Vibroacoustics Random Response Analysis Methodology

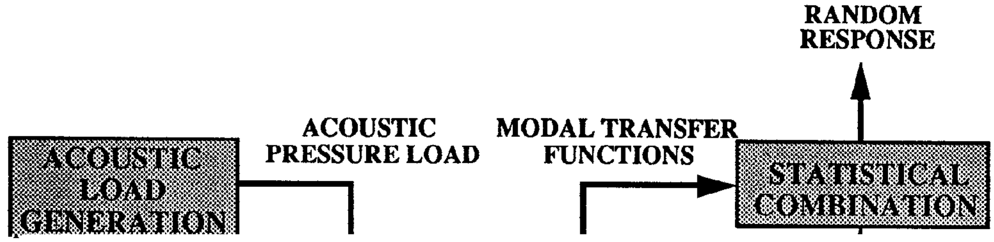
By
Gregory J. Michels
Commercial & Government Systems
Eastman Kodak Company
Rochester, NY 14650-3118

Abstract

The theory, application, and assumptions of a dynamic modal analysis based technique are presented to familiarize users with a method to use MSC/NASTRAN to determine the random response of a structure subjected to acoustic loads. Emphasis is placed on computing the mechanical response of a structure due to applied acoustic loads with worst case spatial correlation. MSC/NASTRAN solutions 101, 103 and 111 with the random response processor are used in the analysis method.

Introduction

The theory and application of an analysis methodology for determining the random response of a structure exposed to an acoustic environment is presented in this paper. While parts of the algorithm are drawn from a simpler method intended for the analysis of only simple panels, the method presented here can be applied to any structure [1]. The method can be subdivided into three major sections as illustrated by Figure 1.



in the form of transfer function magnitude spectral densities. The final section combines the response contributions from each mode to yield a random response quantity.

Theory

The first equation to be developed is a relation which generates the worst case harmonic pressure profile for each mode. For now it will be assumed that the peak pressure at any frequency is unity (1.0 psi). We shall see that it is easier to deal with the magnitude of the acoustic load in the statistical combination section. Assume that we have meshed a finite element model of a structure whose stiffness matrix is [K], mass matrix is [M], dynamic eigenvalues are λ_{mode} and corresponding eigenvectors are $[\Phi]_{mode,grid,component}$. In addition, assume that for each grid in the model the effective area exposed to the acoustic environment and the normal to this surface has been computed and are expressed as A_{grid} and $n_{grid,component}$, respectively, where *grid* indicates the grid number and *component* indexes one of the three orthogonal components of the normal expressed in the output coordinate system of the grid (CD on the GRID card). At each natural frequency the worst case spatial correlation for the harmonic load would have the same spatial correlation as the load which deforms the structure in the shape of the eigenvector. This is simply

$$[\mathbf{F}]_{\mathbf{mode,grid,component}} = \mathbf{C_0}\ [\mathbf{K}]_{\mathbf{grid,component,grid,component}}\ [\mathbf{\Phi}]_{\mathbf{mode,grid,component}} \qquad (1)$$

where $[F]_{mode,grid,component}$ is a horizontal vector of vertical load vectors whose spatial correlations are such that each will maximize the dynamic response of the $mode^{th}$ dynamic mode of the structure and C_0 is an arbitrary constant which scales the load vector [2]. Although $[F]_{mode,grid,component}$ does possess the worst case spatial correlations for a dynamic load exciting the $mode^{th}$ dynamic mode, it does not represent acoustic pressure profiles because the load vectors do not necessarily act everywhere normal to the structure. Therefore, we need to take the component of each load vector in $[F]_{mode,grid,component}$ which is normal to the structure to determine the worst case acoustic pressure profiles. So we write

$$[P_0]_{mode,grid,component} = C_0 \; (([K]_{grid,component,grid,component} \; [\Phi]_{mode,grid,component}) \cdot \{n\}_{grid,component}) \; \{n\}_{grid,component} \tag{2}$$

where $[P_0]_{mode,grid,component}$ is a horizontal vector of vertical harmonic acoustic pressure load vectors with the worst case spatial correlations for exciting the modeth dynamic mode. Only the translational components are considered in the matrix multiplication and dot product operations performed in the parentheses. Notice that the quantity in parentheses, $(([K]\,[\Phi])\cdot\{n\})$, is a scalar for each mode multiplying the unit normal vector, $\{n\}_{grid,component}$.

