ABSTRACT

The Efficient Calculation of Stress Concentration Factors

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Certain stress concentration problems recur often in the design of torque-carrying shafts. We desired to construct a set of tables similar to those in Peterson's Stress Concentration Handbook to permit rapid manual preliminary sizing of these shafts. Because a large number of parameters were involved, the efficiency of the modeling process was an issue. This study compares a variety of modeling techniques with emphasis on the p-adaptive approach available in MSC/NASTRAN, version 68.

INTRODUCTION

Traditional Strength of Materials formulas can be used for the initial sizing of torque-carrying shafts by using stress concentration factors, as for example from Peterson's handbook, "Stress Concentration Design Factors." A popular geometric configuration not found in Peterson is that of a cylindrical shaft with a countersunk cross-hole. See Fig 1.

Stress concentration factors were often derived from photoelastic studies in the past, but that technology has largely been supplanted by finite element techniques. Because discretization errors are a concern in this type of finite element analysis, the new p-adaptive capability in MSC/NASTRAN Version 68 is especially attractive. The subject of this note is a comparison of the p-method solution with "conventional solutions" based on two arbitrary meshes composed of either tetrahedra or hexahedra.

DISCUSSION OF PROBLEM

It is desired to construct graphs of stress concentration factors for solid and hollow shafts containing thru-holes that are chamfered at each end. See Fig 1. We'll consider torsion here; extension to bending or tension requires only a change in boundary conditions.

St. Venant's principle and preliminary tests suggest that a shaft length of four times the diameter is sufficiently long to avoid interaction of the boundary with the stress concentration. Therefore the stress concentration factor, K, is a function of five geometry parameters, two material parameters (linear elasticity) and the type of load, i.e.,

$K = f(D, d, \Delta, \delta, \phi, E, \nu, T)$ [symbols defined in Fig 2]

We can use dimensional analysis to reduce the problem to:

$$K = f(d/D, \delta/D, \phi, \nu, \Delta/D, T/ED^3)$$

Additionally, there is an eight-fold symmetry in the problem geometry. We had presumed that we would use cyclic symmetry to solve the problem, but p-adaptive elements currently are only available for Solutions 101 and 103, i.e., statics and normal modes. Fortunately, we can exploit the problem symmetry by using boundary conditions as illustrated in Fig 3.

These boundary conditions are not especially straight-forward. One is tempted to think of the torsional loads as having mirror symmetry, especially if the vector convention of Fig 3a is used. The sketch of Fig 3b is perhaps more appropriate and suggests that the loads are indeed anti-symmetric. (By way of contrast, Fig 3c shows symmetric loads.) For anti-symmetric loading, $u_x = u_y = 0$ at the center of the cut section. For convenience and efficiency, we choose $u_r = u_\theta = 0$ across the cut section, and justify this assumption by the lack of forces of constraint (SPCForces). There is a danger here of overconstraining the deformation. This same assumption would not work for bending because of the anticlastic effect.

MSC/NASTRAN V68 does not support MPCs or RBEs for quadratic or cubic order edges. We therefore used the equation option of GMBC to specify the angular displacements. That is for a 0.1 radian twist, we specified:

```
delta x = -Y*0.1
delta y =  X*0.1
```

where X and Y are the coordinates of any point lying on the loaded face.

A listing of the input file is provided as an appendix.

THE FINITE ELEMENT DISCRETIZATION

The MSC/ARIES solid modeler was used to generate the finite element mesh. The problem illustrated is a solid shaft, i.e., d = 0. The p-element mesh was generated by regioning the MSC/ARIES solid as illustrated in Fig 4. MSC/ARIES will not yet generate entries unique to p-elements, e.g. FEFACE, GMBC, etc., but these are not difficult to manually generate for a small model. FEFACE and FEEDGE entries are quite similar to constructs in MSGMESH having zero, one or two intermediate points along an edge to define its curvature. We also created a tetrahedral or free mesh, and a hexahedral mesh as illustrated in Figs 5a and 5b. A contour plot of stresses is shown in Fig 6. A XY-plot of the maximum Von Mises stresses versus p-adaptivity cycle is also shown in Fig 6 demonstrating the converging process.

Element Type	**Remarks**	**Maximum Von Mises**
CTETRA	h-elements	6990
CHEXA	h-elements	7139
CHEXA	p-elements, Error = 0.1	8073
CHEXA	p-elements, Error = 0.01	8525
CHEXA	p-elements, Error = 0.001	8607

CONCLUSIONS

The stress results for three models are shown in the Table. The models are homogeneous and consist of either 1) conventional CHEXA's 2) p-element CHEXA's or 3) CTETRA's.

