Optimum Design of a Lightweight Telescope

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ABSTRACT

The sizing and shape capability of MSC/NASTRAN was applied to the design of an orbiting lightweight telescope. Design variables included dimensions of the primary mirror, mounts, and metering structure. Constraints were applied to optical performance measures such as image motion and surface distortion, as well as the conventional stress, frequency, and buckling behavior.

INTRODUCTION

Precision optical structures are usually stiffness limited designs, as compared to stress limited designs in many other industries. By the time a design meets the mirror surface distortion, system pointing accuracy, and image jitter requirements, the stress levels are often quite low. However due to tight weight limits and high launch loads applied to orbiting telescopes, portions of the system may be stress or buckling limited. The system level structural design is a complex trade of weight versus performance in a variety of load conditions including ground tests, launch, and on-orbit configurations.

Good design practice for optical structures utilizes kinematic supports to isolate mirrors from their support structures and to isolate metering structures from their surroundings. A kinematic support is a statically determinate (non-redundant) structure, usually composed of six ball-ended rods or flexures, which allows the base structure to distort without inducing any loads into the optic. A metering structure is the "optical bench" which holds the optics in precise location relative to each other. The use of kinematic supports has allowed the design process to be subdivided into individual components in the past. Even though supports are rarely "exactly" kinematic, this is still a valid first order design approach to components. However, as performance levels are increased, a system level design approach is required where components cannot be designed in isolation.

The new optimization capabilities released in Version 68 of MSC/NASTRAN allow a system level approach to the design of a telescope shown in Figure 1. Both sizing and shape variables were used with a variety of analysis types, including statics, buckling, and natural frequency. Optical performance measures were included through a series of multipoint constraints (MPC) and equation (DEQATN) entries.

DESIGN PROBLEM STATEMENT

The overall system level design problem can be stated as follows: Find the values of the design variables (X_j) which minimize the objective function Φ subject to all behavior constraints ($g_k \leq 0$) and side constraints ($XL_j \leq X_j \leq XU_j$) being satisfied.

DESIGN VARIABLES

The design variables can be divided into major components. The most important optical element is the primary mirror (Figure 2) which has it's diameter and curvature set by optical requirements. The overall height was fixed by a rather tight space allocation. The mirror has both sizing variables such as plate thickness and shape variables such as core plate location. The front and back faceplates were linked to a common thickness for thermal and fabrication considerations. The core struts were composed of 12 separate thickness variables. The only shape variable for the glass was the radius of core struts which moved with the mount location. Because of the six equally spaced mount pads, the mount location was determined with a single radius variable.

Since the primary mirror is such a key item in the telescope performance, it was optimized separately to see if the could provide adequate stiffness to meet optical test requirements in a 1g vertical configuration. During this component optimization, it was determined which design variables were significant and what the best values were in this idealized setup. These values were then used as starting points for the system level optimization.

The primary mirror mount has a direct relationship on the design of the mirror. Design variables include sizing variables on mountpad thickness, strut nominal diameter and strut neckdown diameter. Frictionless, ball-ended struts would be perfectly kinematic, but ball joints have unpredictable friction and rattle. To prevent ball joint problems, flexures which have necked down ends are used (Figure 3). The six mount struts act as a highly predictable, nearly kinematic, support. There is an important design trade on the neckdown diameter. If the diameter is too large, then too much moment is passed to the mirror. If the diameter is too small, then stress levels are too high, or the strut may buckle. Shape design variables on the primary mirror mount included the radial location of the mount and the geometric orientation of the struts both of which affect mirror distortion.

The secondary mirror can be considered structurally a flat, uniform thickness plate with a single sizing variable. The focus adjustment mechanism is composed of 3 flexures which again are driven by a trade on the neckdown diameter. The secondary mirror spider assembly has both sizing and shape variables to describe it structurally. The spider is driven by a design trade to keep it small for obscuration and weight versus keeping it large for static deflection and natural frequency considerations.

The metering structure is a stiffened shell of fixed diameter and length. The only variables are sizing on the shell thickness and stiffener dimensions. The main mount ring geometry was fixed based on space allocation, but thicknesses were variable. The important shape variable was the orientation of the main mount struts to the spacecraft. The aft trusswork which holds the imaging optics and electronics had only sizing variables on tube diameter and thickness. The geometry was predetermined by the location of the optical elements.

RESPONSES / CONSTRAINTS

Many standard responses were constrained in the design process. Under quasi-static launch loads stress was limited throughout the structure to be within the allowable range. For the metal structures the stress limit was based on Von Mises stress, but for the glass optics, the stress limit was based on maximum principal stress. Displacements are known to be relatively small during launch and were not included as design constraints. Relative displacements were checked in a post-processing step to be sure the assumption was still valid. Buckling also was constrained under launch loads since portions of the structure, such as the primary mirror mount struts, the secondary mirror spider and the metering shell, could be buckling limited. In all of these constraints the factors of safety were included in the response limits.

A minimum natural frequency constraint was imposed on the system to prevent vibration problems during launch and during on-orbit operation. For this structure the minimum frequency limit was 60 Hz.

