# Dynamic Analysis by the Fourier Transform Method with MSC/NASTRAN

by

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*Abstract*

This paper briefly describes the Fourier transform capability using MSC/NASTRAN, demonstrates its application to several examples without including any aerodynamic effects and provides an explanation for general application within MSC/NASTRAN.

The Fourier transform capability in MSC/NASTRAN allows transient response analysis to be performed with a frequency response solution. Time dependent loads are transformed into the frequency domain and all frequency dependent calculations are performed with the modal frequency response modules. The frequency response results are then transformed back into the time domain with an inverse Fourier transform module as an integral part of the solution sequence.

Fourier transform methods have been implemented in MSC/NASTRAN to solve the equations of motion for the aeroelastic response of fixed wing aircraft. This capability is vital to this analysis technique because the unsteady aerodynamic matrices are calculated in the frequency domain. The Fourier transform method is easily accessible with SOL 146 and the aerodynamic input becomes unnecessary when a DMAP alter avoids the aeroelastic coupling. The forward Fourier transform is also included in the frequency response analysis solution sequences, e.g., SOL 108. This provides a method for calculation of Fourier series coefficients from a function of time as specified on any combination of TLOAD1 and/or TLOAD2 bulk data entries.

# Introduction

When a capability is implemented in MSC/NASTRAN to meet a specific client need, the capability often contains a methodology feature that is applicable to other analysis disciplines. One example is the aeroelastic analysis capability which is used by a small group of clients and may be "hidden" (publicized only in aeroelastic documents) from other user groups. One "hidden" feature of MSC/NASTRAN is the Fourier transform method which is commonly used to perform aircraft aeroelastic response analysis. The Fourier transform method has been a part of the aeroelastic solution since the mid 1970s, then it was SOL 46 and now it is SOL 146. The aeroelastic response solution allows simulation of fixed wing aircraft response to discrete, harmonic or random gust excitation. If one ignores the aerodynamic considerations and proceeds to this little explored capability, an interesting and beneficial analysis tool can be easily accessed to assist in determining the dynamic behavior of a structure. The Fourier transform method provides an alternative to determining a steady state transient solution to the equations of motion. This technique avoids the lengthy integration times of the equations of motion and repetitive input of a periodic load history. Furthermore, most people familiar with MSC/NASTRAN are unaware that a forward Fourier transform can be obtained from any frequency response solution sequence (SOL 108, 111, etc.) by simply specifying a time dependent loading function. The Fourier transform method may be used as a step in the calculation of the Fourier series coefficients which are used to calculate input power spectral density functions for random analysis calculations. A basic understanding of Fourier series and integrals is required to use this methodology in MSC/NASTRAN.

This paper presents background information, describes guidelines and outlines examples for the Fourier transform usage, input power spectral density function generation and steady state transient response analysis. The background information section provides basic information related to and references for Fourier series and integrals. The guidelines section of the paper outlines items for preparing input files and topics related to results evaluation. The examples section of the paper examines four cases which aid in the understanding and usage of the Fourier transform method. Case 1 calculates the Fourier series coefficients for a combination of trigonometric functions. Case 2 uses several basic series to examine the behavior of the method. Case 3 computes the Fourier series coefficients of an earthquake acceleration history with a direct frequency response solution. An input power spectral density function derives from a relationship to the Fourier transform. Case 4 examines a steady state transient response from SOL 146 with a DMAP alter for comparison with a direct transient analysis, SOL 109, after it achieves steady state motion. This paper is completed with conclusions, references and appendices.

# Background Information

A complete theoretical treatment of Fourier series and transforms is beyond the scope of this paper and is unnecessary since these topics are treated in current literature and text books. For example, Reference 1 describes the transformation of time dependent loads supplied in the input file to MSC/NASTRAN. The equations presented in Section 2.7 of Reference 1 provide an exact calculation of the Fourier coefficients which represent piecewise linear tabular functions (TLOAD1) and the general purpose function (TLOAD2). Additional and analogous information is presented in References 2–5. Reference 5 details the initial design of the Fourier transform method implemented in NASTRAN and discusses the Fourier series and integrals for the implementation. Even though the capability has been available for a quarter of a century it is primarily relegated to aeroelastic analysis, and few users attempt to use the Fourier transform methods of MSC/NASTRAN just for dynamic analysis. The author hopes that this paper encourages the dynamic user community to explore this alternative solution strategy. This alternative provides a method for using time dependent information in a frequency dependent solution to gain insight to structural dynamic characteristics.

Application of the Fourier transform method is not limited to constant (equal) frequency spacing; however, it is easier to determine the interval of the periodicity with constant spacing. Constant frequency spacing is used in all the results calculated for this paper. Until the interval of the period for the loading function is known and values for the Fourier series coefficient are determined the user is urged to maintain constant frequency spacing. Reference 6 describes a method that uses the features of SOL 76 while avoiding the aerodynamic modules with a DMAP alter to calculate a steady state transient response. Reference 6 also shows a comparison with a direct transient solution and describes techniques for SOL 76 to obtain the same result as a transient analysis solution. References 7 and 8 provide the mathematical substance of the Fourier transform (series and integrals) theory. The previously mentioned references provide information that will assist in the understanding and usage of the Fourier transform method while performing a dynamic analysis with MSC/NASTRAN.

An assumption of the paper is that the reader is familiar with the dynamic solution sequences of MSC/NASTRAN. Reference 9 provides the reader with details of the programs basic dynamic capabilities.

# Guidelines

## Considerations

Two solution strategies offer the Fourier transform methodology in MSC/NASTRAN. All of the frequency response solution sequences use the FRLG module to calculate the loading function at specified frequency values. An assumption of the paper is that the reader is familiar with the dynamic solutions of MSC/NASTRAN. Internal to the FRLG module, is an algorithm to evaluate the Fourier integrals that convert the time dependent functions to frequency dependent functions. The output of the FRLG module, when a time dependent load (TLOAD1 and/or TLOAD2) is input, represent the Fourier series coefficients. To properly setup the time dependent input, the following information should be considered:

a) the interval of the periodic function, T
b) the function with respect to time, f(t)
c) use constant frequency spacing, $\Delta f$, set so that $\Delta f = 1/T$
d) do **not** mix loading functions types of RLOAD1 or RLOAD2 with TLOAD1 or TLOAD2
e) with modal methods at least three degrees of freedom should be used, eigenvalue input files of fewer degrees of freedom are degenerate problems and require special handling.

## Frequency Response Analysis, SOL 108 or 111

Frequency response analysis solutions use frequency as an independent variable in the system of equations. The transformation to the frequency domain is made by assuming a steady state harmonic motion of the system. For more details of frequency response analysis the reader can refer to Chapter 5 of Reference 9.

Frequency response analysis solutions use the following list of case control commands and bulk data entries. These input items are included in an input file to perform a structural dynamic analysis with SOL 108 or 111 while applying a time dependent loading function with a TLOAD1 or TLOAD2. Other input items, e.g., virtual mass or acoustics, may be included with a dynamic solution; however, the items in the following list are generally needed to perform a frequency response analysis solution.

| Case Control | Bulk Data | |
|---|---|---|
| METHOD | EIGRL or EIGR | (optional) |
| FREQ | FREQ1 | (required) |
| DLOAD | DLOAD | (optional) |
| | TLOAD1, TABLED1 or TLOAD2 | (required) |
| | DAREA | (required) |
| | DELAY | (optional) |

### Steady State Transient Response Analysis, SOL 146

The Aeroelastic Response solution, SOL 146 or SOL 76, provides a methodology that is not available in the other solution sequences of MSC/NASTRAN. The inverse Fourier transform does not calculate the constant term in the series and arbitrarily sets it equal to zero in the IFT module.

