# Model Reduction and Model Correlation Using MSC/NASTRAN

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### **ABSTRACT**

Dynamic mathematical models used in the launch vehicle verification loads analysis for predicting the flight loads and assessing the structural integrity are required to be testverified. The test-verified model is usually developed after conducting a modal survey on a structural test article and correlating the measured frequencies and mode shapes with the analytical prediction. However, it is not practical to instrument a test article in all degrees of freedom corresponding to those of the analytical model. Therefore, it is extremely difficult to correlate each analytical mode shape with the measured data. A systematic approach using MSC/NASTRAN version 67.5 direct matrix algorithm program (DMAP) is developed to minimize the effort for test-analysis model reduction and correlation. Four model reduction methods are available and can be selected by the user to generate a test analysis model (TAM). The size of the TAM is equal to the number of accelerometers mounted on the test article. This provides a direct comparison of the analytical prediction with the measured data. The orthogonality matrix, the cross-orthogonality matrix, and the modal assurance criteria between the analytical modes and the test modes are computed automatically by the DMAP to assess the correlation of the TAM with the modal test results. An analytical model is test-verified if the cross-orthogonality matrix and the frequency comparison meet the launch vehicle payload verification requirements. An example is presented to demonstrate the implementation of this MSC/NASTRAN DMAP for payload model verification.

#### INTRODUCTION

Dynamic mathematical models used in the launch vehicle verification loads analysis for predicting the flight loads and assessing the structural integrity are required to be testverified. The test-verified model is usually developed after conducting a modal survey on a structural test article and correlating the measured data with the analytical prediction. This process requires certain pre-established correlation criteria [1] in order to obtain a meaningful test-verified model. In addition, the modal density within a frequency range of interest of a typical spacecraft is generally high and it is not practical to instrument a test article in every degree of freedom corresponding to those of the analytical model. Consequently, it is extremely difficult to correlate each analytical mode with the measured data. From a structural load point of view, only those modes which are major contributors to the dynamic loads are critical to the accuracy of the loads analysis. Therefore, criteria and methodology to identify the significant modes and the instrument locations [2-4] for measuring these significant modes should be established prior to the start of a modal survey. A pre-modal test analysis approach to identify the significant modes of the structure and the best accelerometer locations to measure these modes is presented and demonstrated by a typical payload model.

For complex structures an incomplete set of measured modes is usually determined from the modal survey data. To assess the correlation of the mathematical model predictions which in general do not have dynamic degrees of freedom uniquely one to one with the modal test measurements, a reduction of the mathematical model to the test-analysis model (TAM) is required. The TAM can be generated from a analytical model if the significant modes and the optimal accelerometer locations of the structure have been determined from the pre-modal test analysis. To ensure an analytical model is indeed a proper representation of the test article, several assessment criteria are developed to examine the quality of the measured modal data and the correlation between the analytical predictions and the test results. A version 67.5 MSC/NASTRAN DMAP is developed for generating a testanalysis model of a payload and performing model correlation based on modal test data. Four model reduction approaches, static reduction [5], improved reduced system method [6], modal reduction [7,8], and hybrid reduction [9] are available and can be selected by users. A cross orthogonality matrix between the mode shapes of a full model and its reduced model may be computed to assess the model reduction method and the selection of the accelerometer locations. Three matrices, orthogonality, cross-orthogonality, and modal assurance criteria between analytical and test modes are computed automatically by the DMAP. The orthogonality matrix is used to assess the quality of the measured data and is used widely as a criterion for tearing down the test article in the aerospace industry; the cross-orthogonality matrix is used to measure the degree of correlation between analytical model and the test article; and the modal assurance criteria is used to evaluate the independence between modes. In addition, the modal effective masses associated with the test modes are also calculated and compared with those computed from analytical modes. An analytical model is test-verified if the cross-orthogonality matrix and the analytical/test frequency comparison meet the launch vehicle payload verification requirements [10].

### MODEL REDUCTION

A math model may be used to identify the target modes [11] and the optimal accelerometer locations [2-4]. A reduction of a mathematical model is performed to assist proper selection of accelerometer locations prior to performing a modal survey. A test-analysis-model (TAM) can then be derived based on the selected accelerometer locations.

## Static (Guyan) Reduction

Static or Guyan reduction [5] is the most common approach used to reduce the system mass matrix, [MA], and stiffness matrix, [KA], of the finite element model to the test degrees of freedom (number of accelerometers) using the transformation matrix derived from the analytical stiffness matrix. The number of accelerometers, a, available for use during the modal survey is considerably small compared to the size of the analytical model, A. The system equations of motion can be written as

$$\begin{bmatrix} \mathbf{M}_{dd} & \mathbf{M}_{da} \\ \mathbf{M}_{ad} & \mathbf{M}_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_{d} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{dd} & \mathbf{K}_{da} \\ \mathbf{K}_{ad} & \mathbf{K}_{aa} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_{d} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_{d} \\ \mathbf{F}_{a} \end{Bmatrix}$$
(1)

where subscripts a represents the degrees of freedom to be kept in the reduced model and d represents the degrees of freedom to be condensed out.