Clearly the use of CTETRA's permits the simplest and quickest generation of a model, while the p-element implementation is the most labor intensive, at least until MSC/ARIES generates p-element input.

The p-element implementation in MSC/NASTRAN, version 68, has two important advantages. It provides assurance by local measures that discretization error is below some user-specified level. Secondly, the elements are relatively insensitive to distortion. This allows convenient paramaterization of the solid model as well as the mesh generated from that solid model.

If we had only one specific shaft problem to solve, we might choose to use free meshing (i.e. tetrahedra) for convenience and rapid turnaround. We would construct a finer model than used here. For the analysis of an entire family of shafts, where accuracy and consistency are especially important, p-elements offer a better alternative.

If we had only one specific shaft problem to solve, we would choose to use free meshing, i.e. tetrahedra. For an entire family of shafts covering a wide range of parameters, we choose to use p-elements, because of the relative insensitivity to distorted elements and because of the consistency provided by p-element technology.

ACKNOWLEDGEMENTS

The authors would like to thank their colleagues who contributed to this effort: Chris Pawlicki, Sreedhar Vangavolu, Nagendra Palle and Jay Fash at the Ford Motor Co. and to John Schiermeier at the MacNeal-Schwendler Corporation.

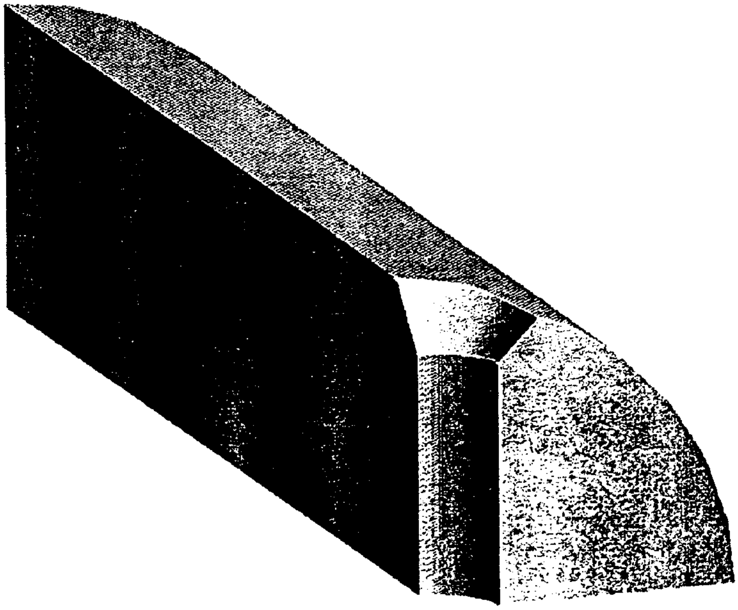


Fig. 1 1/8 Symmetric Shaft

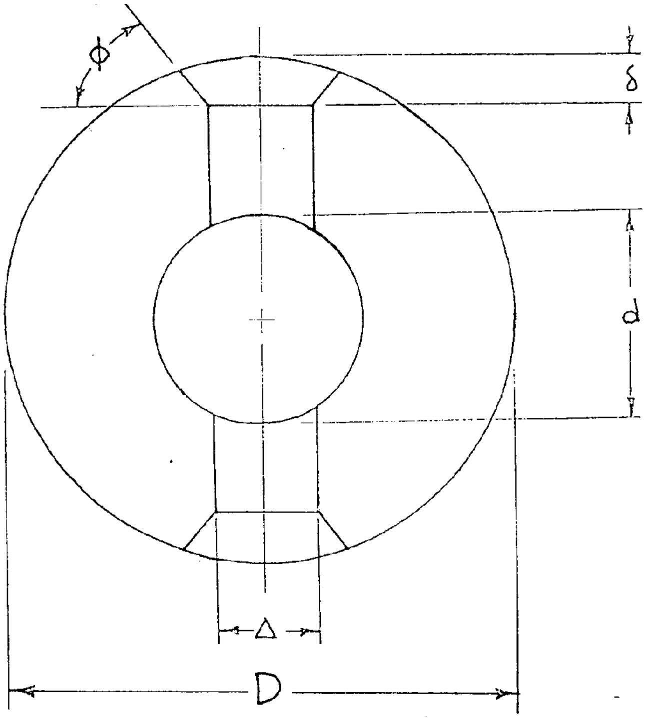


Fig. 2 Symbol Definition

