)

OF ELEMENTS = 800

8 NODE SOLID

RADIAL DISTANCE FROM THE CENTER OF THE HOLE
4 ELEM/45 DEGREES; 8 MESH; 5 FIRST; 5 2ND; 5 3RD; 5 4TH; 5 TO EDGE

Plot 5: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 2 Plane Strain



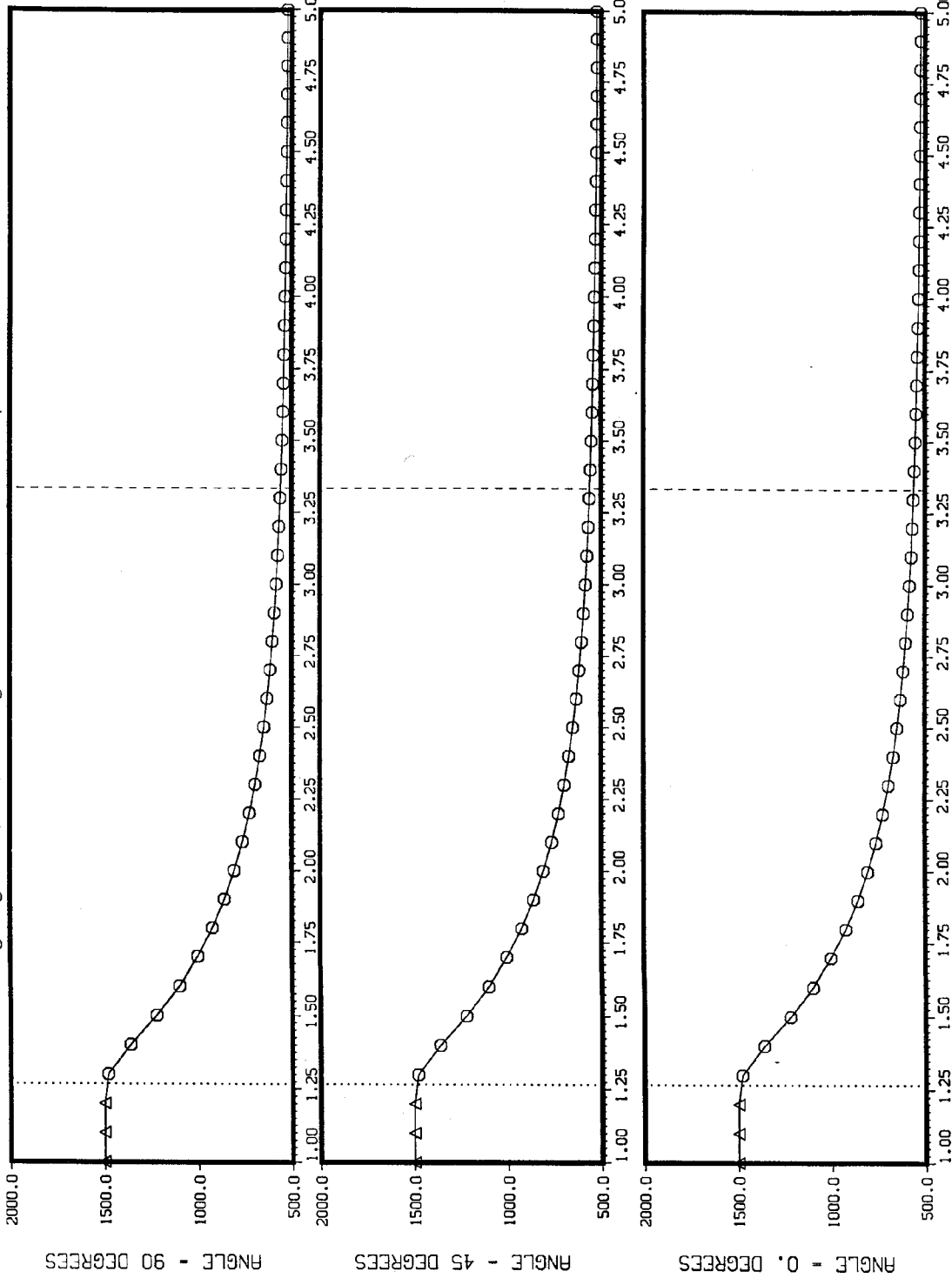
40. INCH BY 40. INCH BY
 1. INCH FLAT PLATE WITH
 1. INCH RADIUS HOLE
 LOADING (A) IS 1275.
 LOADING (B) IS 1275.