For a certain value of C_0, say C_{mode}, $[P_0]_{mode,grid,component}$ represents pressure profiles with peak pressures of 1.0 psi, $[P]_{mode,grid,component}$. To compute C_{mode} so that $[P]_{mode,grid,component}$ have peak pressures of 1.0 psi we write

$$C_{mode} = 1/\mathrm{Max}([K]_{grid,component,grid,component} \; [\Phi]_{mode,grid,component} \; \{n\}_{grid,component} \, / \, A_{grid}) \tag{3}$$

where A_{grid} is the area of the gridth grid. Equation (3) sets C_{mode} for the modeth mode equal to the reciprocals of the maximum pressures of the pressure profiles given by equation (2) for C_0=1.0. Therefore, when $C_0 = C_{mode}$ worst case pressure profiles, $[P]_{mode,grid,component}$, with peak pressures of unity are obtained. Such worst case pressure profiles can be expressed as

$$[P]_{mode,grid,component} = C_{mode} \; [K]_{grid,component,grid,component} \; [\Phi]_{mode,grid,component} \; \{n\}_{grid,component} \tag{4}$$

where there is no sum on mode. Figure 2 gives a sketch of the vectors {n}, [F], $[P_0]$, and [P] to clarify the physical meaning of equations (1)-(4).

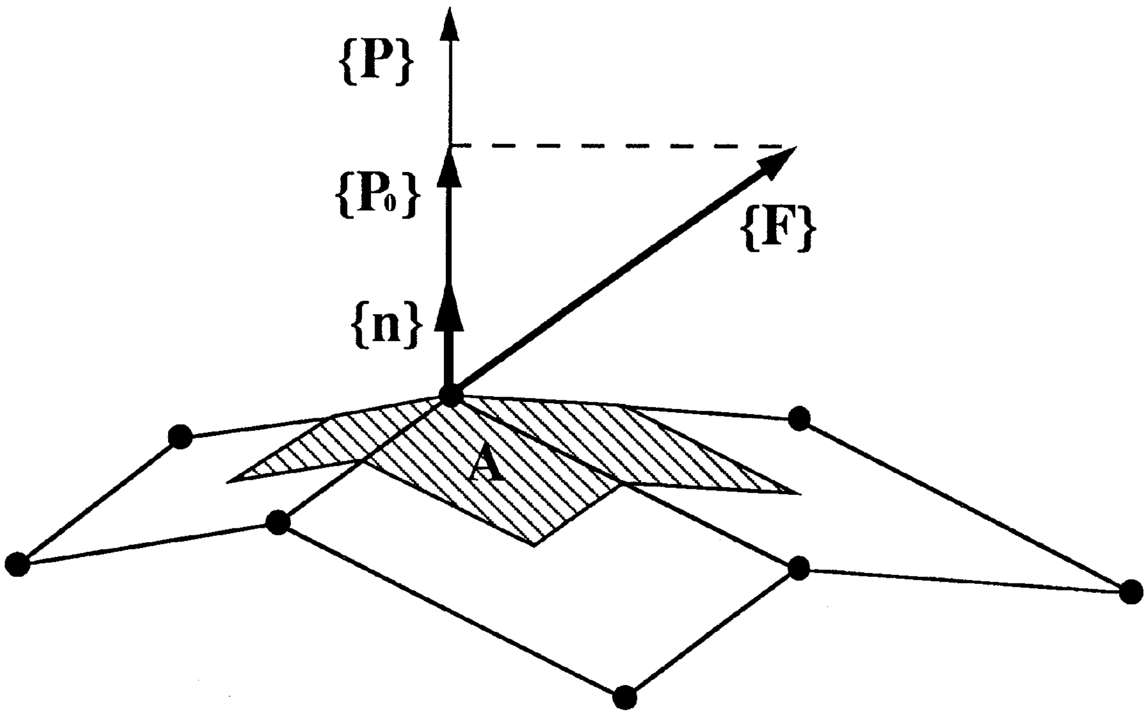


Figure 2
Sketch of vectors {n}, {F}, $\{P_0\}$, and {P}.