Steady state transient response analysis solutions use the following list of case control commands and bulk data entries for processing a solution of a common structural dynamic analysis. Other input items, e.g., virtual mass or acoustics, may be included with a dynamic solution; however, the items in the following list are generally needed to perform a steady state transient response solution. SOL 146 requires both a FREQ and TSTEP case control c

| Case Control | Bulk Data | |
|---|---|---|
| METHOD | EIGRL or EIGR | (required) |
| FREQ | FREQ1 | (required) |
| TSTEP | TSTEP | (required) |
| DLOAD | DLOAD | (optional) |
| | TLOAD1, TABLED1 or TLOAD2 | (required) |
| | DAREA | (required) |
| | DELAY | (optional) |

## Examples

Anyone planning to use the Fourier transform methods in MSC/NASTRAN is urged to perform exercises to supplement their understanding of the method. The exercises should involve familiar functions that have known results or a alternative solution methodology. As a validation of the Fourier transform method, four cases illustrate a method for validating the accuracy of Fourier transform processing or produce acceptable results.

### Case 1: Four Trigonometric Sine Functions and Frequency Response, SOL 108

An input file with three loading conditions shown in Listing 1 illustrates a frequency response solution simulation that demonstrates load input processing and combinations. Subcase 2000 uses a combination of four TLOAD2 bulk data entries to provide a sine function excitation with four frequencies requested on the selected FREQ1 entry. The four TLOAD2 entries specify that constant amplitude sine function excitation is to be applied with frequencies of 1.0, 1.5, 2.0 and 2.5 Hz during a time interval of 2.0 seconds. Subcase 2001 illustrates the conventional frequency response RLOAD2 form of load specification. The DPHASE entry is used to obtain a sine function excitation consistent with the TLOAD2 input of Subcase 2000. To obtain the same amplitude of loading with the TLOAD2, load set 2000, as the RLOAD2, load set 2001, the value of the A field of the DAREA entry must be specified to be equal to two divided by the interval of periodicity. In this instance, the value of A is unity since the interval of the period is 2.0 units. All four frequency values, field 8 of each TLOAD2, is periodic within this interval. If other frequencies are to be included in the loading function, ensure that the A field of the DAREA entry, the DF field of the FREQ1 entry and the T2 field have compatible values to maintain a consistent periodicity. The DF field of the DAREA entry determines the interval of periodicity, e.g., the interval is 1/DF, and is based on the DF value irrespective of the loading function specification. The response and corresponding output load vectors of subcases 2000 and 2001 in Listing 2 show the same values to seven significant digits except for the computed zero values. In subcase 2002, the TLOAD2 and RLOAD2 are combined with a DLOAD entry to obtain output results that are the sum of subcases 2000 and 2001 for displacements and load vector. This was done to illustrate that the two types of loads can be used together; however, care must be exercised to obtain the proper load vector. In Listing 2, the complex load vector output shows computed zero values for the real part and a unit negative value for the imaginary part. The load vector may be represented in the time domain as given by equation (1).

# Listing 1. Four Combined Sine Functions Input File

```
ID TLOAD2 EXAMPLE $ 1995 WUC
TIME   10
SOL   108 $ Direct Frequency Response
CEND
TITLE = FREQUENCY RESPONSE ANALYSIS WITH TRANSIENT LOAD INPUT
SUBTI = TLOAD2 - FOUR FREQUENCY INPUT SINE LOAD FUNCTION.
LABEL = 1 DOF SYSTEM - OLOAD OUTPUT IS FOURIER TRANFORM OF TLOAD
  OLOAD = ALL
  DISP(PHASE,SORT2) = ALL
  ACCELERATION = ALL
  FREQ  = 2100
 SUBCASE 1
  DLOAD = 2000
 SUBCASE 2
  DLOAD = 2001
 SUBCASE 3
  DLOAD = 2002
BEGIN BULK
CELAS2    101       1.        100       0
CMASS2    102       1.        100       0
CDAMP2    103       0.02      100       0
$
$       DATA INPUT FOR A FOURIER TRANSFORM
$         OF A TIME DEPENDENT FUNCTION SPECIFIED ON A TLOAD2
$
$ The interval of period of a function is T and is related to the
$ frequency increment delta-f, DF, of the FREQi entry.
$
$ Set the A  field on the DAREA  entry to 2*DF
$ Set the DF field on the FREQ1  entry to 1/T
$ Set the T2 field on the TLOAD2 entry to T
$
$  Use an overall period of 2.0
$
$FREQ1    SID       F1        DF        NF
$TLOAD2   SID       DAREAID   DELAY     TYPE      T1        T2        F         P
$DAREA    DAREAID   GID       CID       A
FREQ1     2100      1.0       0.5       3
$ TIME DEPENDENT LOADING
$ F=1.0, T=1.0
TLOAD2    2011      2011                0         0.        2.0       1.        -90.
DAREA     2011      100       0         1.0
$ F=1.5, T=2/3 -> OLOAD=(0.0,-1.0) at F
TLOAD2    2021      2021                0         0.        2.0       1.5       -90.
DAREA     2021      100       0         1.0
$ F=2.0, T=0.5
TLOAD2    2031      2031                0         0.        2.0       2.        -90.
DAREA     2031      100       0         1.0
$ F=2.5, T=0.4
TLOAD2    2041      2041                0         0.        2.0       2.5       -90.
DAREA     2041      100       0         1.0
DLOAD     2000      1.0       1.0       2011      1.0       2021      1.0       2031
          1.0       2041
$ FREQUENCY DEPENDENT LOADING
$         P(F) = A * B(F) * EXP[I(phi(f)+theta-2*pi*f*tau)]
$RLOAD2   SID       DAREA     DELAY     DPHASE    TB        TP
RLOAD2    2001      2001                2001      2001
$DAREA    SID       P1        C1        A1        etc
DAREA     2001      100       0         1.
$DPHASE   SID       P1        C1        TH1
DPHASE    2001      100       0         -90.
$TABLED1 TID
$         X1        Y1        X2        Y2        etc.      ENDT
TABLED1   2001
          0.0       1.0       100.0     1.0       ENDT
$ COMBINED LOADING
DLOAD     2002      1.0       1.0       2011      1.0       2021      1.0       2031
          1.0       2041      1.0       2001
ENDDATA
```

# Listing 2. Results of Four Combined Sine Function Input File

```
   POINT-ID =      100                                                                                SUBCASE 1
                                           C O M P L E X   D I S P L A C E M E N T   V E C T O R
                                                         (MAGNITUDE/PHASE)
    FREQUENCY   TYPE        T1            T2            T3            R1            R2            R3
 1.000000E+00     S      2.598845E-02
                          90.1871
 1.500000E+00     S      1.138607E-02
                          90.1229
 2.000000E+00     S      6.372923E-03
                          90.0917
 2.500000E+00     S      4.069336E-03
                          90.0732

   POINT-ID =      100                                                                                SUBCASE 2
                                           C O M P L E X   D I S P L A C E M E N T   V E C T O R
                                                         (MAGNITUDE/PHASE)
    FREQUENCY   TYPE        T1            T2            T3            R1            R2            R3
 1.000000E+00     S      2.598845E-02
                          90.1871
 1.500000E+00     S      1.138607E-02
                          90.1230
 2.000000E+00     S      6.372923E-03
                          90.0918
 2.500000E+00     S      4.069336E-03
                          90.0732

   POINT-ID =      100                                                                                SUBCASE 3
                                           C O M P L E X   D I S P L A C E M E N T   V E C T O R
                                                         (MAGNITUDE/PHASE)
    FREQUENCY   TYPE        T1            T2            T3            R1            R2            R3
 1.000000E+00     S      5.197691E-02
                          90.1871
 1.500000E+00     S      2.277213E-02
                          90.1229
 2.000000E+00     S      1.274585E-02
                          90.0917
 2.500000E+00     S      8.138672E-03
                          90.0732

   POINT-ID =      100                                                                                SUBCASE 1
                                                C O M P L E X   L O A D   V E C T O R
                                                         (REAL/IMAGINARY)
    FREQUENCY   TYPE        T1            T2            T3            R1            R2            R3
 1.000000E+00     S      3.139167E-07
                        -9.999999E-01
 1.500000E+00     S     -6.397579E-07
                        -9.999999E-01
 2.000000E+00     S     -6.397580E-07
                        -1.000000E+00
 2.500000E+00     S     -6.397577E-07
                        -9.999999E-01

   POINT-ID =      100                                                                                SUBCASE 2
                                                C O M P L E X   L O A D   V E C T O R
                                                         (REAL/IMAGINARY)
    FREQUENCY   TYPE        T1            T2            T3            R1            R2            R3
 1.000000E+00     S     -4.371139E-08
                        -1.000000E+00
 1.500000E+00     S     -4.371139E-08
                        -1.000000E+00
 2.000000E+00     S     -4.371139E-08
                        -1.000000E+00
 2.500000E+00     S     -4.371139E-08
                        -1.000000E+00

  POINT-ID =      100                                                                                SUBCASE 3
                                                C O M P L E X   L O A D   V E C T O R
                                                         (REAL/IMAGINARY)
    FREQUENCY   TYPE        T1            T2            T3            R1            R2            R3
 1.000000E+00     S      2.702053E-07
                        -2.000000E+00
 1.500000E+00     S     -6.834693E-07
                        -2.000000E+00
 2.000000E+00     S     -6.834694E-07
                        -2.000000E+00
 2.500000E+00     S     -6.834691E-07
                        -2.000000E+00
```