MSC 67.7
 CHEXR8
 DEFAULT

△ Nodes which are
 plastic
 ○ Nodes which are
 linear elastic

Theory:
 - - - SpkoloV
 (Plane Strain)

OF ELEMENTS = 1600



RADIAL DISTANCE FROM THE CENTER OF THE HOLE
 4 ELEM/45 DEGREES; 8 MESH; 10 FIRST; 10 2ND; 10 3RD; 10 4TH; 10 TO E

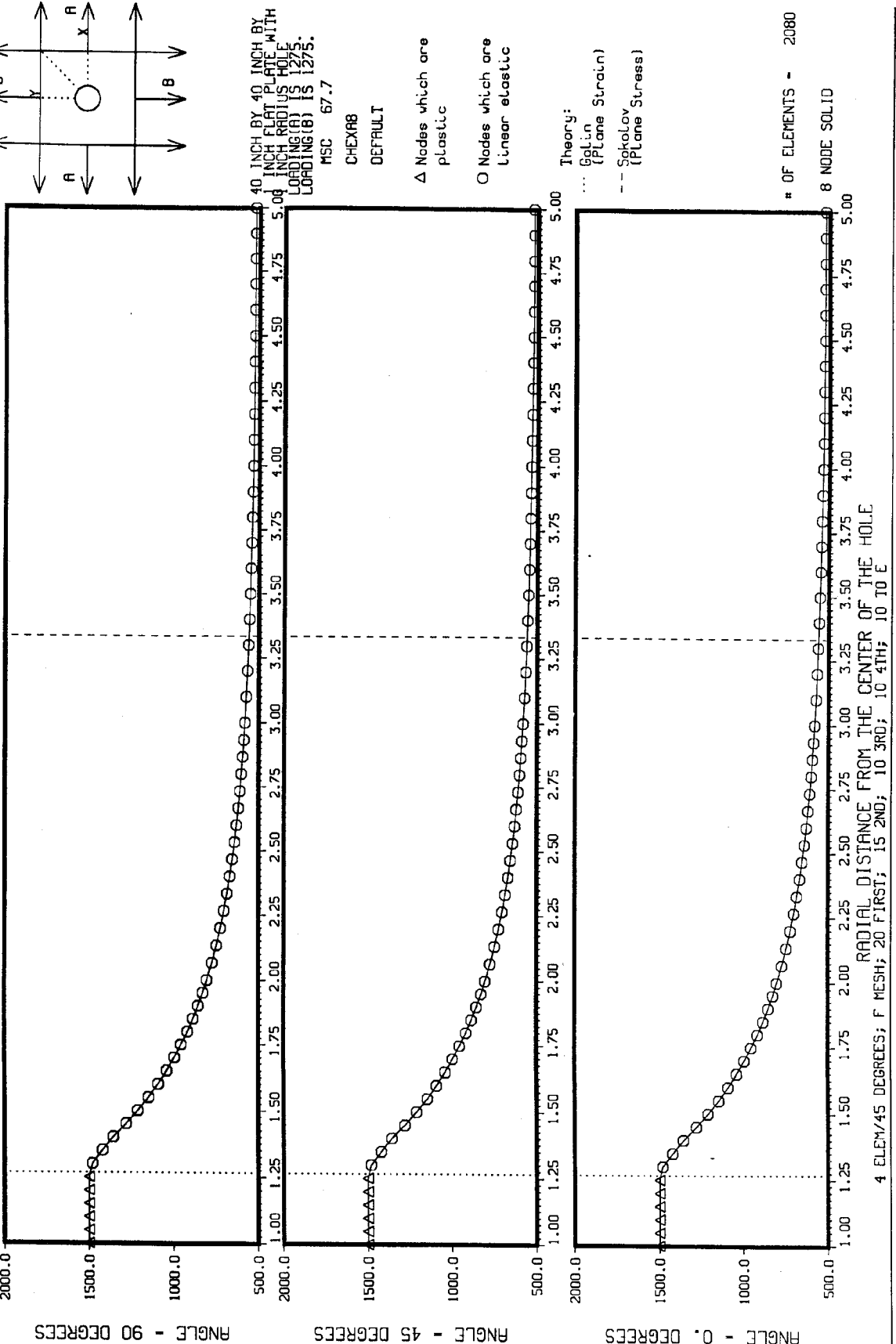
8 NODE SOLID

ANGLE = 90 DEGREES

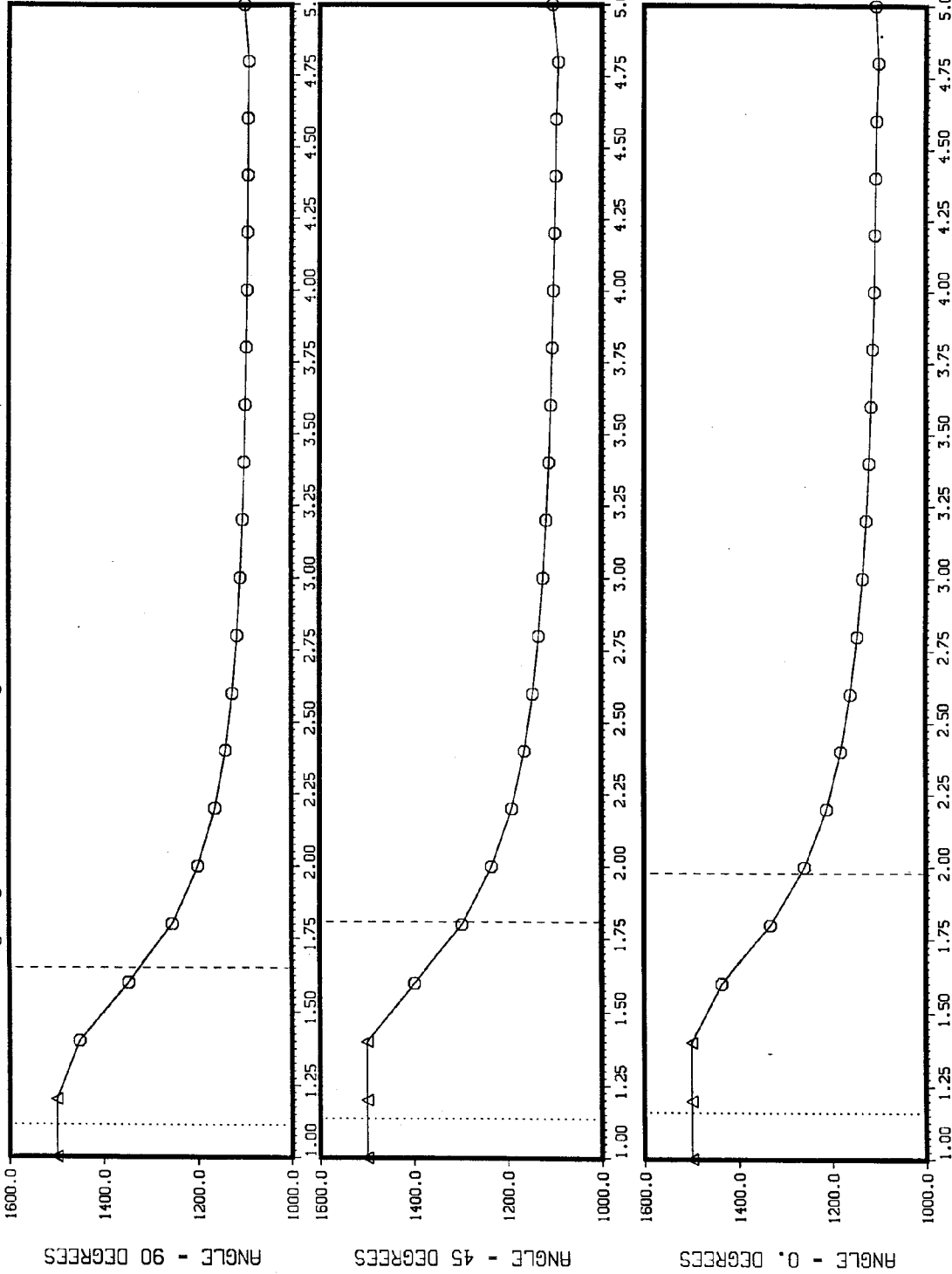
ANGLE = 45 DEGREES

ANGLE = 0. DEGREES

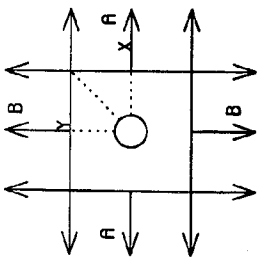
Plot 6: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 2 Plane Strain