The Guyan transformation is derived based on a static solution which neglects the inertia effects,

$$\begin{bmatrix} K_{dd} & K_{da} \\ K_{ad} & K_{aa} \end{bmatrix} \begin{Bmatrix} X_{d} \\ X_{a} \end{Bmatrix} = \begin{Bmatrix} F_{d} \\ F_{a} \end{Bmatrix}$$
(2)

The dependent degrees of freedom,  $\{x_d\}$ , can be expressed in terms of independent degrees of freedom,  $\{x_a\}$ , by solving the upper partition of Eq. (2),

$$\{X_{d}\} = -[K_{dd}]^{-1}[K_{da}]\{X_{a}\} + [K_{dd}]^{-1}\{F_{d}\}$$
(3)

The transformation matrix,  $[T_G]$ , to condense the degrees of freedom (DOF's) from A-set to a-set can be obtained by noting that,

where

$$[T_{G}] = \begin{bmatrix} -\left[K_{dd}\right]^{-1}\left[K_{da}\right] \\ \left[I_{aa}\right] \end{bmatrix}$$
(5)

is the static transformation matrix. The main assumption of Guyan reduction is that there are no forces applied on the omitted degrees of freedom, therefore Eq. (4) can be reduced to

$$\{X_A\} = \left\{\begin{array}{c} X_d \\ X_a \end{array}\right\} = [T_G] \{X_a\} \tag{6}$$

The corresponding reduced mass and stiffness can be expressed as,

$$\left[\mathbf{M}_{G}\right]^{*} = \left[\mathbf{T}_{G}\right]^{\mathsf{T}} \left[\mathbf{M}_{A}\right] \left[\mathbf{T}_{G}\right]$$
$$\left[\mathbf{K}_{G}\right]^{*} = \left[\mathbf{T}_{G}\right]^{\mathsf{T}} \left[\mathbf{K}_{A}\right] \left[\mathbf{T}_{G}\right] \tag{7}$$

# Improved Reduced System (IRS)

The Guyan reduction produces an exact reduction on the stiffness if no forces are applied on the omitted degrees of freedom. If this is not the case, then the Guyan reduction is only approximate. The improved reduced system (IRS) proposes an algorithm [6] to correct this approximation in a static analysis by including the second term of Eq. (3). The procedure to derive the correction term is obtained by first formulating the standard eigenvalue equation for Eq. (1),

$$[K_A]\{X_A\} = [M_A]\{X_A\}[\Omega^2]$$
(8)

Using the Guyan transformation matrix shown in Eq. (5), Eq. (8) can be reduced to the asset degrees of freedom

$$[K_{G}]^{*}\{x_{a}\} = [M_{G}]^{*}\{x_{a}\}[\omega^{2}]$$
(9)

or

$$\{x_a\}[\omega^2] = \{[M_g]^*\}^{-1}[K_g]^*\{x_a\}$$
(10)

The A-set mode shapes may be recovered by employing the mode expansion method as shown in Eq. (11),

$$\{\widetilde{\mathsf{X}}_{\mathsf{A}}\} = [\mathsf{T}_{\mathsf{G}}]\{\mathsf{X}_{\mathsf{a}}\}\tag{11}$$

The mode shapes and frequencies of Eq. (11) are not identical to those of Eq. (8) due to the approximation of mass matrix using static reduction. The system eigenvalue equation for the A-set using Guyan reduction can be derived by substituting Eqs. (9) and (11) into Eq. (8),

$$[K_A]\{\widetilde{x}_A\} = [M_A]\{\widetilde{x}_A\}[\omega^2]$$
$$= [M_A][T_G]\{x_a\}[\omega^2]$$
(12)

Combining Eqs. (2), (10), and (12), it can be shown that the forces can be expressed in terms of the mode shapes of the reduced model as

$$[F_{A}] = [K_{a}]\{\tilde{x}_{A}\}$$

$$= [M_{A}]\{\tilde{x}_{A}\}[\omega^{2}]$$

$$= [M_{A}][T_{G}]\{x_{a}\}[\omega^{2}]$$

$$= [M_{A}][T_{G}]\{[M_{G}]^{*}\}^{-1}[K_{G}]^{*}\{x_{a}\}$$
(13)

The IRS transformation matrix can be obtained by substituting Eq. (13) into the second term of Eq. (4) as shown in Eq. (14),

$$\left\{ X_{A} \right\} = \left[ T_{G} \right] \left\{ X_{a} \right\} + \left[ \begin{array}{cc} K_{dd}^{-1} & 0 \\ 0 & 0 \end{array} \right] \left\{ F_{a} \right\}$$

$$= \left[ T_{IRS} \right] \left\{ X_{a} \right\}$$

$$(14)$$

where

$$[T_{IRS}] = \begin{bmatrix} \left\{ - \left[ K_{dd} \right]^{-1} \left[ K_{da} \right] \right\} + \left\{ \left[ K_{dd} \right]^{-1} \left( \left[ M_{da} \right] - \left[ M_{dd} \right] \left[ K_{dd} \right]^{-1} \left[ K_{da} \right] \right) \left\{ \left[ M_{G} \right]^{\cdot} \right\}^{-1} \left[ K_{G} \right]^{\cdot} \right\}$$