A ground test of the telescope was required to verify its optical performance. In order for this test to be conducted accurately, limits were imposed on the surface distortion of the primary mirror and the image motion due to the difference in a zero gravity on-orbit condition versus a 1g on-earth vertical orientation test. These were very important constraints on the design which could not be described by simple DRESP1 entries.

In an optical element such as the primary mirror, the displacements of the surface can be decomposed into rigid body motions and aberrations, known as Zernike polynomials. This decomposition can be performed by a least squares fit of the selected polynomials to the deformed surface displacements [1]. This leads to the solution of a linear system of the form:

$$[H]\{C\} = \{F\}$$

where H contains the geometry of the surface, F is the deformed shape of the surface, and C is the vector of polynomial coefficients solved for (See Appendix). The rigid body and power terms describe pointing and focusing errors which can be adjusted out of the system through simple rigid body motions of alignment and focusing optics. The remaining higher order aberrations result in image quality degradation which cannot be easily corrected. Thus an important measure of an optical elements performance is the amount of distortion in a surface after the rigid body terms have been removed. As a key optical measure of the system, this was introduced as DRESP2 entries utilizing equations and multi-point constraints.

Since the equation above is linear, the magnitude of the polynomial coefficients can be written as MPC equations [2] in the form:

$$\{C\} = [H^{-1}]\{F\} = [A]\{U\}$$

where U is the displacement vector for grid points on the optical surface. This equation appears simple in matrix form, but the calculation of the coefficient matrix A involves several operations requiring a large preprocessing program. Note that H contains surface geometry so that it is only constant if the grids do not move as shape design variables. This can be a significant restriction in some design problems.

The coefficients defining rigid body motion for the primary mirror are combined with the rigid body motions of the other optics to obtain image motion response, again as an MPC equation. For simple flat (fold) mirrors the image motion can be written as:

$$d_i = (I' - I) - d_p = (L_2 + L_3)2\,\Theta_A - L_3\,2\Theta_B - d_p$$

where L is the optical pathlength from the mirror to the image plane (Figure 4). For lenses and curved mirrors the first order image motion equations have the same simple MPC form, but the coefficient calculation may require an optical analysis program. In most optical systems the displacements are extremely small so linear theory applies. For small optics represented as a single grid point and lumped mass, the rotation above is a single grid point rotation. Slightly larger optics, such as the secondary mirror, can use RBE3 elements to calculate the average tilt. The primary mirror's tilt is calculated from the least squares fit represented as the MPC equation for C. Note that the absolute motion of the image is not the significant item, but rather the image motion relative to the focal plane. The last term (d_p) in the above equation subtracts the motion of the focal plane. The image motion is limited in a 1g static optical test and in steady-state harmonic response for jitter analysis on-orbit.

In addition to the rigid body motions calculated above, the surface distortion of the primary mirror is an important design constraint. There are two commonly used measures of the surface distortion: peak-to-valley (P-V) and root-mean-square (RMS). P-V is the single highest point minus the lowest point of the deformed surface. RMS on the other hand is an average measure of the whole surface. Both of these must be calculated on the deformed surface after the rigid body and focus terms have been removed. Zernike polynomials can be used to represent the rigid body, focus and aberrations in the form as

$$Z_k = \Sigma_j C_j \Phi_{jk}$$

for grid point (k) where the form of the polynomials (Φ) is given in the Appendix. The net surface displacement (D_k) after bias ($C_1\Phi_{1k}$), tilt about x ($C_2\Phi_{2k}$), tilt about y ($C_3\Phi_{3k}$) and defocus ($C_4\Phi_{4k}$) are subtracted from the original surface (U_k) is:

$$D_k = U_k - C_1\Phi_{1k} - C_2\Phi_{2k} - C_3\Phi_{3k} - C_4\Phi_{4k}$$

Since this is a linear equation, it can be easily written as an MPC equation in the same preprocessing program which wrote the MPC equations for the Zernike coefficients C.

The response D_k was placed into a second set of "dummy" grid points coincident with the existing grids on the optical surface. These dummy grids had a constant GID offset, making it easy to create contour plots of either the full surface displacement U or the modified displacement D.

The P-V is the maximum difference of $(D_k - D_j)$ over all grid points (k,j) on the surface. A DRESP2 with a DEQATN entry using MAX and MIN functions was used for this calculation. The RMS calculation is nonlinear and requires the use of DRESP2 with DEQATN entries to evaluate:

$$RMS^2 = (1/N) \Sigma_k D_k^2$$

where N is the number of grid points on the surface. Since the magnitudes of the responses were so small, they were arbitrarily scaled by a large factor to assure their significance during the optimization process.

Both the P-V and RMS calculations require a significant number of bulk data entries. The writing of the MPC, DRESP2, and DEQATN entries was automated in the program which calculates the Zernike terms. A version 68 limit of 100 entries in the DEQATN parameter list was quite bothersome. To avoid this limit, the analysis used 1/2 of the grids uniformly spaced on the surface.

DESIGN RESULTS

The overall problem was broken down into components for model creation, checkout and suboptimization. Not only was the model checkout conducted, but the component optimization allowed verification of the many design optimization entries. The RMS and P-V calculations via MPCs and DRESP2 were verified against a post-processing program for fitting Zernike polynomials to deformed surfaces.