Plot 7: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 1 Plane Stress

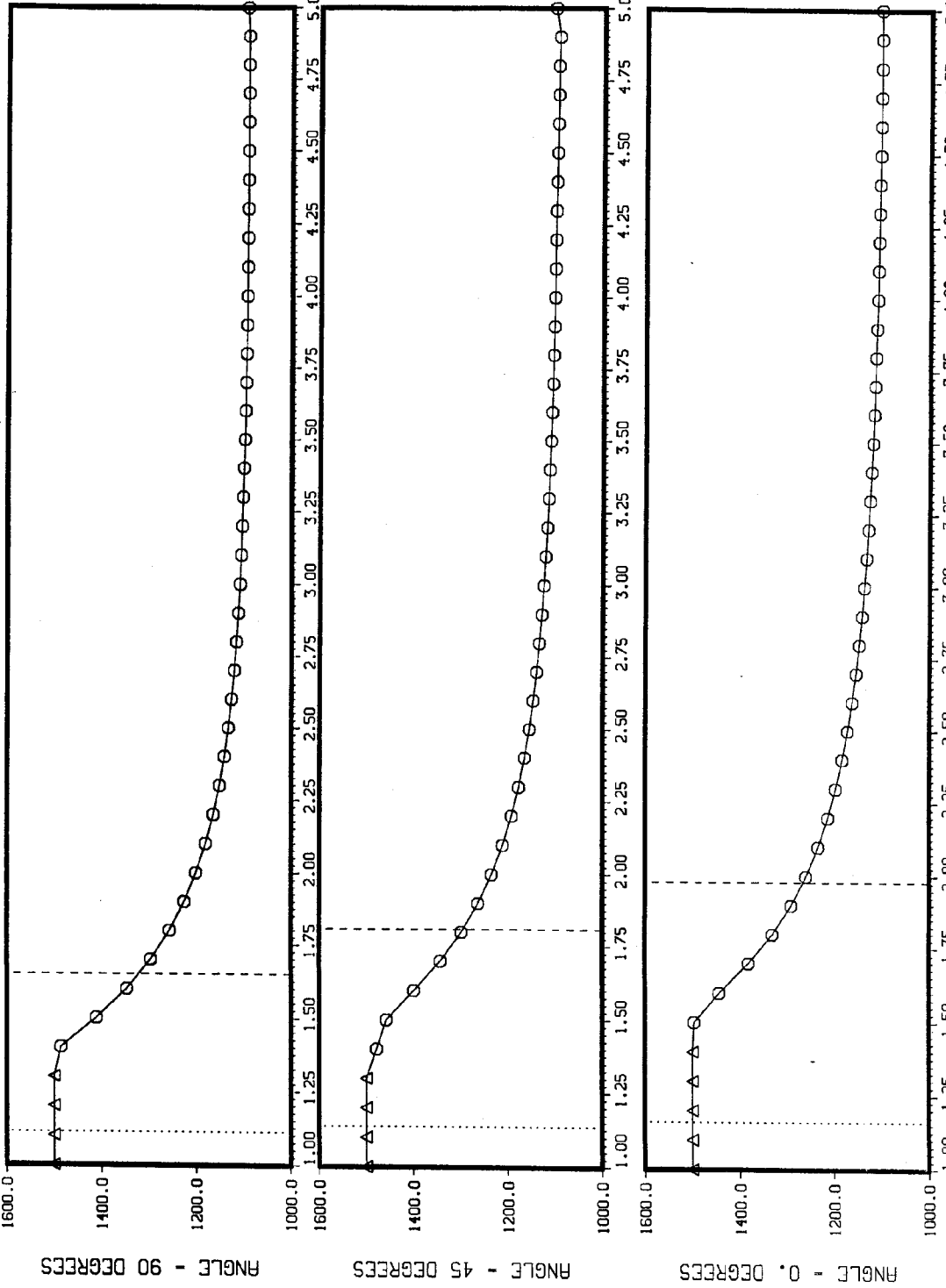


Plot 8: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 1 Plane Stress



40 INCH BY 40 INCH BY
1 INCH FLAT PLATE WITH
1 INCH RADIUS HOLE
LOADING(A) IS 1068.75
LOADING(B) IS 1106.25
MSC 67.7
CHEXAB
DEFAULT

△ Nodes which are
plastic
○ Nodes which are