$$[I_{aa}]$$

$$(15)$$

#### **Modal Reduction**

The reliability of the statically reduced system matrices,  $[K_G]^*$  and  $[M_G]^*$ , tends to deteriorate if the structural behavior is dominated by relatively soft but heavy components. For those cases the dynamic contribution has to be taken into account. The modal reduction [7] technique utilizes the mode shapes of the analytical model as the transformation matrix for reducing the full scale finite element model to a test-analysis model representation. Assume there exists a Rayleigh-Ritz representation of the physical displacements

$$\{X_A\} = \left\{\begin{array}{c} X_d \\ X_a \end{array}\right\} = \left[\Phi_A\right] \{q_A\} = \left[\begin{array}{c} \Phi_d \\ \Phi_a \end{array}\right] \{q_A\}$$
 (16)

where  $[\Phi_A]$  are the mode shapes of the structure and  $\{q_A\}$  are the generalized coordinates. The generalized coordinates  $\{q_A\}$  can be expressed in terms of the dependent coordinates  $\{x_a\}$  by solving the lower partition of Eq. (16),

$$\{q_{A}\} = \left[\Phi_{a}^{\mathsf{T}}\Phi_{a}\right]^{\mathsf{T}}\left\{X_{a}\right\} \tag{17}$$

Consequently,

$$\{X_{d}\} = \left[\Phi_{d}\left[\Phi_{a}^{\mathsf{T}}\Phi_{a}\right]^{\mathsf{T}}\left[\Phi_{a}\right]^{\mathsf{T}}\right] \{X_{a}\}$$
 (18)

The transformation matrix [T<sub>D</sub>] for the modal reduction can be obtained as

$$\{X_A\} = [T_D]\{X_A\} \tag{19}$$

where

$$\begin{bmatrix} \mathsf{T}_{\mathsf{D}} \end{bmatrix} = \begin{bmatrix} \Phi_{\mathsf{a}} \begin{bmatrix} \Phi_{\mathsf{a}} \end{bmatrix} & \Phi_{\mathsf{a}} \end{bmatrix}^{-1} \Phi_{\mathsf{a}}^{\mathsf{T}} \\ \mathsf{I}_{\mathsf{a}\mathsf{a}} \end{bmatrix} \tag{20}$$

The reduced mass and stiffness matrices using the modal reduction approach have the form

$$\begin{bmatrix} \mathbf{M}_{\mathsf{D}} \end{bmatrix}^{*} = \begin{bmatrix} \mathbf{T}_{\mathsf{D}} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} \mathbf{M}_{\mathsf{A}} \end{bmatrix} \begin{bmatrix} \mathbf{T}_{\mathsf{D}} \end{bmatrix}$$
$$\begin{bmatrix} \mathbf{K}_{\mathsf{D}} \end{bmatrix}^{*} = \begin{bmatrix} \mathbf{T}_{\mathsf{D}} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} \mathbf{K}_{\mathsf{A}} \end{bmatrix} \begin{bmatrix} \mathbf{T}_{\mathsf{D}} \end{bmatrix}$$
(21)

Note that the dynamically reduced system exactly predicts the mode shapes and frequencies, as well as the orthogonality matrix of the original analytical model that are used in the model reduction process. Let [OR] represent the orthogonality matrix of the reduced system, then

$$[OR] = [\Phi_{a}]^{\mathsf{T}} [\mathsf{M}_{\mathsf{D}}] [\Phi_{a}]$$

$$= [\Phi_{a}]^{\mathsf{T}} [\mathsf{T}_{\mathsf{D}}]^{\mathsf{T}} [\mathsf{M}_{\mathsf{A}}] [\mathsf{T}_{\mathsf{D}}] [\Phi_{a}]$$

$$= [\Phi_{\mathsf{d}}^{\mathsf{T}} \Phi_{\mathsf{a}}^{\mathsf{T}}] [\mathsf{M}_{\mathsf{A}}] [\Phi_{\mathsf{d}}^{\mathsf{d}}]$$

$$= [\Phi_{\mathsf{A}}]^{\mathsf{T}} [\mathsf{M}_{\mathsf{A}}] [\Phi_{\mathsf{A}}]$$
(22)

# **Hybrid Reduction**

The modal reduction approach uses the mode shapes of an analytical model to derive the model reduction transformation matrix. In other words, this approach assumes the analytical model reflects the test article exactly. This assumption is invalid for most cases due to the model simplification, model linearization, and the truncation of higher modes [8]. A hybrid method [9] is developed to include the effects of the residual modes for improving the representation of the residual dynamics. To take into account the contribution of the residual modes, Eq. (16) is expanded to include the higher modes

$$\begin{aligned}
\left\{X_{A}\right\} &= \left\{\begin{array}{c}X_{d}\\X_{a}\end{array}\right\}^{m} + \left\{\begin{array}{c}X_{d}\\X_{a}\end{array}\right\}^{s} \\
&= \left[\Phi_{A}\Phi_{R}\right] \left\{\begin{array}{c}q_{A}\\q_{R}\end{array}\right\} \\
&= \left[\begin{array}{c}\Phi_{dA}\Phi_{dR}\\\Phi_{aA}\Phi_{aR}\end{array}\right] \left\{\begin{array}{c}q_{A}\\q_{R}\end{array}\right\} \\
&= \left[\begin{array}{c}\Phi_{d}\left[\Phi_{a}^{\mathsf{T}}\Phi_{a}\right]^{-1}\Phi_{a}^{\mathsf{T}}\\I_{aa}\end{array}\right] \left\{X_{a}\right\}^{m} + \left\{\begin{array}{c}X_{d}\\X_{a}\end{array}\right\}^{s} \\
&= \left[T_{D}\right] \left\{X_{a}\right\}^{m} + \left\{\begin{array}{c}X_{d}\\X_{a}\end{array}\right\}^{s}
\end{aligned} \tag{23}$$

where the superscript m represents the dynamic response contributed by the kept modes and s represents the response contributed by the residual modes. The residual dynamics can be approximated by the static analysis as