$$\{P(t)\} = \{\overline{A}(\omega)\}\, e^{i\omega t} \tag{1}$$

$$= \{Re(\overline{A})\} \cos(\omega t) + i \{Im(\overline{A})\} \sin(\omega t)$$

Figure 1 provides a graphical illustration of the loading function generated by the four sine function combination with the DLOAD 2000 in Listing 1. Since the complex loading function is expected to be periodic over the interval of 2.0 seconds, two interval periods were plotted to confirm the actual periodicity.

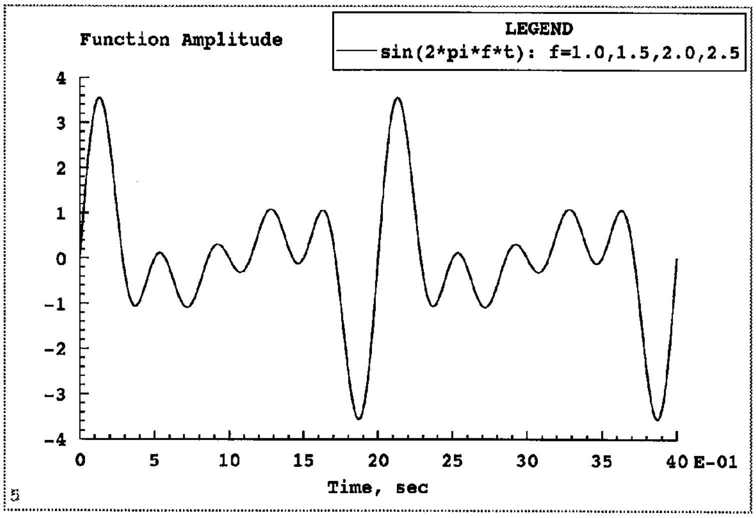


Figure 1. Four Combined Sine Function Versus Time

### Case 2: Three Fourier Series Examples, SOL 108

Three functions (a basic Fourier series, a Fourier cosine series and a Fourier sine series) demonstrate the calculation of Fourier series coefficients with a frequency response solution, SOL 108. Details of Fourier series are presented in Chapter VI of Reference 7. The calculated coefficients from analytical expressions of Reference 7 are compared to the MSC/NASTRAN output results. The analytical expressions for the Fourier series coefficients are given in equation 2. Listing 3 displays the input file used to calculate the results shown in Listing 4. The three funcitons are input with TABLED1 bulk data entries which are selected by TLOAD1 entries. The data comparison of Listing 4 validates the accuracy of the coefficients calculated with SOL 108. Note that the coefficients are accurate to all digits shown. Figure 2 provides a graphical representation of the three series, and the results were obtained by using SOL 146 to perform an inverse Fourier transform. This allows the function to be plotted with respect to time. Note that Figures 2(b) and 2(d) show a value of zero for the constant term

$$b_n = -\frac{1}{n\pi}$$

$$a_n = \frac{2}{n^2\pi^2}\left[ (-1)^n - 1 \right] \tag{2}$$

$$b_n = \frac{1}{n\pi}(-1)^{n+1}$$

## Listing 3. Three Fourier Series Input File

```
ID BASIC SERIES $ WUC 1995
TIME   10
SOL   108 $ Direct Frequency Response
CEND
TITLE = FOURIER SERIES COEFFICIENTS FOR THREE BASIC FUNCTIONS
SUBTI = PIECEWISE LINEAR TLOAD1 - REF. 3 SECTION 6.5
LABEL =
  OLOAD = ALL
  DISP(PHASE,SORT2) = ALL
  ACCELERATION = ALL
  FREQ  = 2100
SUBCASE 2001
  DLOAD = 2001
SUBCASE 2002
  DLOAD = 2002
SUBCASE 2003
  DLOAD = 2003
BEGIN BULK
CELAS2  101     1.      100     0
CMASS2  102     1.      100     0
CDAMP2  103     0.02    100     0
$
$       DATA INPUT FOR FREQUENCY ANALYSIS USING FOURIER TRANFORM
$
$ The period of a function is T which is related to the
$ frequency increment delta-f, DF, on the FREQi entry.
$
$ Set the A  field on the DAREA  entry to 2*DF
$ Set the DF field on thc FREQ1  entry to 1/T
$ Set the T2 field on the TLOAD2 entry to T
$
$  Use an overall period of 1.0
$
$FREQ1  SID     F1      DF      NF
FREQ1   2100    0.0     0.5     100
$DAREA  DAREAID GID     CID     A
DAREA   2011    100     0       1.0
$TLOAD1 SID     DAREAID DELAY   TYPE    TID
$ Fourier coefficient value: b-sub-n=-(1/n*pi)
TLOAD1  2001    2011            0       1001
TABLED1 1001
        0.0     0.0     1.0     1.0     1.0     0.0     2.0     1.0
        ENDT
$ Fourier coefficient value: a-sub-n=2*[(-1)^n-1]/(n^2*pi^2)
TLOAD1  2002    2011            0       1002
TABLED1 1002
        0.0     0.0     1.0     1.0     2.0     0.0     ENDT
$ Fourier coefficient value: b-sub-n=2*((-1)^(n+1))/(n*pi)
TLOAD1  2003    2011            0       1003
TABLED1 1003
        0.0     0.0     1.0     1.0     1.0     -1.0    2.0     0.0
        ENDT
ENDDATA
```

of the Fourier series. Shifting both figures by adding 0.5 to the ordinate values yields the same functions as shown in Figures 2(a) and 2(c) except for the influence of the Gibbs phenomenon.