Linear elastic

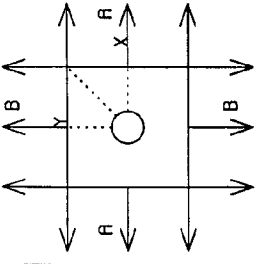


Theory:
Gpl.in
(Plane Strain)
-- SakoLav
(Plane Stress)

OF ELEMENTS - 1600
8 NODE SOLID

4 ELEM/45 DEGREES; B MESH; 10 FIRST; 10 2ND; 10 3RD; 10 4TH; 10 TO E
RADIAL DISTANCE FROM THE CENTER OF THE HOLE

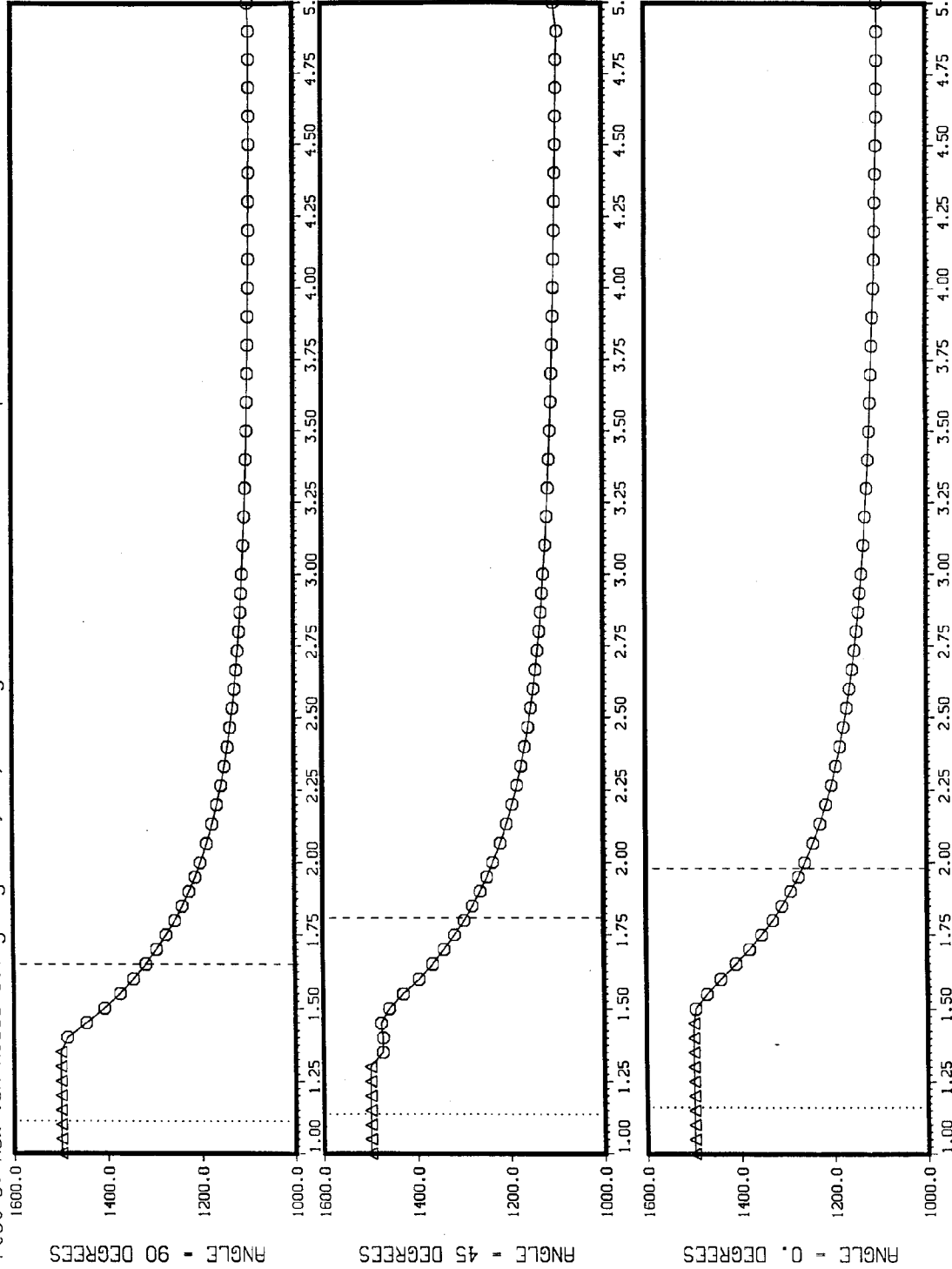
Plot 9: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 1 Plane Stress