Assume an idempotent projector, P, exists such that a space can be divided into two complimentary subspaces [12]. Then by definition

$$P^2 = P \tag{25}$$

and

$$P_c = I - P \tag{26}$$

where P<sub>C</sub> is the complimentary of P and I is an identity matrix. A projector P can be constructed such that

$$\{x_a\}^m = P\{x_a\}$$

$$\{x_a\}^s = P_c\{x_a\}$$

$$= (I - P)\{x_a\}$$
(27)

Substituting Eqs. (24) and (27) into Eq. (23),

$$\{x_A\} = [T_D]P\{x_a\} + [T_G](I - P)\{x_a\}$$

$$= [[T_G] + ([T_D - T_G)]P\{x_a\}$$

$$= [T_{Hybrid}]\{x_a\}$$
(28)

where

$$[T_{Hybrid}] = [[T_G] + ([T_D - T_G)]P$$
(29)

is the transformation for the hybrid reduction approach. It has been shown [9] that an oblique projector can be expressed as

$$P = [\Phi_{a}][\Phi_{a}]^{\mathsf{T}}[\mathsf{M}_{\mathsf{D}}]^{\cdot} \tag{30}$$

where  $[\Phi_a]$  and  $[M_D]^*$  can be obtained from Eqs. (16) and (21), respectively.

### MODEL CORRELATION

The purpose of the modal survey is to verify the dynamic mathematical model of a structure which is used in the verification load analysis. Since the modal density of a typical spacecraft is generally high it is extremely difficult to compare each mode in the frequency range of interest. From a structural load point of view, only significant modes are critical to the prediction of the structural member loads. Therefore, modes which are the major contributors to the dynamic loads should be identified before the modal survey. Reference 11 investigates several methods to identify the target modes and proposes an approach to select proper target modes for the modal test. The measured frequency response functions are used to identify the frequencies, mode shapes and damping of the test article. To ensure the analytical model is indeed a proper representation of the test article, several criteria [10,13] have been developed to assess the quality of the measured data and the correlation between the analytical prediction and the test results.

# Orthogonality

The orthogonality matrix, [OR], of the test modes  $[\Phi_t]$  with respect to the analytical reduced mass matrix  $[M_a]$  as shown in Eq. (31) is widely used to assess the quality of the measured test modes,

$$[OR] = [\Phi_t]^{\mathsf{T}} [\mathsf{M}_{\mathsf{a}}] [\Phi_t]$$
(31)

The test data is acceptable if the off-diagonal terms of the orthogonality matrix are less than 0.1 when the matrix diagonal is normalized to 1.0. If the orthogonality matrix fails to meet the requirements, a thorough examination of the test article and the analytical model or additional tests such as sine dwell may be required to identify the problems. The test article may not be torn down before the requirements are met or the discrepancies have been justified.

# **Cross-Orthogonality**

Measured frequencies are widely accepted as the most accurate test data and can be compared directly with the analytical prediction. The requirement is to have frequency discrepancy within  $\pm 5\%$ . The measured mode shapes are usually assessed by their orthogonality with respect to the analytical reduced mass matrix. Quite often, it is difficult to identify the corresponding test mode that associates with an analytically predicted mode due to high modal density. One relatively simple way for assessing correlation between the mathematically predicted modes,  $[\Phi_a]$ , and the measured modes,  $[\Phi_t]$ , is the cross-orthogonality matrix (XOR) as shown in Eq. (32),

$$[XOR] = [\Phi_a]^{\mathsf{T}} [\mathsf{M}_a] [\Phi_t]$$
(32)

Each element of the [XOR] is a scalar value between zero and one representing the degree of correlation between the test mode and the referenced analytical mode. A XOR value near one indicates a high degree of correlation or consistency between two mode shapes. A generally accepted requirement for the cross-orthogonalty matrix is to have all diagonal terms larger than 0.9 and all the off-diagonal terms less than 0.1.

#### Modal Effective Mass

A target mode, by definition, has significant modal effective mass associated with it and is a major contributor to the dynamic load. The modal effective mass is a measurement of how much mass of a payload participates in the dynamic response within the frequency range of interest. Therefore, a good correlation between test modes and analytical modes should demonstrate similar modal effective mass. The goal is to have the variation of the modal effective mass using measured target modes and the analytical target modes within 10% for those target modes having system effective mass larger than 10% in any translational direction.