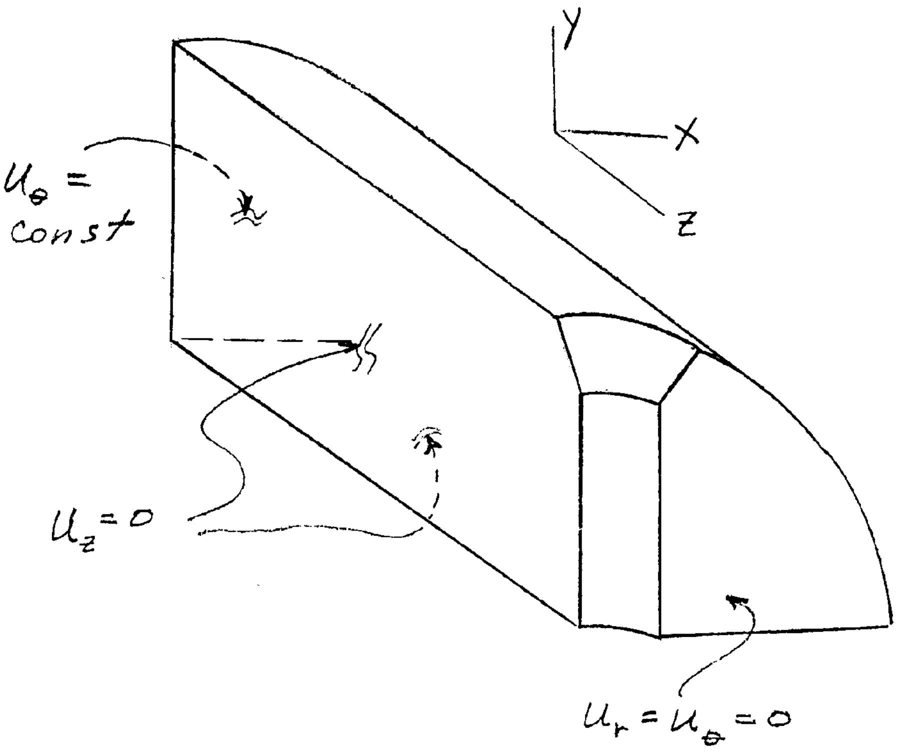


Fig. 3 Boundary Conditions

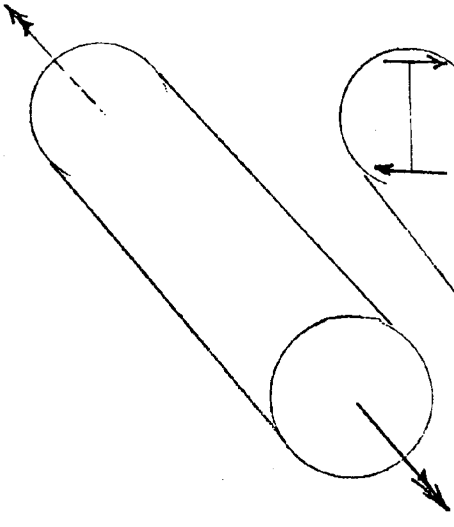


Fig. 3A

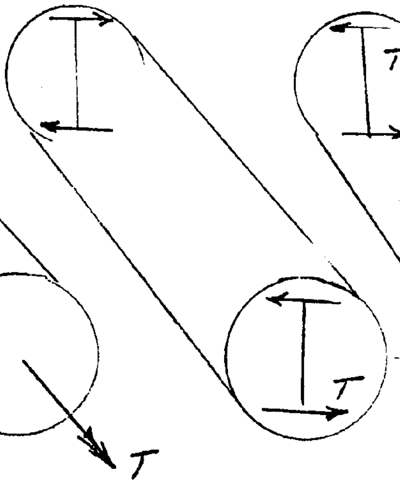


Fig. 3B

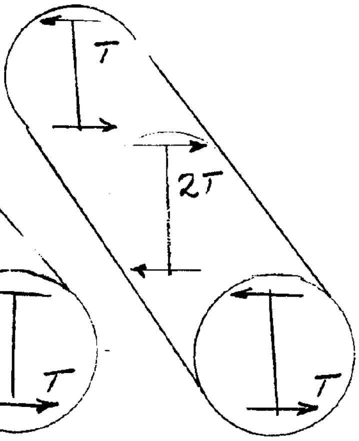


Fig. 3C

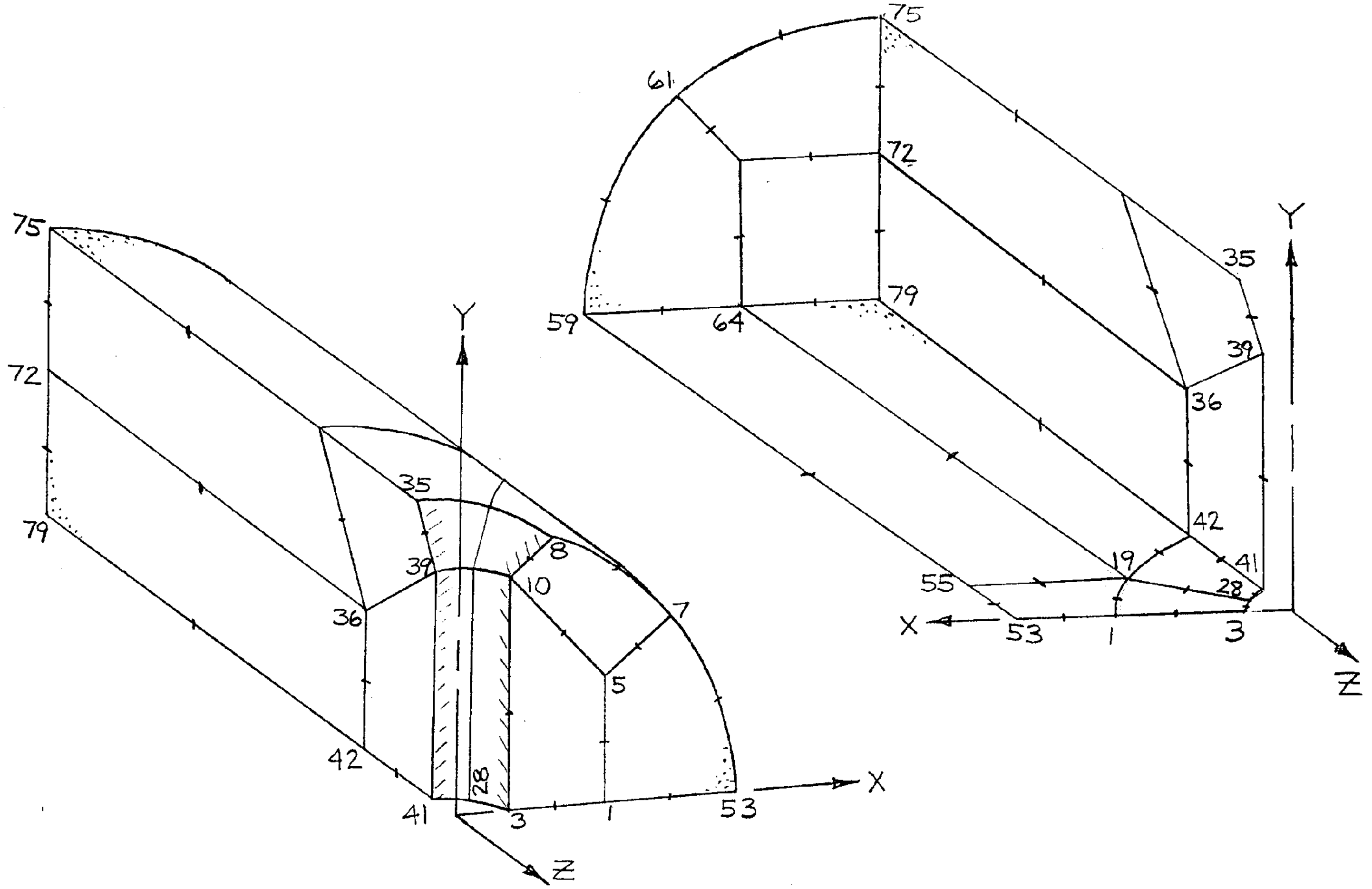


Fig 4. Layout of P-Elements

Fig 5a   CTETRA Mesh

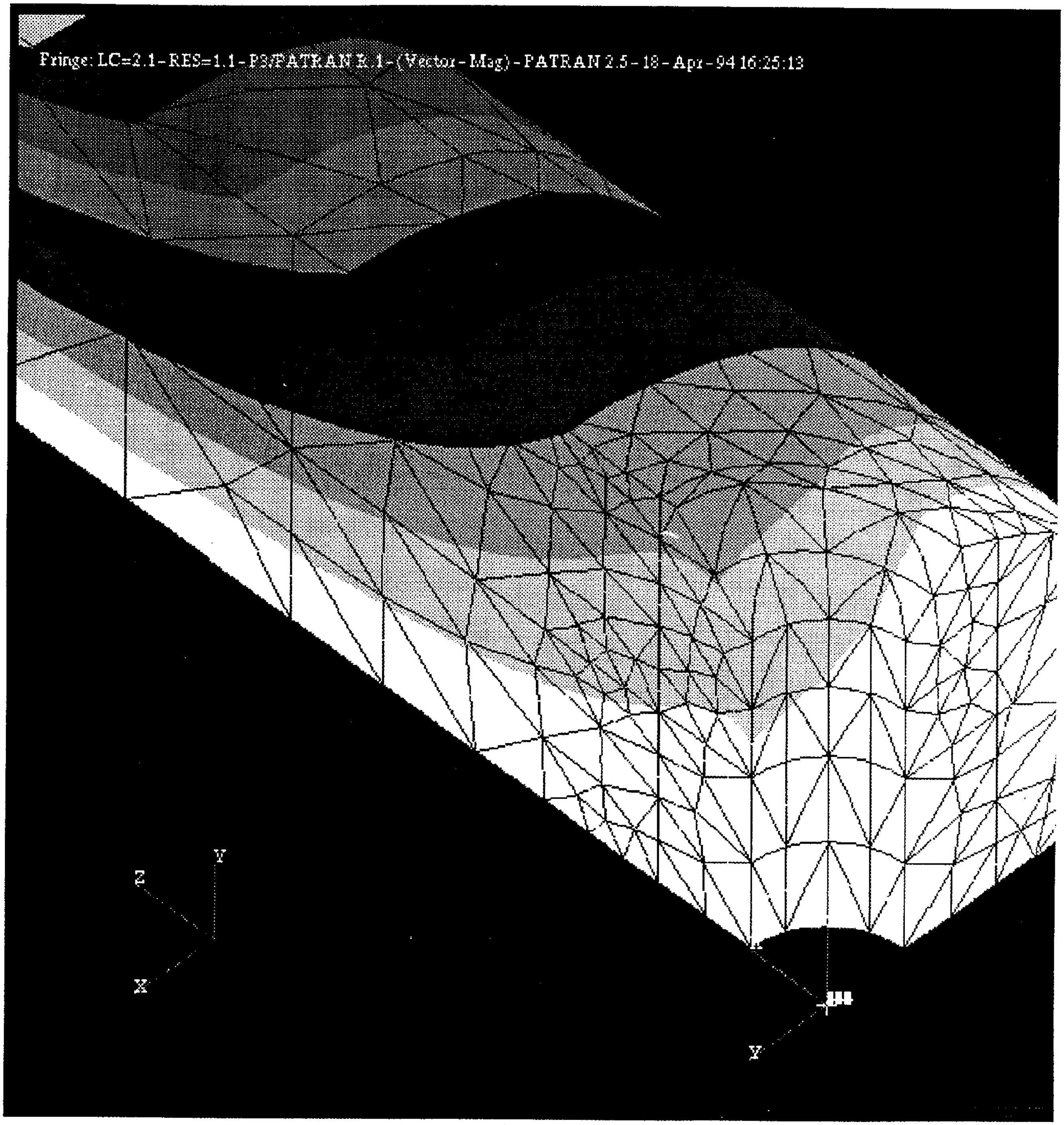
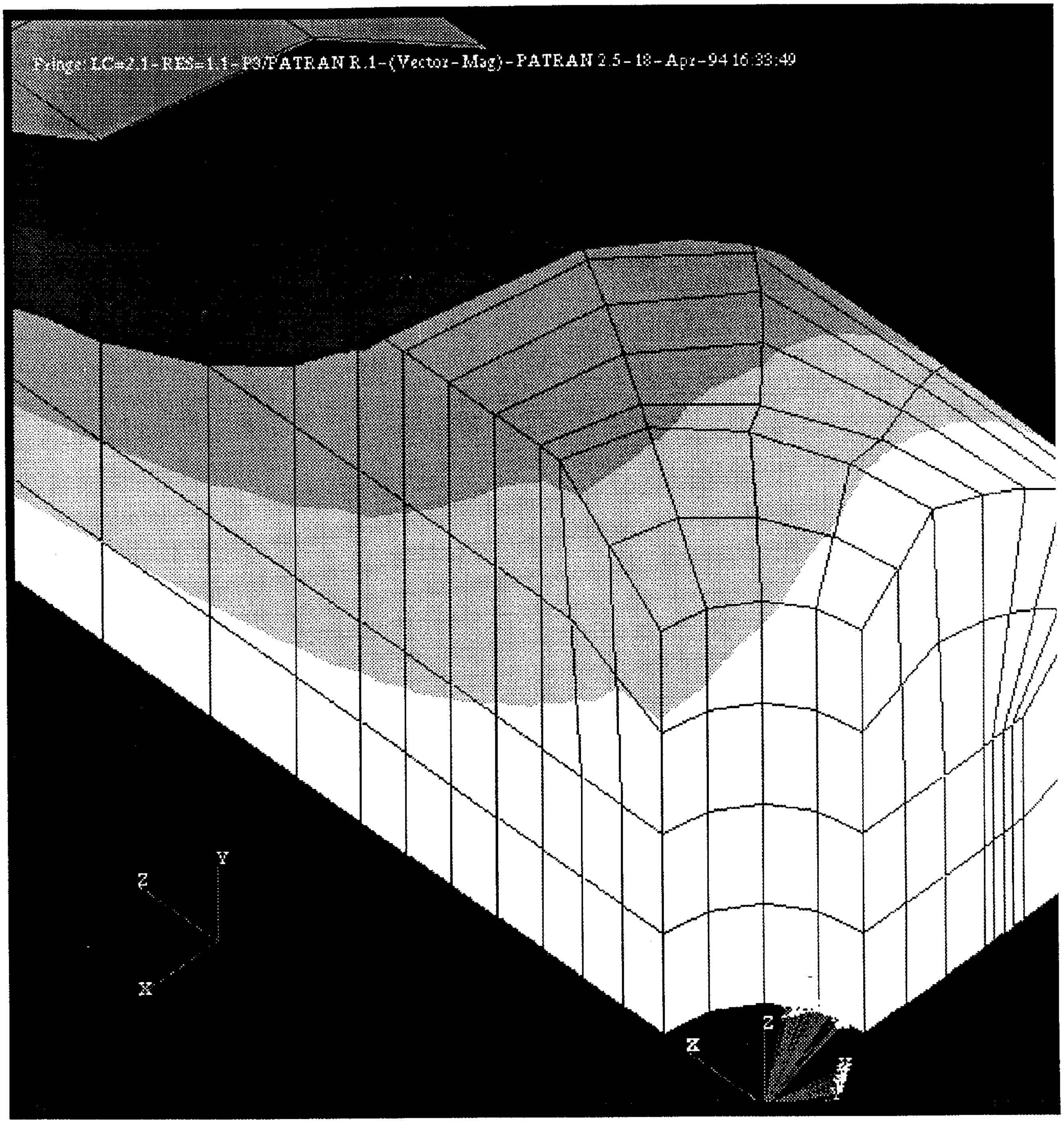