40 INCH BY 40 INCH BY
1 INCH FLAT PLATE WITH
1 INCH RADIUS HOLE
LOADING (A) IS 1068.75
LOADING (B) IS 1106.25
MSC 67.7
CHEXR8
DEFAULT

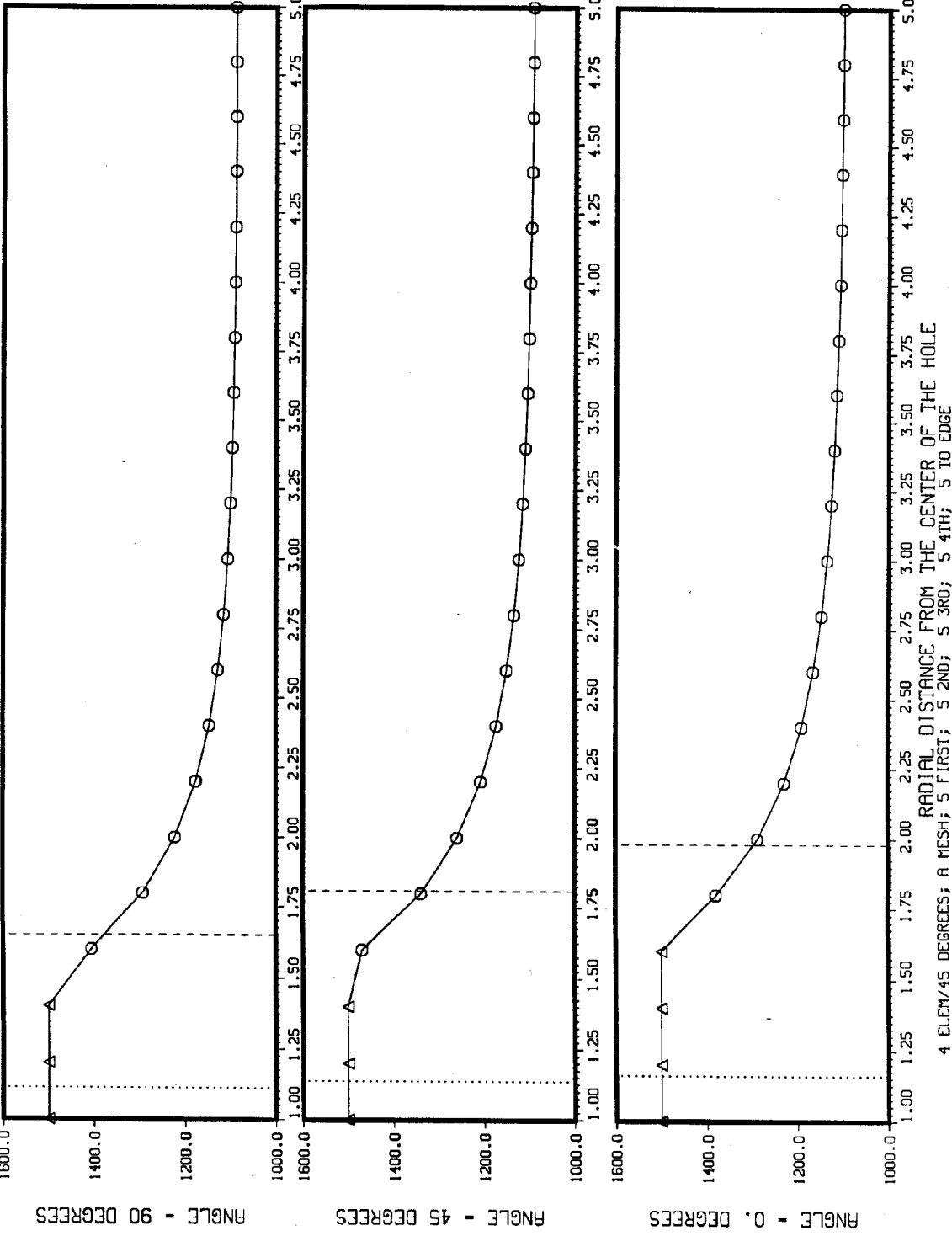
△ Nodes which are
plastic
○ Nodes which are
Linear elastic