#### Modal Assurance Criterion

The modal assurance criterion (MAC) is used to evaluate the correlation between two modes ignoring the effects of the system mass. The MAC is a secondary criterion and has been used primarily to check the independence of two modes. The cross MAC can be computed by the following equation

$$[XMAC] = \frac{\left[ \left[ \Phi_{a} \right]^{\mathsf{T}} \left[ \Phi_{l} \right] \right] \otimes \left[ \left[ \Phi_{a} \right]^{\mathsf{T}} \left[ \Phi_{l} \right] \right]}{\left\{ \mathsf{Diag} \left( \left[ \Phi_{a} \right]^{\mathsf{T}} \left[ \Phi_{l} \right] \right) \right\} \left\{ \mathsf{Diag} \left( \left[ \Phi_{t} \right]^{\mathsf{T}} \left[ \Phi_{l} \right] \right) \right\}}$$
(33)

where

$$Diag(A) = \begin{cases} a_{11} \\ a_{22} \\ . \\ . \\ a_{nn} \end{cases}$$
 (34)

contains the diagonal terms of [A],  $\otimes$  and – represent element-by-element multiplication and division, respectively.

# **DMAP IMPLEMENTATION**

The model reduction and model correlation technique is implemented in MSC/NASTRAN version 67.5 using a rigid format alter. The rigid format alter for SOL 103 is included in Appendix A. Several user defined parameters are required to use the model reduction and model correlation DMAP. The available parameters and their functions are as follows.

### Parameters:

TAM Generate test analysis model. The master degrees of freedom are

specified by A set, or U1 set, or DMIG cards. The reduction

options available are:

GUYAN Static reduction

IRS Improved reduced system

MODAL Modal reduction HYBRID Hybrid reduction

GRIDPART Partition mode shapes of selected degrees of freedom (DOF).

The DOF can be defined by PART and SET in the case control deck,

or U1 set, A set, or DMIG in the bulk data deck.

XORTHO Compute model correlation matrices. The matrices automatically

computed by the DMAP are the orthogonality, cross-orthogonality

and the modal assurance criteria matrices.

TAMFREQ Compute TAM frequencies. This feature is used to validate the

model reduction method.

TMONLY Use target modes only.

#### **EXAMPLE**

A general purpose spacecraft (GPSC) shown in Figure 1 is used to demonstrate the application of the model reduction and model correlation DMAP. The baseline frequencies,  $[\Omega_A]$ , and mode shapes,  $[\Phi_A]$ , are computed with the interface (grid points 44, 45, 48, 49) fully constrained for. The accelerometer locations and directions listed in Table 1 are selected for measuring the response functions. A total of 30 accelerometers are selected. To compare with the test modes directly, the mode shapes,  $[\Phi_a]$ , associated with the accelerometer locations are partitioned out from the full model mode shapes  $[\Phi_A]$ . The baseline frequency matrix  $[\Omega_A]$  and  $[\Phi_a]$  which are punched out by the application DMAP in DMI and DMIG formats are referenced as test data.

The TAM is generated using the Guyan reduction to assess the validity of the accelerometer locations. The frequencies,  $[\Omega_{TAM}]$ , and the mode shapes,  $[\Phi_{TAM}]$ , of the TAM are computed and referenced as analysis data. The accelerometer locations selected for the modal test are assessed by computing the cross-orthogonality matrix between  $[\Phi_a]$  and  $[\Phi_{TAM}]$  using the application DMAP. Table 2 shows the comparison of the frequencies while Figure 2 shows the cross-orthogonality of the mode shapes graphically. The cross-orthogonality matrix indicates that all the diagonal terms are greater than 0.95 and all the off-diagonal terms are less than 0.02. This implies the motion of the GPSC test article can be described properly by the selected accelerometers.

# CONCLUSIONS

A version 67.5 MSC/NASTRAN DMAP has been developed for generating a test-analysis model of a payload and performing model correlation based on modal test data. Four model reduction approaches, static reduction, improved reduced system method, modal reduction, and hybrid reduction are available to the user. The orthogonality matrix, cross-orthogonality matrix, and the modal assurance criteria are computed and used to assess the model correlation. The orthogonality matrix is used to assess the quality of the measured data, the cross-orthogonality matrix is used to measure the degree of correlation between the analytical model and the test article, and the modal assurance criteria is used to evaluate the independence between modes. A typical payload model is used to demonstrate the flexibility and efficiency of the algorithm.

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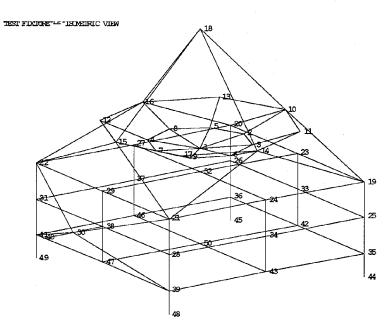


Figure 1. General Purpose Spacecraft Model

Table 1. Accelerometer Locations

Table 1.	Accelei oilletei Lo	cations
Accelerometer	MSC/NASTRAN	Direction
ID	Model Grid ID	
1	3	Z
2	4	$\mathbf{Z}$
3	4 5 9	Z
4	9	Z
5	11	Z
6	12	Z
7	13	Z
2 3 4 5 6 7 8 9	17	Z
9	18	XY
10	32	Z
11	33	Y
12	34	$\mathbf{X}$
13	37	X
14	39	X
15	40	XYZ
16	41	$\mathbf{X}$
17	42	XY
18	43	XY
19	46	XY
20	47	XY
21	50	XYZ