Fig 5b. CHEXA Mesh



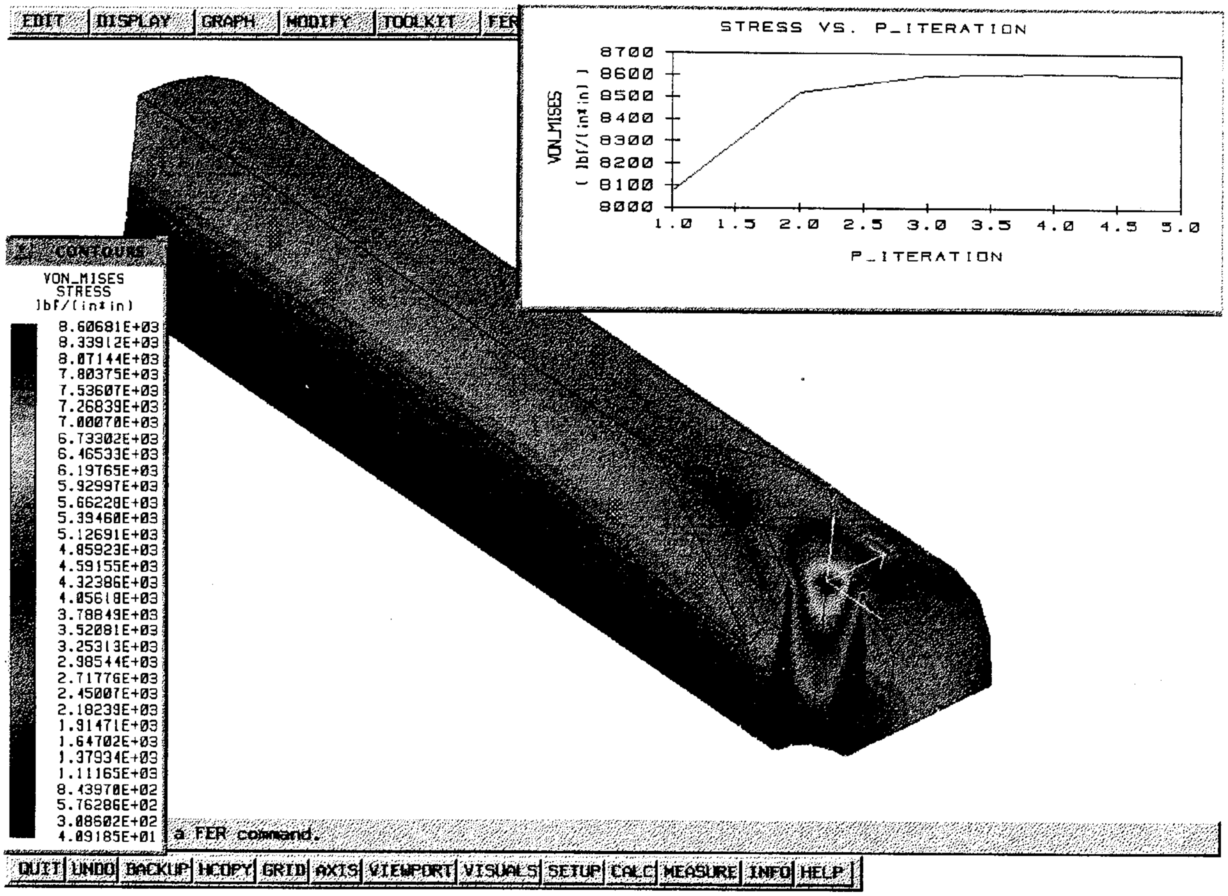


Figure 6. Von-Mises Stress Contour and XY-plot for the p-Element Model with 0.001 Error Tolerance