Theory:
--- Golln
(Plane Strain)
--- Sokolov
(Plane Stress)

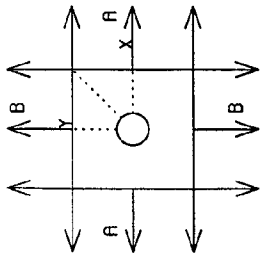
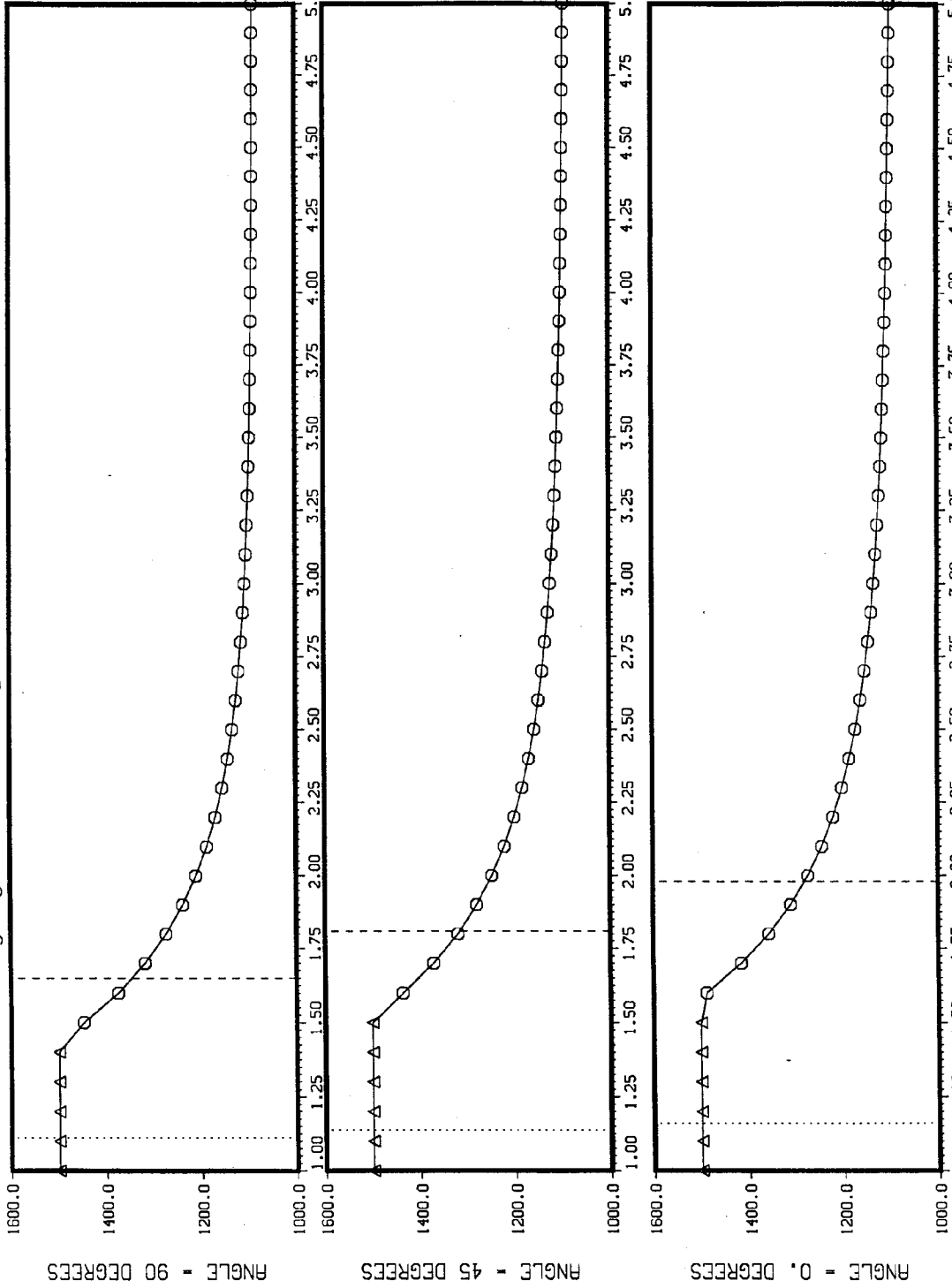


4 ELEM/45 DEGREES; F MESH; 20 FIRST; 15 2ND; 10 3RD; 10 4TH; 10 TO E
OF ELEMENTS = 2080
8 NODE SOLID

Plot 10: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 1 Plane Stress



Plot 11: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 1 Plane Stress



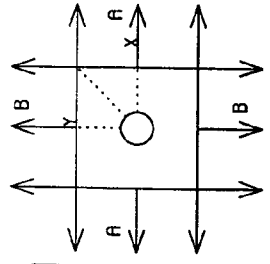
△ Nodes which are plastic
 ○ Nodes which are linear elastic