If analytical expressions for the acoustic pressure are available as functions of frequency and spatial variables, they may be used to modify equation (4) to generate loads with less conservatism. In such a case we would write

$$[P]_{mode,grid,component} = p(\omega_{mode}, x, y, z)\ A_{grid}\ \{n\}_{grid,component} \qquad (5)$$

where $p(\omega_{mode},x,y,z)$ is the pressure field description with a peak pressure of 1.0 psi and there is no sum over the grids.

Once a pressure load profile has been generated for each dynamic mode, the response of each mode to its corresponding harmonic load profile can be computed [3]. Beginning with Newton's second law of motion for harmonic nodal displacements and applied loads we write

$$(-\omega^2[M] + i\omega[B] + [K])\ \{U(\omega)\} = \{P\} \qquad (6)$$

where ω is the forcing frequency of the load vector {P}, i is the square root of -1, [B] is the damping matrix, and {U(ω)} is the complex vector of nodal displacements as a function of the forcing frequency. By making the following substitution

$$\{U(\omega)\}_{grid,component} = [\Phi]_{mode,grid,component}\ \xi(\omega)_{mode} \tag{7}$$

where $\xi(\omega)_{mode}$ is the transfer function for the modeth mode as a function of the forcing frequency. If we substitute equation (7) into equation (6) and pre-multiply each term by $[\Phi]_{mode,grid,component}$, then equation (6) becomes

$$\xi(\omega)_{mode} = p_{mode} / (-\omega^2 m_{mode} + i\omega b_{mode} + k_{mode}) \tag{8}$$

where

$$p_{mode} = [\Phi]^T_{mode,grid,component}\ [P]_{mode,grid,component}$$

$$m_{mode} = [\Phi]^T_{mode,grid,component}\ [M]_{grid,component,grid,component}\ [\Phi]_{mode,grid,component} = 1.0$$

$$b_{mode} = \omega_{mode}/Q$$

$$k_{mode} = [\Phi]^T_{mode,grid,component}\ [K]_{grid,component,grid,component}\ [\Phi]_{mode,grid,component} = \omega_{mode}^2$$

and there is no sum on mode. Equation (8) is a scalar equation which can be evaluated to give the transfer function $\xi(\omega)_{mode}$ for each mode subjected to its corresponding worst case unit load $[P]_{mode,grid,component}$.

It should be understood that because the pressure profiles for each mode were scaled to possess a peak pressure of unity, these transfer functions do not represent the mechanical response of the system due to the actual acoustic environment. To obtain a properly scaled mechanical response the magnitude of each transfer function is converted to a spectral density of the magnitude of the transfer function. Equation (9) gives the relation which accomplishes this.

$$|\xi(\omega)_{SD}| = |\xi(\omega)|^2 \ \mathbf{PSD}(\omega) \tag{9}$$

where $|\xi(\omega)_{SD}|$ is the spectral density of the magnitude of the transfer function and PSD(ω) is the pressure spectral density of the acoustic environment in units of psi^2/Hz. Once $|\xi(\omega)_{SD}|$ has been computed for each mode, the RMS's of the $|\xi(\omega)_{SD}|$'s can be found by computing the square root of the area under the curve. Each mode's RMS of $|\xi(\omega)_{SD}|$ represents a scale factor which can scale any result from the natural frequency analysis of that mode to give an RMS random response result due to the behavior of that mode. For example, $\{\Phi\}_{3,23,x}$ multiplied by the RMS of $|\xi(\omega)_{SD}|$ for mode three would be the RMS displacement of grid 23 in the x direction due to the behavior of mode three.

Since the mechanical response computation section separately predicts the random response due to the behaviors of each mode, responses from each mode must be combined. The response spectral density due to the behavior of all modes for a particular result would be the sum of the individual response spectral densities from each mode. This is shown pictorially in Figure 3.