170

# Listing 4. Output Results of Fourier Series Examples

```
  POINT-ID =    100                                                     SUBCASE 2001
                               C O M P L E X   L O A D   V E C T O R
                                       (REAL/IMAGINARY)

  FREQUENCY  TYPE        T1         Theory

5.000000E-01   S     2.980232E-08    ---
                     0.0
1.000000E+00   S     8.348260E-08  0.0
                     3.183099E-01 -3.18310E-01    The calculation of the Fourier series terms
1.500000E+00   S     3.725290E-09    ---          from the column T1 data can be made as follows:
                     0.0
2.000000E+00   S     8.348260E-08  0.0            Re[P(f)] * cos( f*t )  -  Im[P(f)] * sin( f*t )
                     1.591549E-01 -1.59155E-01
2.500000E+00   S     4.284084E-08    ---          The Fourier series is made by summing the cosine and
                     0.0                          sine functions.
3.000000E+00   S     7.591614E-09  0.0
                     1.061033E-01 -1.06103E-01    Note: In this example the ramp is input twice so the
3.500000E+00   S     6.007031E-08    ---                the interval of the period is compatible with the
                    -7.450581E-09                       next two functions.
4.000000E+00   S     8.348260E-08  0.0
                     7.957747E-02 -7.95775E-02
4.500000E+00   S     2.561137E-09    ---
                     0.0
5.000000E+00   S     1.290172E-07  0.0
                     6.366197E-02 -6.36620E-02

  POINT-ID =    100                                                     SUBCASE 2002
                               C O M P L E X   L O A D   V E C T O R
                                       (REAL/IMAGINARY)

  FREQUENCY  TYPE        T1         Theory

5.000000E-01   S    -4.052847E-01 -4.05285E-01
                     5.960464E-08  0.0
1.000000E+00   S     0.0           0.0
                     0.0           0.0
1.500000E+00   S    -4.503164E-02 -4.50316E-02
                     0.0           0.0
2.000000E+00   S     0.0           0.0
                     0.0           0.0
2.500000E+00   S    -1.621139E-02 -1.62114E-02
                     1.490116E-08  0.0
3.000000E+00   S     0.0           0.0
                     0.0           0.0
3.500000E+00   S    -8.271116E-03 -8.27112E-03
                     7.450581E-09  0.0
4.000000E+00   S     0.0           0.0
                     0.0           0.0
4.500000E+00   S    -5.003515E-03 -5.00352E-03
                     0.0           0.0
5.000000E+00   S     0.0           0.0
                     0.0           0.0

  POINT-ID =    100                                                     SUBCASE 2003
                               C O M P L E X   L O A D   V E C T O R
                                       (REAL/IMAGINARY)

  FREQUENCY  TYPE        T1        Theory

5.000000E-01   S    -5.960464E-08  0.0
                    -6.366197E-01  6.36620E-01
1.000000E+00   S     5.565506E-08  0.0
                     3.183099E-01 -3.18310E-01
1.500000E+00   S    -3.725290E-09  0.0
                    -2.122066E-01  2.12207E-01
2.000000E+00   S     5.565506E-08  0.0
                     1.591549E-01 -1.59155E-01
2.500000E+00   S    -8.568168E-08  0.0
                    -1.273240E-01  1.27324E-01
```

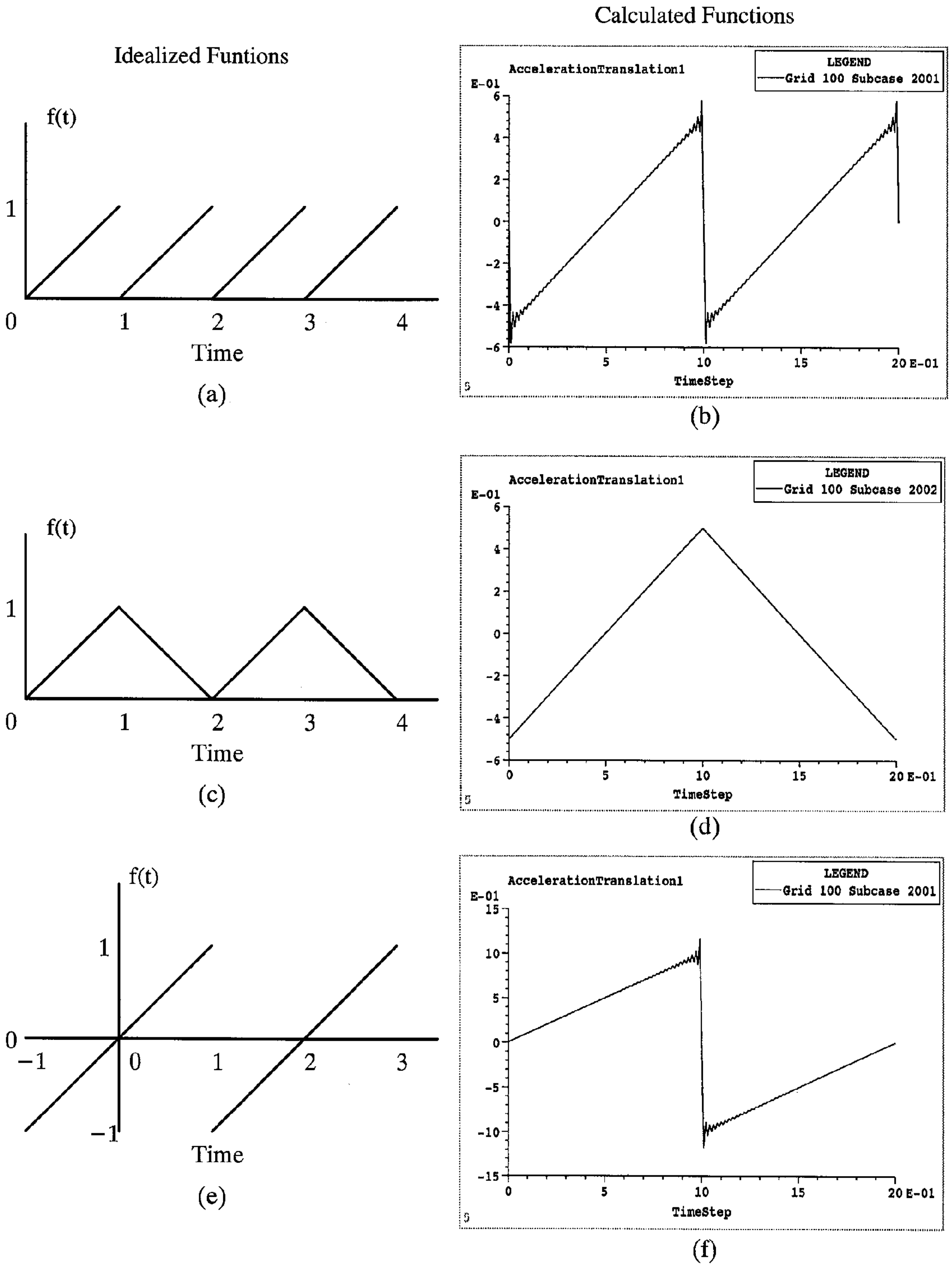


Figure 2. Fourier Series Examples

## Case 3, Earthquake Example SOL 111

In this example, a simple two–dimensional frame structure with 18 bays is subjected to an earthquake base excitation. The model has two masses placed on the roof of the structure to represent elevator and air–conditioning equipment. Transient response results are compared to random response results. The transient data is used to create the input power spectral density function of the random response analysis. Clearly, this example is simplified and presented here to illustrate a processing technique.

A two–dimensional frame structure, Figure 3, is used in this case to illustrate the transformation of a time dependent function to the frequency domain for application in a random response calculation. The input file is provided in Listing 5 and it is used to calculate and plot the data shown in Figure 4(a) 4(b). Listings 6 through 8 represent the remainder of the input data file of Listing 5. These files are brought into the input stream with the "include" feature of MSC/NASTRAN. The time dependent data for this case uses the north–south component of acceleration of the El Centro earthquake which occurred on May 18, 1940. The acceleration history was obtained from MSC/pal2 and reformatted to be compatible with the TABLED1 bulk data format. Figure 4 shows a graphical representation of the acceleration history as well as the input power spectral density which is determined from the modified Fourier transform of the acceleration. The results shown graphically in Figure 4(c) are determined by performing a detailed Fourier transform of the transient input acceleration shown in Figure 4(a) and using relationships of Reference 9 to calculate the power spectral density function. The tabular data is too lengthy to be presented here and an abbreviated version of the data is provided in Listing 8. The power spectral density function of the stress response for element 3001 connecting grid points 1 and 14 is shown in Figure 4(d). A simplified version of the power spectral density function is provided in the xy–plot of Figure 4(e). The two xy–plots, Figures 4(a) and 4(b), show the enforced motion and response characteristics relative to time. The statistical quantities, RMS and $N_0$, from the random analysis used to create Figures 4(c) and 4(d) have been placed on the transient xy–plots for comparison. The RMS=0.09 and $N_0$=5.3 values from the simplified input are somewhat larger values