Theory:
 ... Gal'n (Plane Strain)
 --- Sokolov (Plane Stress)

OF ELEMENTS = 1600
 4 NODE SHELL

RADIAL DISTANCE FROM THE CENTER OF THE HOLE
 4 ELEM/45 DEGREES; 8 MESH; 10 2ND; 10 3RD; 10 4TH; 10 10 E

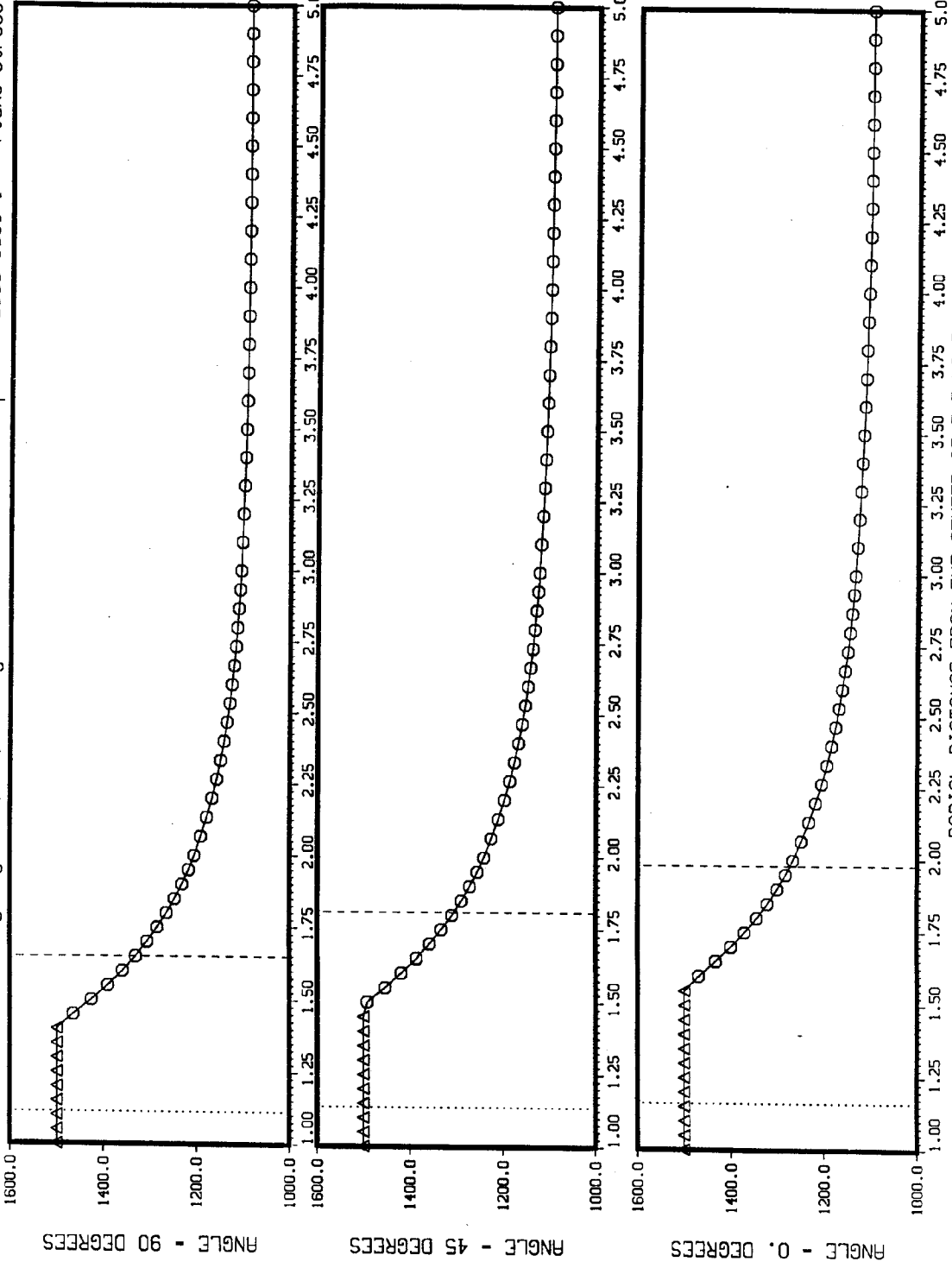
Plot 12: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 1 Plane Stress



40 INCH BY 40 INCH BY
 1 INCH FLAT PLATE WITH
 1 INCH RADIUS HOLE
 LOADING (A) IS 1068.75
 LOADING (B) IS 1108.25
 MSC 67.7
 COURD4
 DEFAULT