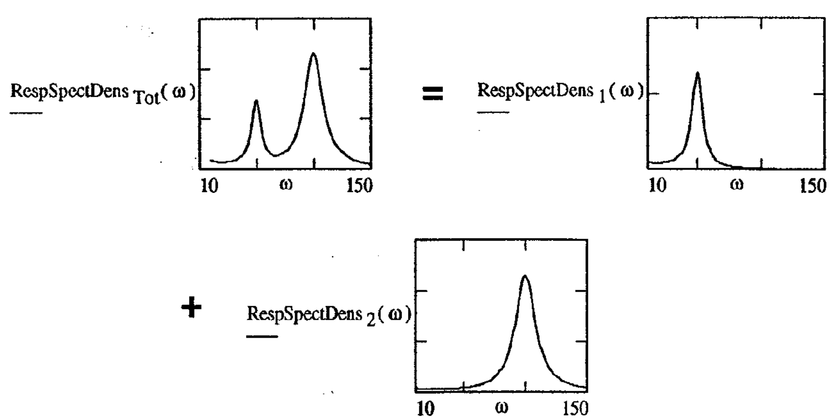


Figure 3
Response contributions from each mode are combined so that the response spectral densities from each mode add together.

Therefore, we can take the RMS of the sum of the individual modes' response spectral densities to be the RMS response due to the behavior of all modes considered. However, we do not need to actually sum the response spectral densities from each mode to compute the RMS response due to the behavior of all modes. Instead we can simply RSS the RMS contributions to the response from each mode. Since we have added the individual response spectral densities on the RHS of the pictorial equation in Figure 3, we can write

$$\text{Area}_{\text{Total}} = \text{Area}_1 + \text{Area}_2 \tag{10}$$

$$\left(\sqrt{\text{Area}_{\text{Total}}}\right)^2 = \left(\sqrt{\text{Area}_1}\right)^2 + \left(\sqrt{\text{Area}_2}\right)^2 \tag{11}$$

$$\text{RMS}_{\text{Total}}^{\;2} = \text{RMS}_1^{\;2} + \text{RMS}_2^{\;2} \tag{12}$$

$$\text{RMS}_{\text{Total}} = \sqrt{\text{RMS}_1^{\;2} + \text{RMS}_2^{\;2}} \tag{13}$$

Notice that equation (13) states that the total RMS due to the behavior of all the modes is the RSS of the individual RMS responses from each mode.

Method Application

The first step in running this analysis is to compute the normal and associated area for each grid in the model exposed to the acoustic environment. This is done by applying in SOL 101 a static uniform 1.0 psi pressure load (PLOAD4 or PLOAD1) to all elements which are exposed to the acoustic environment. The OLOAD(PUNCH) = ALL output request is included to output the effective force vectors at each grid which are expressed in the grids output coordinate system. These force vectors have a direction parallel to the grid normal and have a magnitude equal to the effective grid area.

The next step is to compute $[K]\{\Phi\}$ for each mode. Therefore, a SOL 103 natural frequency analysis is performed with the following DMAP alter for Version 68:

```
COMPILE SEDRCVR SOUIN=MSCSOU,NOLIST,NOREF
ALTER 1
TYPE DB KJJ
$ Delete print/punch or real eigenvectors.
ALTER 209,209
$Compute & print/punch [K]{Φ}.
```

```
ALTER 209
MPYAD KJJ,UG,/KPHI/
SDR2  CASEDR,CSTMS,,,EQEXINS,,,OL2,BGPDTS,,,
       KPHI,,,OINT,PELSETS,VIEWTB/
       ,,OKPHI,,,/APP1////////////FALSE
OFP  OKPHI,,,,,,
       CSTMS,EPTS,GEOM1VU,ERROR1//
       S,N,CARDNO
$Send [K]{Φ} to XL.
EQUIVX OKPHI/OUGV1/ALWAYS
```

If the natural frequency analysis is expensive, the databases can be saved for restart in the following step.

The third step is to punch the natural frequency eigenvectors and other results which will be of interest (ie. stresses, forces etc.) with a SOL 103 restart of the previous run. Notice that the alter is to be removed for this run.