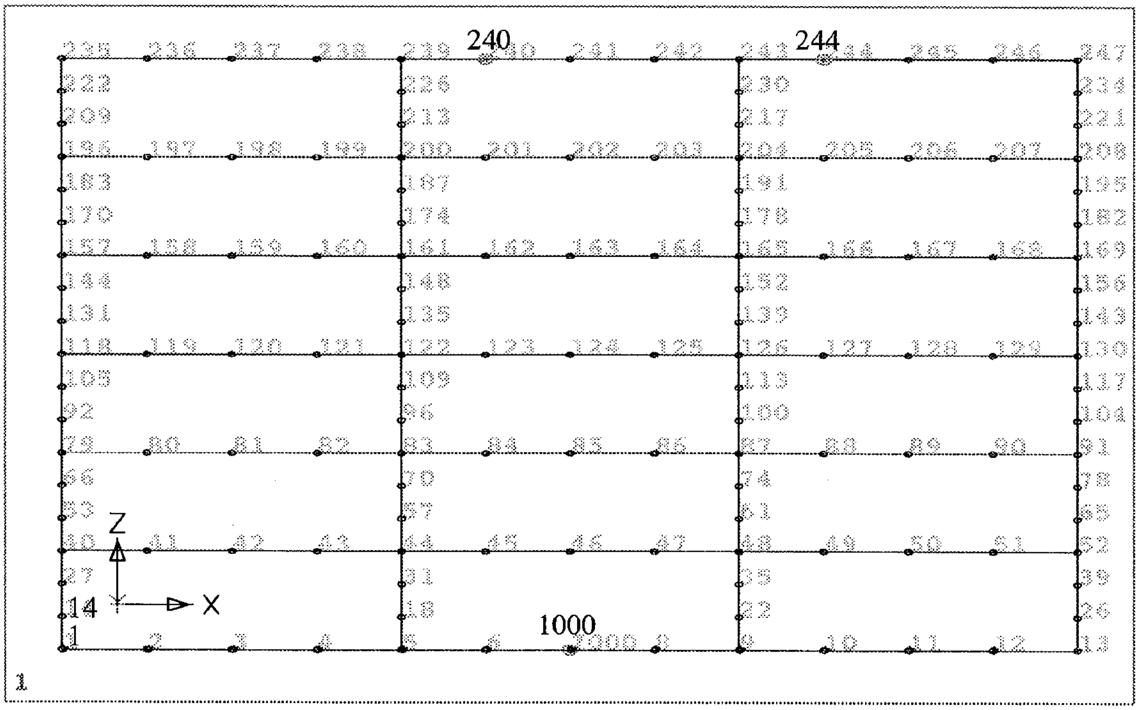


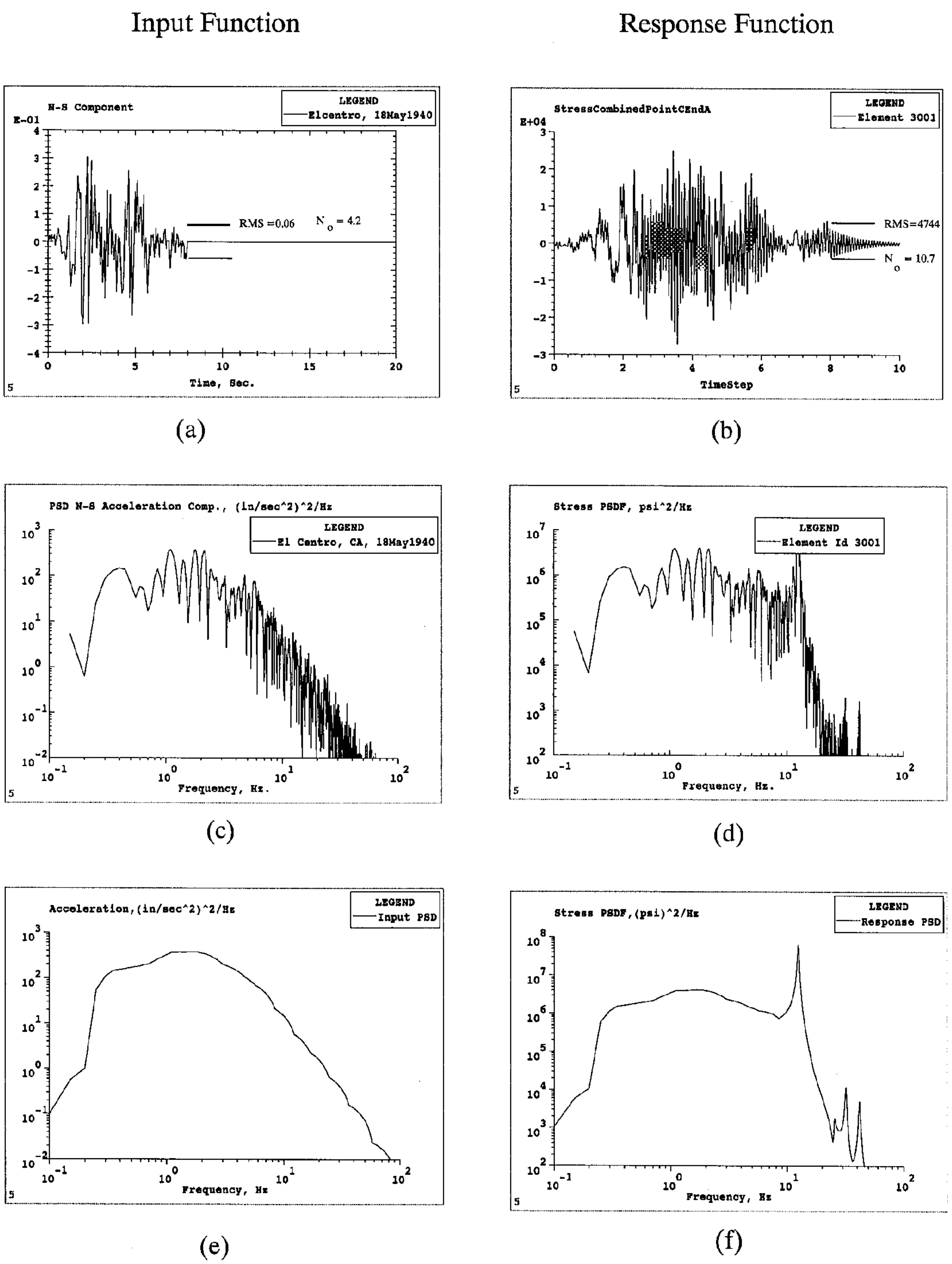
Figure 3. Two Dimensional Frame Strucute

## Listing 5. Earthquake Input File

```
NASTRAN PREFOPT=2
ID MSC, EDB $
TIME 100
SOL  111
CEND
TITLE= TWO Dimensional Beam Frame - 3 bays wide x 6 bays high
SUBTI= May 1940 El Centro Earthquake, N-S Component of Acceleration
LABEL= Modal Frequency Response and Random Analysis
$
ECHO            = NONE
SET 1 = 100,240,244
SET 2 = 1221,1222,1225,1226,3001,3002,3003,3004,3005,3006,3007
  METHOD       =    1
  SPC          =    1
  DLOAD        =  200
  FREQ         =  100
  DISP(PHASE,PLOT)   =  ALL
  ACCEL(PHASE,PLOT)  -    1
  STRESS(PHASE,PLOT) =    2
  RANDOM       =  300
include 'psdf.requests'
BEGIN BULK
MESHOPT,YES $ Let MESH the input bulk data
$           2           3           4           5           6           7           8           9
PARAM       AUTOSPC     YES
PARAM       GRDPNT      0
PARAM       POST        0
PARAM       W4          78.0
EIGRL       1           -0.1                    20
$ Integration Time Steps
FREQ1       100         0.15        0.05        2000
$ Applied Loads
RLOAD1      200         2011                    0           1000
DAREA       2011        1000        1           1.0+7
GRID        1000                    45.0        0.0         0.0                     23456
CONM2       100         1000                    1.+7
RBE2        101         1000        1           1           2           3           4           5
            6           7           8           9           10          11          12          13
TABLED1     1000
            0.0         1.0         100.0       1.0         ENDT
RANDPS      300         1           1           1.0                     2000
include 'elcentro.psdf'
$ boundary condtitions.
SPCG        1           1           23456       AB
SPCG        1           1           246         0100        1812
$ Model Generation
include 'frame.ipf'
ENDDATA
```

because the simplified input power spectral density represents an outer envelop of the detailed input power spectral density.