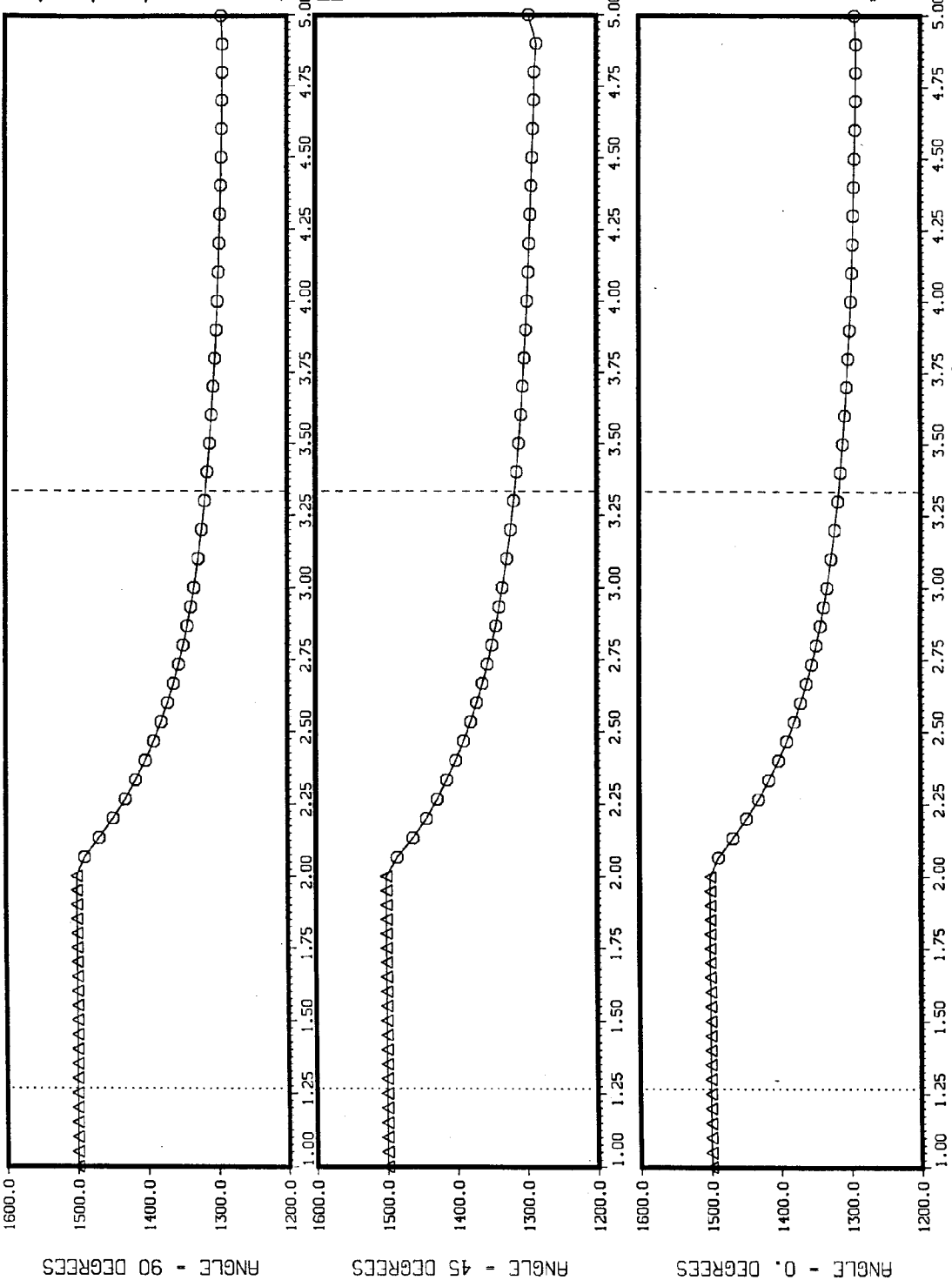
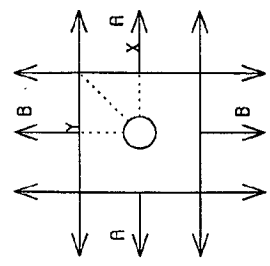
△ Nodes which are plastic
 ○ Nodes which are linear elastic

Theory:
 --- Gellin (Plane Strain)
 --- Spkolav (Plane Stress)



4 ELEM/45 DEGREES; F MESH; 20 FIRST; 15 2ND; 10 3RD; 10 4TH; 10 TO E
 # OF ELEMENTS = 2080
 4 NODE SHELL

Plot 13: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 2 Plane Stress



40 INCH BY 40 INCH BY
1 INCH THICK PLATE WITH
1 INCH RADIUS HOLE
LOADING (R) IS 1575.
LOADING (B) IS 1275.
MSC 67.7

CHEXRB
DEFAULT
△ Nodes which are
plastic
○ Nodes which are
linear elastic

Theory:
- - - Golun
(Plane Strain)
- - - Sokolov
(Plane Stress)

OF ELEMENTS - 2080

8 NODE SOLID

4 ELEM/45 DEGREES; F MESH; 20 FIRST; 15 2ND; 10 3RD; 10 4TH; 10 TO E
RADIAL DISTANCE FROM THE CENTER OF THE HOLE

Plot 14: Max von Mises along angles 0, 45, 90 degrees. Yield is 1500 psi. Load Case 2 Plane Stress