APPENDIX: LIST OF p-ELEMENT ASCII INPUT FILE

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SOL 101
TIME 9999
CEND
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SPCF = ALL
STRE = ALL
STRAIN(FIBER) = ALL
SUBCASE 1
TITLE=LINEAR_STATIC_1
$
SUBTITLE = p-adaptivity
$
$   p-elements Case Control commands
$
ADAPT    = 150
DATAREC = 301
$
$     Select output options for p-elements
$
OUTRCV = 401
$
SPC= 1
LOAD = 998
$
$     Select set(s) of elements for adaptivity and stress output
$
SETS DEFINITION
set 501 = all
set 101 = 1
$
$
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PARAM,PRGPST,NO
PARAM,POST,0
$*** *NODES:
$*** *BASE UNIT: MM
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+A3,14,27
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+A7,61,75
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+A8,64,19,
$
$*** *PIDS
PSOLID,1,2,0
$*** *MID
$*** *DENSITY UNITS ARE: N*SEC*SEC/(MM*MM*MM*MM)
MAT1*                  2 2.068388640E+05 8.004600000E+04 2.920000000E-01 MD00000
*MD00000 7.833400000E-09 8.200000000E-06 3.200000000E+01
$
$   p-element data
$
$    Adaptivity control
$
ADAPT   150                     120     120     150
        part=ALL,elset=501,type=EBEP,errest=1,errtol=0.1
$
PVAL    120     3       3       3               SET     501
PVAL    150     8       8       8               SET     501
$
$     Select various output options;
$
OUTPUT  301
        ELSET=501, DISP=PLOT, STRESS=PLOT
        STRAIN=PLOT, PVAL=PRINT, ERROR=PRINT,BY=1
        ELSET=101, DISP=plot, STRESS=print
        STRAIN=PLOT, PVAL=PRINT, ERROR=PRINT,BY=1
$
$     Define various output options;
$
OUTRCV  401     501
        VIEW=3*3*3
OUTRCV  401     101
        VIEW=2*2*2
$
GRDSET,,,,,,,456
CORD2R*                1                0  0.00000000E+00  0.00000000E+00 VZ00082
*VZ00082  0.00000000E+00  0.00000000E+00 1.000000000E+01  0.00000000E+00 VZ00083
*VZ00083  0.00000000E+00  0.00000000E+00 1.000000000E+01
$
$    Use POINT, FEEDGE to describe FEEDGE
$
POINT*               131                0 1.99659E+00      1.00000000E+01 ED00131
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POINT*              132                0 1.99659E+00      0.00000000E+00 ED00132
*ED00132 -1.50454E+00
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*ED00145 -2.12012E+00
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$
$       EDGEID  G1      G2      CID     GEOMIN  ID1     ID2
$
FEEDGE  1101    3       29              POINT   32      132
FEEDGE  1102    10      27              POINT   31      131
$
FEEDGE  1201    8       7               POINT   9
FEEDGE  1202    14      16              POINT   24
FEEDGE  1203    8       14              POINT   13
$
FEEDGE  1301    39      27              POINT   51      151
FEEDGE  1302    35      14              POINT   49
FEEDGE  1303    33      16              POINT   47
$
FEEDGE  1401    41      29              POINT   45      145
FEEDGE  1402    42      19              POINT   50
FEEDGE  1403    36      22              POINT   46
$
FEEDGE  1501    7       53              POINT   52
FEEDGE  1502    16      55              POINT   58
$
FEEDGE  1601    61      59              POINT   60
$
FEEDGE  1701    75      61              POINT   77
$
$
$    Specify FEFACE for BC and loads
$    +Z Faces
$       FACEID  G1      G2      G3      G4      CID     SURFID
FEFACE  2101    1       5       10      3
FEFACE  2201    5       10      8       7
FEFACE  2301    1       5       7       53
$
$    -Z Faces
FEFACE  2602    64      59      61      62
FEFACE  2702    72      62      61      75
FEFACE  2802    79      64      62      72
$
$    -Y Faces
FEFACE  2103    3       29      19      1
FEFACE  2403    29      41      42      19
FEFACE  2503    1       19      55      53
FEFACE  2603    19      64      59      55
FEFACE  2803    42      79      64      19
$
$    -X Faces
FEFACE  2305    39      36      33      35
FEFACE  2405    41      42      36      39
FEFACE  2705    72      36      33      75
FEFACE  2805    42      36      72      79
$*** *RESTRAINT CASES
$
$    Use GMBC ,GMLOAD and FEFACE to specify BC and loads
$      on geometry instead of element data
$
$    Boundary Conditions without z-constraints
$
$               SID     C       ENTITY  ID
GMBC    0       1       1       FEFACE  2101    constant 0.0
GMBC    0       1       1       FEFACE  2201    constant 0.0
GMBC    0       1       1       FEFACE  2301    constant 0.0
GMBC    0       1       2       FEFACE  2101    constant 0.0
GMBC    0       1       2       FEFACE  2201    constant 0.0
GMBC    0       1       2       FEFACE  2301    constant 0.0
$
```

```
$     CONSTRAINT XZ-Plane     (z=0)
$
GMBC     0       1       3       FEFACE  2103    constant 0.0
GMBC     0       1       3       FEFACE  2403    constant 0.0
GMBC     0       1       3       FEFACE  2503    constant 0.0
GMBC     0       1       3       FEFACE  2603    constant 0.0
GMBC     0       1       3       FEFACE  2803    constant 0.0
$
$     CONSTRAINT YZ-Plane     (z=0)
$
GMBC     0       1        3       FEFACE  2305   constant 0.0
GMBC     0       1        3       FEFACE  2405   constant 0.0
GMBC     0       1        3       FEFACE  2705   constant 0.0
GMBC     0       1        3       FEFACE  2805   constant 0.0
$
$     LOADING ON AN ELEMENT FACES
$
$     ENFORCED DISPLACEMENTS
$
$        LID     SPCID     C      ENTITY  ID      METHOD  F1      F2
GMBC     998     1         1      FEFACE  2602    EQUATION201
GMBC     998     1         1      FEFACE  2702    EQUATION201
GMBC     998     1         1      FEFACE  2802    EQUATION201
GMBC     998     1         2      FEFACE  2602    EQUATION202
GMBC     998     1         2      FEFACE  2702    EQUATION202
GMBC     998     1         2      FEFACE  2802    EQUATION202
$
$    equations for deformation
$    disp basic cid=0: r*sin(theta)=r*theta
$     Let theta=0.1 rad
$        x disp = -y * 0.1 rad
$        y disp =  x * 0.1 rad
$
DEQATN   201     D(x,y,z)=-y*0.1
DEQATN   202     D(x,y,z)= x*0.1
$
PARAM    BAILOUT -1
PARAM    MAXRATIO1.0E3
ENDDATA
```