The calculation of the input power spectral density function is based on the Fourier transform of the time dependent data and the relationship presented in Chapter 7 of Reference 10. An analogous relationship is expressed in Section 2.8 of Reference 2. Application of the relationship between the Fourier transform and the power spectral density function allows the straightforward calculation of the power spectral density function. The calculations are made with the Unix tool, "awk," described in Reference 11. Appendix 1 lists the "awk" program used to calculate the input power spectral density function displayed in the xy–plot of Figure 4(c). The power spectral density function is calculated from a Fourier transform of the El Centro time dependent data. A second "awk" program, not presented in this paper, is used to format the input power spectral density function into the TABRND1 format. An explanation of the frequency response analysis in MSC/NASTRAN has been skipped because it is assumed that the reader is already familiar with the solution. Those readers requiring a review or quick refresher of the frequency response solution can find an excellent description in Chapter 5 of Reference 9.



(a) (b)

(c) (d)

(e) (f)

Figure 4. Transient and Random Response of Frame Structure

## Listing 6. frame.ipf Input File

```
$       2       3       4       5       6       7       8       9
EGRID   1               0.0     0.0     0.0
EGRID   2               90.0    0.0     0.0
EGRID   3               90.0    0.0     60.0
EGRID   4               0.0     0.0     60.0
GRIDG   1                       12      -1      -2      -3
        18      -4
GRIDU   1                               1       THRU    247
DELETE  1       0101    0203
=       =       *300    *300
=4
DELETE  1       0105    0207
=       =       *300    *300
=4
DELETE  1       0109    0211
=       =       *300    *300
=4
CGEN    BEAM    1001    1       1       L               1001    1228
                1.0
CGEN    BEAM    3001    2       1       M               3001    3234
                1.0
$ Properties
MAT1    1       30.+6           0.3     7.764-4                 0.03
$ nominal BEAM
PBEAM   1       1       28.22   206.8   1674.7          1881.5  5.0
        5.875   9.08    -5.875  9.08    -5.875  -9.08   5.875   -9.08
PBEAM   2       1       28.22   206.8   1674.7          1881.5  5.0
        5.875   9.08    -5.875  9.08    -5.875  -9.08   5.875   -9.08
$ AC Equipment
CONM2   6240    240             25.0
$ Elevator Equipment
CONM2   6244    244             210.0
```

## Listing 7. 'psdf.requests' Power Spectral Density Requests

```
OUTPUT(XYOUT)
  XAXIS=YES
  YAXIS=YES
  XLOG=YES
  YLOG=YES
  XGRID LINES=YES
  YGRID LINES=YES
  XYPRINT,XYPLOT DISP PSDF /240(T1RM)/244(T1RM)
  XYPRINT,XYPLOT ACCE PSDF /240(T1RM)/244(T1RM)/1000(T1RM)
  XYPRINT,XYPLOT STRESS PSDF /1221(4)/1222(4)/1225(4)/1226(4)/
                              3001(4)/3002(4)/3003(4)/3004(4)/
                              3005(4)/3006(4)/3007(4)
```

## Listing 8. Earthquake Simplified Input Power Spectral Density Function

```
TABRND1 2000
        0.1     0.1     0.2     1.0     0.33    140.0   0.7     200.0
        1.1     360.    1.8     370.0   2.2     330.0   3.0     200.0
        4.7     102.0   5.7     70.0    8.4     21.0    12.3    5.7
        16.8    2.3     24.3    0.66    36.2    0.16    57.4    0.024
        100.0   0.0001  ENDT
```

## Case 4, Vehicle Excited by a Roadbed, SOL 146

Each of us experiences the effects represented by case 4 in one form or another as we travel in our automobiles on our highways. The roadbed displacement history is not associated with any particular highway, and it is created to demonstrate the process of performing the analysis of a vehicle traveling over a roadbed. The roadbed and vehicle response characteristics can be analytically investigated to increase our understanding of the vehicle response and improve the passenger ride quality. The model of this investigation derives from Problem 14 of the MSC/NASTRAN Dynamics Seminar Notes. A listing of the input file is provided in Listing 9; however, notice the function of the enforced motion has been modified from the original in the seminar notes and the NOLINi bulk data entries are removed.

The input file, Listing 9, contains several items that need explanation or are worthy of emphasis. The aeroelastic response solution sequence, SOL 146, is modified with a DMAP alter as listed in Appendix 2. The DMAP alter skips the aerodynamic processing modules and allows SOL 146 to proceed with processing a structural module without any aerodynamic model input. The case control commands are entered as in a conventional dynamics solution. The exception is the requirement that both a FREQUENCY and TSTEP case control command must select corresponding bulk data entries. The TSTEP does not affect the solution accuracy, it provides output times for the inverse Fourier transform calculations. The DLOAD case control command selects a dynamic load as a function of time, a DLOAD combination of TLOAD1s in this case.

The xy−plot of Figure 5(b) shows the roadbed profile of the front suspension (grid point 3). The roadbed profile simulates a 12 inch long bump that is 2.5 inches above the normal roadbed and spaced every 176 inches. A pothole measuring 24 inches long and 2 inches deep follows the bump by 70 inches. Superimposed on the bump and pothole is a series of 1.5x1.5 expansion joint bumps spaced 60 inches apart. Grid point 1 is the attachment point of the suspension system to the frame. Grid point 5 is the location of the passenger or payload. The response of grid points 1 and 5 are displayed in xy−plots of Figures 5(a) and 5(c).

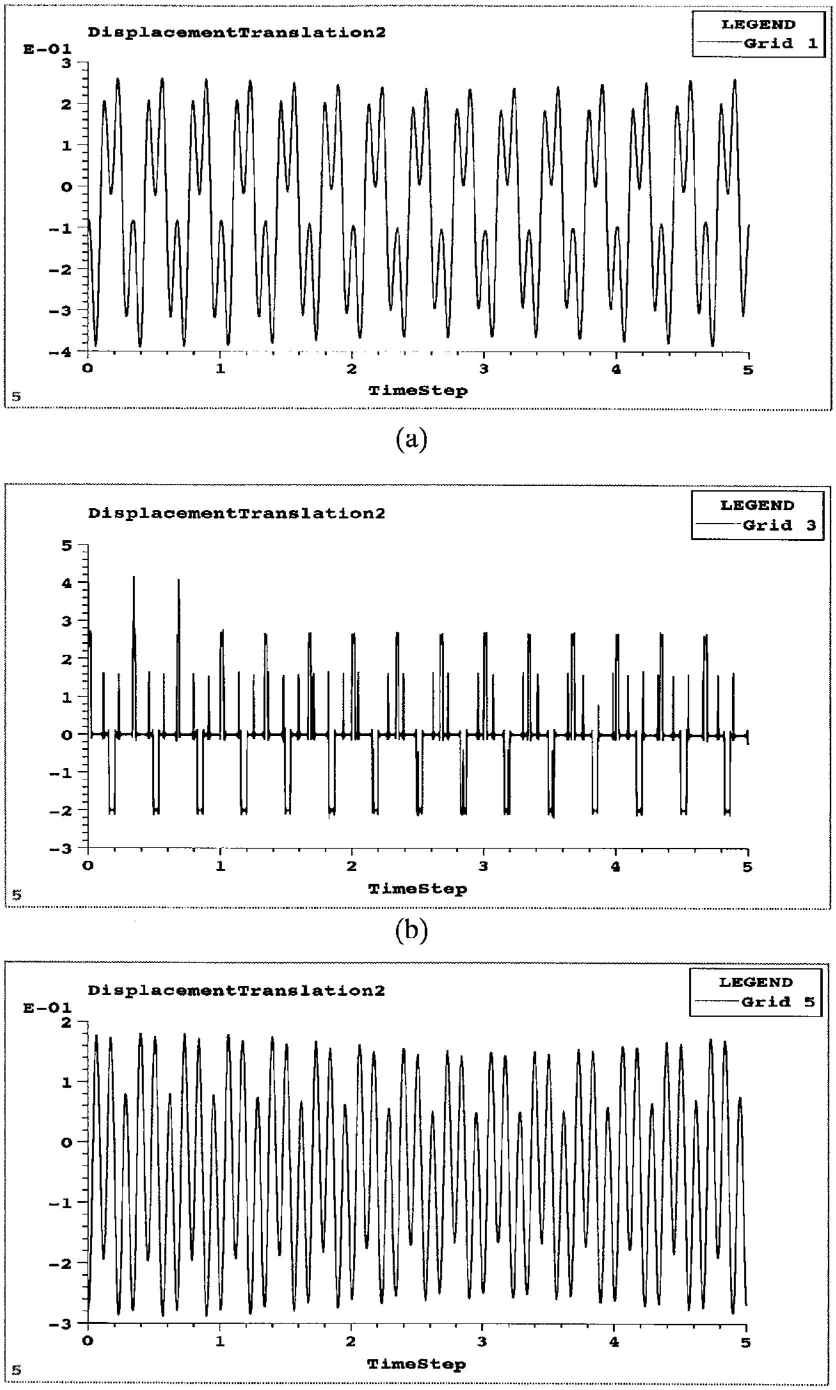
As a measure of the steady state transient response characteristics, a direct transient response analysis was performed using the same enforced motion but applied over a time interval of 6 seconds. The last second, the time between 5 and 6 seconds, repeats the enforced motion applied during the time interval of 0 to 1 seconds. The initial transient response of the vehicle as it encounters the first enforced motion decays sufficiently to a steady state motion. The damping of the suspension system is adequate to cause the vehicle to nearly reach a steady state response after 5 seconds of excitation caused by travelling over the roadbed. The Fourier transform, SOL 146, and direct transient response, SOL 109, results for the enforced motion and other points described above are superimposed on the same xy−plot in Figure 7. The comparison of the two results indicates that their dynamic characteristics are very similar.