Table 2. Analysis and Test Frequency Comparison

Mode	Test (HZ)	Analysis (HZ)	Diff(%)
1	14.272	14.272	0.00
2	16.318	16.318	0.00
3	17.657	17.657	0.00
4	20.269	20.269	0.00
5	24.924	24.962	0.15
6	24.943	24.979	0.15
7	37.408	37. <b>445</b>	0.10
8	38.318	38.359	0.11
9	40.113	40.149	0.09
10	42.703	42.733	0.07
11	46.440	46.639	0.43
12	49.222	49.446	0.46
13	55.398	55.423	0.05
14	57.619	57.629	0.02
15	60.525	60.912	0.64
16	60.989	60.990	0.00
17	69.179	69.192	0.02

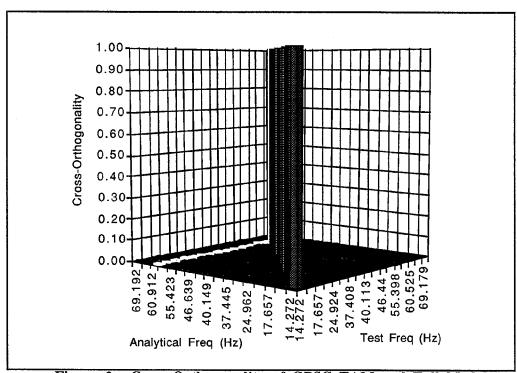


Figure 2. Cross-Orthogonality of GPSC TAM and Full Model

# Appendix A

# Model Reduction and Model Correlation DMAP

\$ \$ ***********************************	**************************************
	*
\$* DMAP FOR COMPUTING CROSS ORTHOGONALITY BETWEEN TWO MODEL \$ *	TWEEN TWO MODELS *
\$ NASTRAN V67.5, SOLJ03 COLD START OR RESTART	* XX
ECHOOFF COMPILE SURDIMAP=IPPL SOUTH-MSCSOUT NOT IST NORFF	*
TYPE PARM, CHAR3, Y. (XORTHO='NO') \$ TYPE PARM, CHAR6, Y. (TAM=' ') \$	
,	
ZUZKI=U \$ EQUIVY DMIZUZRO1/ALWAYS \$ EQUIVY DMIZUZROMATATAS \$	
MINDA/ZUZKUZ/ALWAIS	
IF (TAM $\Leftrightarrow$ ' ) THEN \$ ALTER 87.87	
ENDIF \$ (TAM<>' ') ATTER 80	
F (XORTHO-YES' OR TAM\$\infty\$ ') THEN \$	
ALTER 90,500 ENDIF \$ (XORTH0=YES' OR TAM<'')	
******************	****
\$ * Generate Test Analysis Model (TAM)  ***********************************	* ***
SUBDMAP=MODERS SOUTH=MSCSOU, NOLIST, NOREF	
ALTER 6 TYPE PARM, CHAR6, Y, (TAM=' ') \$	
TYPE DB, PVT PVT CASES/PVTVT &	
F (TAM=GUYAN OR TAM=TRS ) THEN \$	
H.SB \$	
ALTER 95 ENDIF \$ (TAM='GUYAN'OR TAM='IRS')	
SCOMPILE SUBDMAP=SEMODES SOUIN=MSCSOU, NOLIST, NOREF	)REF
ALTER 36	
TYPE PARM, CHAR3, Y, (TAMFREQ='YES') \$	
TYPE PARM NDDL J.N.ZUZR1 \$ TYPE DR 7117PM 7117PM 7117PM 7117PM 7117PM 6	
PVT PVT, CASES/PVTYT \$	
IF(TAM=' ')THEN \$ COPY MMAAAMTAWAIWAYS \$	
MKAA/KTAM/ALWAYS \$	
COPT LAMA/LAMATAM/ALWAYS \$ COPY PHA/PHAYT/ALWAYS \$	
. 2	3 dOXON
//o/ndmig \$ = -1) THEN \$	
VEC USE1/V101SE1A/A/COMP/U1/ \$ ELSE \$	
DIAGONAL ACCEL/DACCEL/COLUMN/1.0 \$ VEC USET/V1ASETG/G/COMP/A' \$	

```
PARTIN DACCEL, VIANETGI, VIUUSETA, JI $

BRUDE $ (NUMGE = 1)

PARAMI. VUUSETA, VIUSETA, VIUSETA, VIUDOF $

MATGEN

MATGEN

MATGEN

MATGEN

MATGEN

MATANINIOP $

AMATGEN

MATGEN

MATGE
```