After these three MSC/NASTRAN runs have been performed, a FORTRAN code can be used to read the OLOAD, $[K]\{\Phi\}$, eigenvalues and eigenvectors from the three punch files. With this information equations (3) and (4) can be evaluated to give the worst case load $\{P\}$ for each mode. The same FORTRAN code can then write a MSC/NASTRAN data deck containing as many one degree of freedom decoupled spring-mass systems as there are modes in the analysis. The spring stiffness of each system is set to be the generalized stiffness, ω_n^2, the mass of each system is set to be the generalized mass, 1.0, and the harmonic load on each grid is set to be the corresponding generalized force, $\{\Phi\}^T\{P\}$. A sample data deck of the generalized dynamics problem for a system whose first three natural frequencies are 10.0 Hz, 20.0 Hz, and 30.0 Hz might look like the following:

```
$ Generalized Dynamics Problem
$ Mode 1
GRID     1       0         0.0    1.0    0.0    0
CELAS2   1       3947.84   1      1
CONM2    1001    1         0      1.0
DAREA    1       1         1      p1
$ Mode 2
GRID     2       0         0.0    2.0    0.0    0
CELAS2   2       7895.68   2      1
CONM2    1002    2         0      1.0
DAREA    1       2         1      p2
$ Mode 3
GRID     3       0         0.0    3.0    0.0    0
```

CELAS2	3	11843.53	3	1
CONM2	1003	3	0	1.0
DAREA	1	3	1	p_3

This extremely inexpensive model can be run in SOL 111, modal forced response, with the DAREA cards referenced by an RLOAD2 card to evaluate equation (8). [3] The TABDMP card referenced by SDAMP in the case control can be used to input modal damping. The FREQ and FREQ1 cards should both be used to specify the natural frequencies and a range of frequencies over which the transfer functions should be computed. If many modes are being included in the analysis, the FREQ card can be written by the same program which writes the generalized dynamics data deck. The displacement of each grid represents the transfer function for the mode which that grid's spring-mass system represents. In addition, MSC/NASTRAN's random response capability can be used to compute the transfer function magnitude spectral densities (as in equation (9)) and evaluate their RMS's [4]. All displacements should be requested in the case control section with DISP(SORT2,PHASE) = ALL and XYPEAK DISP PSDF / [grid number](T1RM) can be used in an OUTPUT(XYOUT) section to get the RMS's of the spectral densities of the transfer function magnitudes.

With the RMS's of the spectral densities of the transfer function magnitudes, a FORTRAN code can be used to scale and RSS the results from the natural frequency run.

Assumptions and Implications

If spatially dependent functions of acoustic pressure are not used and worst case load profiles are assumed, an extreme amount of conservatism can be included in the analysis. Such an assumption introduces conservatism in two ways. Most importantly, since often times 50-100 or more modes must be included in a vibroacoustic analysis, it is highly unlikely that the spatial correlation of the acoustic pressure field corresponds to the dynamic behavior of the structure for this many modes at the same time. This is probably the most conservative aspect of this analysis. In addition, further conservatism is included if the structure possesses close natural frequencies. Natural frequencies which are close enough so that their transfer functions overlap near the peaks will each contribute response due to the acoustic input in the frequency range of the overlap. Because these results are superimposed, this assumes a doubling of the pressure spectral density input in this overlap region.

Because only a limited number of modes can be included in this analysis, the user should perform a convergence study to determine if enough modes have been included.

As a final note, the RMS response obtained after RSSing the RMS contributions from each mode for that response is a one sigma value. This means that it represents the peak ~64% of the time assuming that the behavior of the structure is of Rayleigh distribution in time. A higher sigma quantity may be obtained by scaling the one sigma value.

Acknowledgements

Dr. Victor Genberg
Senior Project Engineer - Eastman Kodak Company
Writing DMAP alter and assisting in developing analysis methodology.

References

[1] "Design and Verification Guidelines For Vibroacoustic and Transient Environments," National Aeronautics and Space Administration, Component Analysis Branch, TM-86538.

[2] Hejal, R., "AXAF-I Vibroacoustic Loads -- Methodology," TRW Space & Electronics Group, M533.2.94-137.

[3] Blakely, Ken, "MSC/NASTRAN Basic Dynamic Analysis: User's Guide," MacNeal-Schwendler Corporation.

[4] Robert S. Lahey et al., "MSC/NASTRAN Reference Manual," Section 13.3, MacNeal-Schwendler Corporation.