# Listing 9. Vehicle on a Bumpy Road.

```
ID MSC WUC
TIME 100
SOL   146 $ AEROELASTIC RESPONSE
include 'skpaero.v68'
CEND
TITLE= SIMPLE VEHICLE MODEL
SUBTITLE= SPRINGS AND DAMPERS RUNNING OVER A BUMP at 528 in/sec
LABEL= SOL 146, CONSTANT FREQUENCY STEPS
SPC      = 100
METHOD   =  10
TFL      = 100
DLOAD    = 100
TSTEP    = 100
FREQ     = 100
DISPLACEMENT(PLOT)= ALL
include 'xyplot.requests'
BEGIN BULK
PARAM, POST, 0
$ EIGENVALUE EXTRACTION
EIGRL     10                            5
$ CARRIAGE POINTS
GRID      1                   0.0       0.0       0.0
GRID      2                   120.0     0.0       0.0
GRID      5                   60.0      0.0       0.0
$ WHEEL POINTS
GRID      3                   0.0       -10.0     0.0
GRID      4                   120.0     -10.0     0.0
$ CAR CARRIAGE
CBAR      5         11        1         5         0.0       1.0       0.0
CBAR      6         11        5         2         0.0       1.0       0.0
PBAR      11        12        10.0      10.0      10.0
MAT1      12        3.0E+7              0.33
$ CONSTRAINTS TO ELIMINATE RIGID#BODY MODES
SPC1      100       1345      1         2         5
SPC1      100       13456     3         4
$ SYSTEM WILL HAVE A NATURAL FREQUENCY OF 1 HZ
$ WITH CRITICAL DAMPING OF 1 PERCENT
CONM2     10        1                   2.5
CONM2     12        2                   2.5
CONM2     13        3                   1000.0
CONM2     14        4                   1000.0
CONM2     20        5                   5.0
$
CELAS2    30        197.4     1         2         3         2
CELAS2    40        197.4     2         2         4         2
$
CDAMP2    50        1.88      1         2         3         2
CDAMP2    60        1.88      2         2         4         2
$
$ USE LAGRANGE MULTIPLIERS TO IMPOSE WHEEL DISPLACEMENTS
$ 103= V3
$ 104= V4
$
EPOINT    103       104
$
TF        100       103       0         0.0       0.0       0.0
          3         2         1.0       0.0       0.0
TF        100       3         2         0.0       0.0       0.0
          103       0         1.0       0.0       0.0
$
TF        100       104       0         0.0       0.0       0.0
          4         2         1.0       0.0       0.0
TF        100       4         2         0.0       0.0       0.0
          104       0         1.0       0.0       0.0
$ FREQUENCY LIST
FREQ1     100       0.0       0.2       1000
```

# Listing 9. Vehicle on a Bumpy Road (Concluded)

```
$ MOVE WHEELS OVER ROADBED
DLOAD   100     1.0     1.0     1001    1.0     1002    1.0     1003
        1.0     1004    1.0     1005    1.0     1006    1.0     1007
        1.0     1008    1.0     1009    1.0     1010    1.0     1011
        1.0     1012    1.0     1013    1.0     1014    1.0     1015
        1.0     2001    1.0     2002    1.0     2003    1.0     2004
=       =       *4      =       *4      =       *4      =       *4
=9
DLOAD   101     1.0     1.0     1001
$ Bumps and Potholes - 70 in. separation 2.5x12 bump, 2x24 pothole
TLOAD1  1001    1001    1001    0       1001
DAREA   1001    103     0       1.0
DAREA   1001    104     0       1.0
DELAY   1001    104     0       .227273
TABLED1 1001
        0.0     0.0     0.001   0.0     .002994 2.5     .021833 2.5
        .023727 0.0     .156303 0.0     .158197 -2.0    .199864 -2.0
        .201758 0.0     .334333 0.0     ENDT
TLOAD1  1002    1001    1002    0       1001
=       *1      =       *1      ==
=12
DELAY   1002    103     0       .333333
=       *1      =       =       *.333333
=12
DELAY   1002    104     0       .560606
=       *1      =       =       *.333333
=12
$ Expansion Joints 1.5x1.5 with 60 in. separation
TLOAD1  2001    2001    2001    0       2001
DAREA   2001    103     0       1.0
DAREA   2001    104     0       1.0
DELAY   2001    104     0       .227273
TABLED1 2001
        0.0     0.0     0.001   0.0     0.002   1.5     .004841 1.5
        .005841 0.0     .113636 0.0     ENDT
TLOAD1  2002    2001    2002    0       2001
=       *1      =       *1      ==
=41
DELAY   2002    103     0       .113636
=       *1      =       =       *.113636
=41
DELAY   2002    104     0       .340909
=       *1      =       =       *.113636
=41
$ TRANSIENT DATA RECOVERY INFORMATION
TSTEP   100     5000    0.001   1
$
ENDDATA
```