Form Hybrid Transformation Matrix, TITAM SMPYAD PHAUI, PHAUITMMODAL...,POB/3 \$

PIC CMI CMI BAC UGSV	IF (GRIDPAR IF (GRIDPAR LAMX, OI MATMOD	IF (TMONLY ZUZR1=O \$ DBVIEW DI DRVIEW DI	DMIN DN  \$ MATPRN  \$ PARTN FI PARTN G	PARAMIL PARAMI TRNSP TY MATMOD TRNSP VN	ADD VW MATGEN, , MERCH, V LAMX EN OPP LAN	MINDE S (1M MINDE S (1M PARAMI PARAMI PARAMI P (NDING P (	ENDIF \$ (N ELSE \$ DIAGONAL ENDIF \$ (NI	ENDJF \$ (GR \$ ********** \$ ********* IF (GRIDPAR) IF (TMONL.Y	PAKIN UQ COPY FR ELSE \$ PAKTN UQ COPY FR	ENDIF \$ (TM MERGE P) MATGEN E	_09#	\$ READ AN \$ ******** IF(XORTHO- DMIII DI	z,
ADD TMODAL,TGUY/DIF!/(-1.0,0.0) \$ MPYAD DIFFPOB,TGUY/TITAM \$ SMPYAD TITAM,MMAA,TITAM,/MTAM/3///// \$ SMPYAD TITAM,MKAA,TITAM,/KTAM/3//// \$ ENDF \$ (TAM=MODAL) ENDF \$ (TAM=GUYAN )	IF (1AMPRE)=YES) THEN \$ MATHYPE/S.N.LANCZOS \$ MATHYPE/S.N.LANCZOS \$ MATHAD CASES, DYNAMICS,	ELSE \$ REIGL KTAM,MTAM,DYNAMICS,CASES,[LAMATAM,PHITAM,MITAM, OFIGSTAM,?MODE/S,N,NEIGTAM \$ BANDE &	DATOL **  MPYAD TITAM,PHITAM,PHAYT \$ ENDIF \$ (TAMFREQ=YES) ENDIF \$ (TAMFREQ=YES)		\$ ITER 60 ITER (0) IF (TAM > ') THEN \$ EQUIVX LAMATAM/OLB/ALWAYS \$ CALL SUPER: CASECC, PHAYT, OLB, , , OLB, , ,	PCDB, X"YCDB, POSTCDB, FORCE, EMAP, MAPS, HQEXINS, PVTS, CASIS, STT, FETT, GOAT, GOPE, SUSET, SILS, EDT, GOAT, THE GOAD, STATE TO GOAT, GOAD, STATE STATE TO GOAT, STATE GOAD SECTS, HETS, INDTA, KEH, MAP, MAP, SILX, HQUEXINX, HETS, MOTA, KEH, MAP, SILX, HQUEXINX, HETS, MOTA, SATE STATE ST		#I.SB \$ ALTER 62  ALTER 62  ***********************************	S ALTER 66 TYRE PRRM, CHAR3,Y,(NASTIRS=NO 'PRTBOTH=NO') \$	TYPE PARM, CS, Y, CSEID \$ TYPE DB, MATPOOL, MCG, MLAA, MAA \$ STYPE DB, DM, DMINDX \$ TYPE PARM, NDD, L. NUSETS \$	TYPE PARM, CHARG, Y, (TAM=' ) \$ TYPE PARM, CHARS, Y, (GRIDPART=NO', XORTHO=NO', TMONLY=NO') \$ TYPE PARM, NDD, I, NZUZRI	PVT PVT/PVTY/ \$ PRTPARM //O/NASTIES' \$ PRTPARM //O/NASTIES' \$	ALTER 144, 144, 144, 144, 144, 144, 144, 144

CALAMA, CABERRUIS, EDIT, YS. GM, PSS. KRS. KSS. GR. CARPHO, OLZ, CALAMA, CABHA, PPI, PSI, MAR. MEA, XYCDBIRG, CRX, BACK, BG, QKG, U. G. PPI, SI, MAR. MEA, XYCDBIRG, CRX, BACK, BG, QKG, U. G. PPI, SI, MAR. MEA, XYCDBIRG, CRX, BACK, BG, QKG, U. G. PPI, SI, MAR. MEA, XYCDBIRG, CRX, G. LALAMA, OLZHAM, 1 s. MATMOD LAMA, "JALAMA, 1 s. MATMOD LAMA", "JARAMA, 1 s. MATMOD LAMA", "JARAMA, 1 s. MATMOD LAMA", "JARAMA, 1 s. MATRON CANAGE, TAVECCI, TREGC, TAVECCI, TREG, SI, NOMDER, SI, SA, MATRON TAVECCI, TREGC, TAVECC

ZUZRI = 0 \$  DUZRI = 0 \$  DBVTEW MASSCX=ZUZRO3 (WHERE ZUZRI=0) \$  DBVTEW KSTIFFCX=ZUZRO4 (WHERE ZUZRI=0) \$  DBVTEW PHCX=ZUZRO5 (WHERE ZUZRI=0) \$  COPY MASSCXIMASSC/ALWAYS \$  COPY KSTIFFCXIETE/ALWAYS \$  COPY PHCX/PHC/ALWAYS \$  FOR PHCX/PHC/ALWAYS \$  FOR PHCX/PHC/ALWAYS \$	ADDS MAAMAA, MASSC \$ COPY ULPHICKLIWSYS \$ ELSE \$ COPY MGGMASSC/ALWAYS \$ COPY UCAPTICKLIWAYS \$ COPY UCAPTIFICKLIWAYS \$ COPY UCAPTIFICKLIWAYS \$ COPY UCAPTIFICKLIWAYS \$	\$\(Tright of the property of the propert	DP18Q.DP18GT_AP1P18Q \$  MP18Q.AP1P18QMACC///192 \$  PHIFFORM_PHIFFORM_P2SQ\\$  4L P2SQMP2SQ/WHOLE/2.0 \$  AL PSQQMP2SQ/WHOLE/2.0 \$  DP2SQ.DP2SQ(T_AP2P2SQ \$  MP2SQ.DP2SQ(T_AP2P2SQ \$  MP2SQ.DP2SQ(T_AP2P2SQ \$  MP2SQ.DP2SQ(T_AP1P2SQ \$  AL XPIPZKPIPSSQ/WHOLE/2.0 \$  DP1SQ.DP2SQ(T_AP1P2SQ \$  XPIPZSQAAP(TP1 \$  PHCPHIFFORM_XYPIPZ \$  AL XPIPZXPAPP2SQ(XAACC/I) \$  XPIPZSQAAP(TP2SQ)	F(TMONL Y=YBS) THEN \$ PARTN ORF.TMAVEGY_XORTM! \$ PARTN ORF.TMAVEGY_TAVECQ_XORTM! \$ PARTN MACCTMAVEGY_TAVECQ_XORTM! \$ PARTN MACCTMAVEGY_TAMACTM!! \$ PARTN MACCTMAVEGY_TAMACTM! \$ PARTN MACCTMAVEGY_TAMACTM! \$ PARTN MACCTMAVEGY_TAMACTM! \$ MATPRN PREOCTM_REOPTM_I \$ MATPRN PREOCTM_REOPTM_I \$ MATPRN PREOCTM_MACCTM_MACTM_I \$ MATPRN PREOCTM_I \$ MATP	MATPRN   FREQC,FREQE   \$
. \$ ADSCX=Z STEFCX= HCX=ZUZ ASSCX/MA ASSCX/MA ITHFCX/KS ICX/PHIC/ GPART,,V1	MASET) TH MLAA, MAZ JL/PHIC/AI UGPART,, GGYT/PHIC JGYT/PHIC	AM \$\leq\$ In Extended to the partial text of the partial tex	DP1SQ,DP PURYORI PURYORI PURYORI PURYORI DP2SQ,DP PHIC,PHII XP1PZJQ,APZ XP1PZJQ,APZ	= YES) TH ORE, TMVI XOR, TMVI XOR, TMVI XMAC, TM XMAC, TM FREQE, TV FREQE, TV FREQCTA ORFTAC, TY FREQCTA ORFTAC, TY ORFTAC, TY ORFTAC, TY	FREQCIA ORF,XOR (AONLY=YI AONLY=YI THEN \$ PLX(-1.0,0 PLX(SED) (BDONE/I/I SNE,SED) = YES) TH EYES TH SED, SNED/ SNED, SNED/ SNE
ZUZRI=0 DBVEW N DBVEW R COPY MA COPY KS COPY PH PARTN U	HLSE \$ IF (NESETS) ADD5 N COPY PARTN ELSE \$ COPY U COPY U	* <u>∞</u> 92,99,22	MPYAD ADD NPYAD DIAGONAL DIAGONAL TRINSP TRINSP TRINSP MPYAD ADD MPYAD DIAGONAL MPYAD ADD ADD ADD ADD ADD ADD ADD ADD ADD	F (TMONLY PARTIN PARTIN PARTIN PARTIN PARTIN PARTIN MATPRN	HALSE & MATPRIN MATPRIN MATPRIN MATPRIN (SEID=0) CSEID=CM ENDIF \$ (SI ENDIF \$

OUTPUT2 REQCIM/FREQFTM,ORFTM,XONTM,/90/NASTAPE///
MATRIX/MATRIX/MATRIX/MATRIX \$

OUTPUT2 MACCTM,MACTEM, MACTM, //90/NASTAPE///
MATRIX/MATRIX/MATRIX/MATRIX \$

OUTPUT2 REDC,FREQF,ORF XOR, //90/NASTAPE///
MATRIX/MATRIX/MATRIX/MATRIX \$

OUTPUT2 REDC,FREQF,ORF XOR, //90/NASTAPE///
MATRIX/MATRIX/MATRIX/MATRIX \$

OUTPUT2 REDC,FREGF,ORF XOR, //90/NASTAPE///
MATRIX/MATRIX/MATRIX/ \$

OUTPUT2 MACC,MACF,XOMAC, //90/NASTAPE///
MATRIX \$

OUTPUT2 REQC,FREGF,ORF XOR, //90/NASTAPE///
MATRIX/MATRIX/MATRIX/ \$

OUTPUT2 MACC,MACF,XOMAC, //90/NASTAPE///
MATRIX/MATRIX/MATRIX/ \$

OUTPUT2 MACC,MACF,XOMAC, //90/NASTAPE///
MATRIX/MATRIX/MATRIX/ \$

ENDIF \$ (NASTIPS-TES)

OUTPUT2 REGCONPY (ORE) ORE) COSTIP ORE)

SECTION ORDON SET IN AND ORTHER SET IN AND OUTPUTS ENDIF OR NOT SET IN AND OUTPUTS ON AND OUTPUTS ON AND OUTPUTS ON AND OUTPUTZ SET IN AND OUTPUTZ SET IN AND OUTPUTZ SET IN AND IN AND OUTPUTS ON AND OUTPUTZ SET IN AND OU