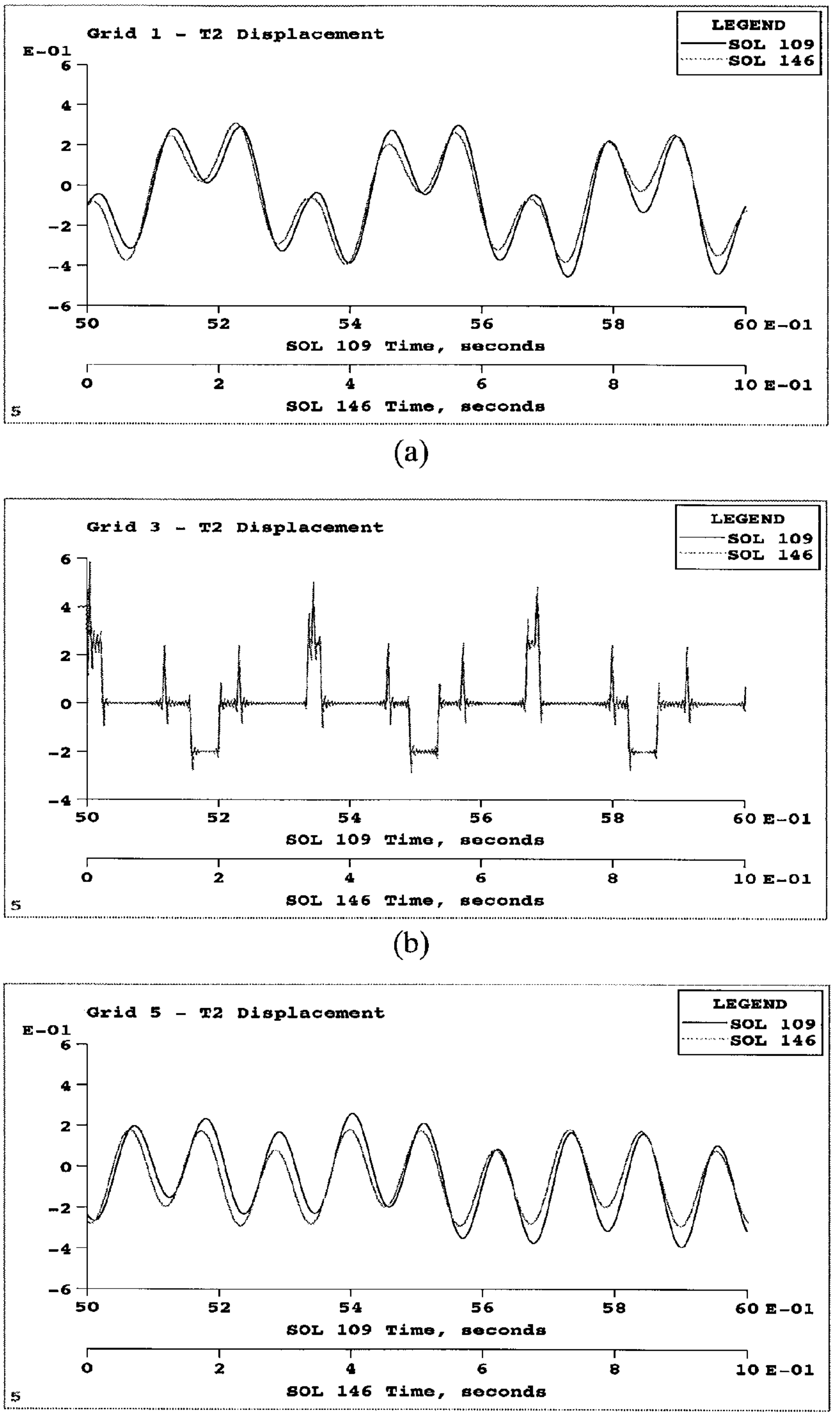


(a)

(b)

(c)

Figure 5. Fourier Transform Solution of Vehicle on a Road at 30 mph.



(a)

(b)

(c)

Figure 6. Comparison of Direct Transient and Fourier Transform Solutions.

## Conclusions

The Fourier transform methodology provides a supplemental method for investigating dynamic behavior of structures. It allows one to migrate between analysis in the time and frequency domains. The Fourier series coefficients can be calculated using a frequency response solution sequence. The Fourier series coefficients are determined with no loss of accuracy other than the precision of the machine. With the assistance of a Unix utility tool (a simple Fortran or c–program could be used too) the Fourier series coefficients from MSC/NASTRAN can be used to calculate input power spectral density functions from a prescribed transient function. A steady state transient solution of a structure due to a periodic loading can be obtained from the aeroelastic response solution sequence, SOL 146; however, the solution sequence must be modified with a DMAP alter to avoid the aerodynamic modules.

## Acknowledgements

The author would like to express his appreciation and acknowledgements to the following people. Dave Bella for supplying a reference to random analysis functions and answering general questions regarding the relationships in the Reference 10. Mike Wong for answering questions about "awk" and supplying a copy of Reference 11.

## References

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12 *Handbook for Dynamic Analysis MSC/NASTRAN Version 63*, The MacNeal–Schwendler Corporation, Los Angeles, CA, MSR–64, June 1983.

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# Appendices

## Appendix 1. 'awk' File to Build an Input PSD.

```
#
#       awk program to reformat complex load vector for importing to MSC/XL.
#       Program moves the IMAGINARY/PHASE line of the complex output to the end of
#       the same line as the REAL/MAGNITUDE output.  Formatting of the columns will
#       follow the pattern:
#
#       GID  T1RM  T1IP  PSDF
#
#       limitations:
#       1.      program will not work if there is no
#               spaces between the columns of data (i.e., load vectors)
#       2.      program does not support superelements
#       3.      program does not support scalar or extra points
#       4.      User must set the value of NumOfFrequencies
#      5.      All of the load vector output must be on one page
#              the case control command: LINE = 3*NumOfFrequencies + 10
#
#       to use the program a one line command is needed:
#
#               awk -f load.awk filename.f06 > filename.ext
#
#               where:  load.awk -       is the following program containing awk statements
#                       filename.f06 -   is editted MSC/NASTRAN printed output containing
#                                        only the load vectors.
#                       filename.ext -   contains the reformatted load vectors.  BEGIN{
        NumOfFrequencies  = 2000  # Change this value
        GridId = 100 # set to grid id no. of loaded grid point
#       Change these print statements to be consistent with the number of
#       complex load vectors.
#       If fewer or more load vectors are in the f06 file then the print statements
#       should be modified.  In the event that you have not modified the print
#       statements,  the end of the program generates the correct lines for all
#       the SectionAlias for all complex eigenvectors.  Any text editor may be used
#       to move the lines at the end of the awk program output to the beginning.
#       print "!"
        print "!"
        print "!#Title: Test Case of run"
        print "!"
        print "!#SectionAlias: 2 PointId"
        print "!"
        print "!#ColumnAlias: 1 Frequency"
        print "!#ColumnAlias: 2 T1"
        print "!#ColumnAlias: 3 PhaseT1"
```

```
        print "!#ColumnAlias: 4 PSDF"
        print "!"
# initialization of variable, NumOfLoads
        NumOfLoads = 0
}
$0 ~ /POINT-ID =/{
        if ( $3=GridId ) {
                firstline=$0
                getline
                secondline=$0
                }
        if ( secondline ~ /C O M P L E X   L O A D   V E C T O R/ ){
        print "!"
        print "!" firstline
        while ($1 != "FREQUENCY"){
                print "!" $0
                getline
        }
        printf("! %5s %15s %15s %15s %15s %15s %15s\n", $1,$3,$4,$5,$6,$7,$8)
        FreqCount = 1
        while (FreqCount <= NumOfFrequencies){
                getline
#                  The following PSDF calculation assumes OLOAD(PHASE)
#                  If real/imaginary the equation must be changed to do
#                  complex arithematic, e.g., PSDF=(2*g^2/T)*( $4*$4 - $5*$5 )
#                  constant coefficient is:
                PSDF=14930.5*$4*$4
                printf("  %5s %15s", $2,$4)
                getline
                printf("%15s %15.6e\n", $1,PSDF)
                FreqCount = FreqCount + 1
        }       NumOfLoads = NumOfLoads + 1
        }
}END{
        print "!"
        print "!"
        print "!"          NCount = 1
        while (NCount <= NumOfLoads){

                print "!#SectionAlias: " NCount+1 " Load" NCount
                NCount = NCount + 1
        }
}
```

# Appendix 2. DMAP Alter for SOL 146

```
COMPILE SEAERO SOUIN=MSCSOU NOREF NOLIST
$ALTER 64 $ V675
ALTER 67 $ V68
TYPE  PARM,,I,N,SKPAERO $
TYPE  PARM,,I,N,EDTRECNO $
PARAML  EDT//'PRESENCE'////S,N,SKPAERO $
IF (SKPAERO=0) THEN
$SEARCH EDT FOR AERO BULK DATA ENTRY
 SKPAERO = -1
 EDTRECNO = 1
 DO WHILE ( EDTRECNO > 0) $
   PARAML   EDT//'DTI'/S,N,EDTRECNO/1//S,N,CTYPE $
   IF ((EDTRECNO<>-1) AND (CTYPE=3202)) SKPAERO=0
   EDTRECNO=EDTRECNO+1
 ENDDO $ end loop over the EDT
IF ( SKPAERO<>-1 )THEN
$ALTER 65 $ V675
ALTER 72 $ V68
ENDIF $ AERO bulk data not included
ENDIF $ EDT is not present
IF ( SKPAERO=-1 ) SKPAMP=-1
```