The optimization capability within solution 200 allowed the objective function and constraints to be easily altered to investigate the sensitivity of the final design. In one case, the weight was minimized while the optical surface RMS and P-V were constrained. This could easily be compared to an objective of minimum RMS with the weight limited. The impact of relaxing certain constraint limits was also investigated as possible design alternatives.

CONCLUSIONS AND RECOMMENDATIONS

There are several conclusions that can be drawn about about the system level design of this lightweight telescope which are specific to this application. However the general conclusion is that the MSC/NASTRAN optimization capability provides a very useful design tool.

The optical performance measures of surface RMS and P-V require a large number of bulk data entries. It would be very useful if MSC/NASTRAN would allow the calling of a user supplied subroutine to calculate complex responses. This subroutine must have as possible inputs any DRESP1 or DRESP2 responses and the current values of all design variables, including grid point locations. This could be in the form of a DRESP3 where the user supplied subroutine returns a response value, similar to the capability described in Reference [3].

The current limit of 100 parameter entries in the DEQATN should be increased. Most users will never hit the current limit, especially if the entries are coded manually. However, automated coding of entries can reach this limit without difficulty as shown in this paper.

REFERENCES

[1] Genberg, V., "Optical Surface Evaluation", Proc. of SPIE, Vol 450, Nov 1983.

[2] Bella, D., "MSC/NASTRAN Preprocessor for Performing Zernike Polynomial Analysis", Proc. of SPIE, Vol 748, Jan 1987.

[3] Thomas, H., and Genberg, V., "Integrated Structural/Optical Optimization of Mirrors", AIAA Paper No. 94-4356CP, Sept 1994.

APPENDIX

The displacement of a point located at (r,Θ) can be written as an infinite series

$$Z(r,\Theta) = \Sigma\ \Sigma\ [A_{nm}P_{mn}(r)\ Cos(m\Theta)\ +\ B_{nm}\ P_{mn}(r)\ Sin(m\Theta)]$$

where

Z = normal displacement to surface
r = radial position
Θ = circumferential position
n = radial wave number
m = circumferential wave number
P = polynomial term as function of (m,n,r)
A = polynomial cosine coefficient
B = polynomial sine coefficient

For Zernike Polynomials (r = radial position)

$$P_{mn}(r) = \Sigma_j\ C_p(n,j)\ r^{n-2j}$$

$$C_p(n,j) = [\ (-1)^j\ (n-j)!\ ]\ /\ [\ j!\ ((n+m)/2-j)!\ ((n-m)/2-j)!\ ]$$

For Zernike polynomials, the series are summed for $m \leq n$ and for alternate values of n. Since the series are orthogonal and complete, this representation is exact. If the series is truncated, then it becomes an approximation in general.

The terms may be represented as coefficients of cosine and sine (A,B) or as magnitude and phase (M,Ψ) (where Ψ uses FORTRAN function ATAN2)

$$M_{nm} = SQRT\ [\ A_{nm}^2\ + B_{nm}^2\ ]$$

$$\Psi_{nm} = (1/m)\ Arc\ Tan\ [\ A_{nm}\ /\ B_{nm}\ ]$$

Given a deformed shape (U=normal displacement) from an MSC/NASTRAN solution, the set of best-fit coefficients (A,B) can be determined as an approximate representation of the surface as follows. Define the error as:

$$E = \Sigma_k\ W_k\ (U_k - Z_k)^2$$

where

k = increment on grid number
U_k = MSC/NASTRAN displacement of kth grid
Z_k = polynomial displacement of kth grid

W_k = weighting of kth grid = area associated with kth grid

The summation is the difference in the MSC/NASTRAN and polynomial displacements If the polynomial series is written as:

$$Z_k = \Sigma_j C_j \Phi_{jk} \qquad j = \text{inc on polynomial term}, \; k = \text{inc on grid \#}$$

where $\{C_j\} = \{ A_{mn,} B_{mn}\}$

and $\{\Phi_{jk}\} = \{ P_{mn}(r_k) \cos(m\Theta_k), \; P_{mn}(r_k) \sin(m\Theta_k) \}$

The error for displacements can be written as:

$$E = \Sigma_k W_k (U_k - Z_k)^2 = \Sigma_k W_k (U_k - \Sigma_j C_j \Phi_{jk})^2$$

Minimizing the error E with respect to the polynomial coefficients {C}

$$dE / dC_i = 2 \Sigma_k W_k U_k \Phi_{ik} - 2 \Sigma_k \Sigma_j W_k C_j \Phi_{jk} \Phi_{ik} = 0$$

results in a linear system solved by Gauss elimination (Ref 1):

$$[H] \{C\} = \{F\}$$

where $F_i = \Sigma_k W_k U_k \Phi_{ik}$

$$H_{ji} = \Sigma_k W_k \Phi_{jk} \Phi_{ik}$$

Note that H is a symmetric, full, positive-definite, square matrix composed of terms calculated from geometry and the selected polynomials. F contains the deformed surface information.

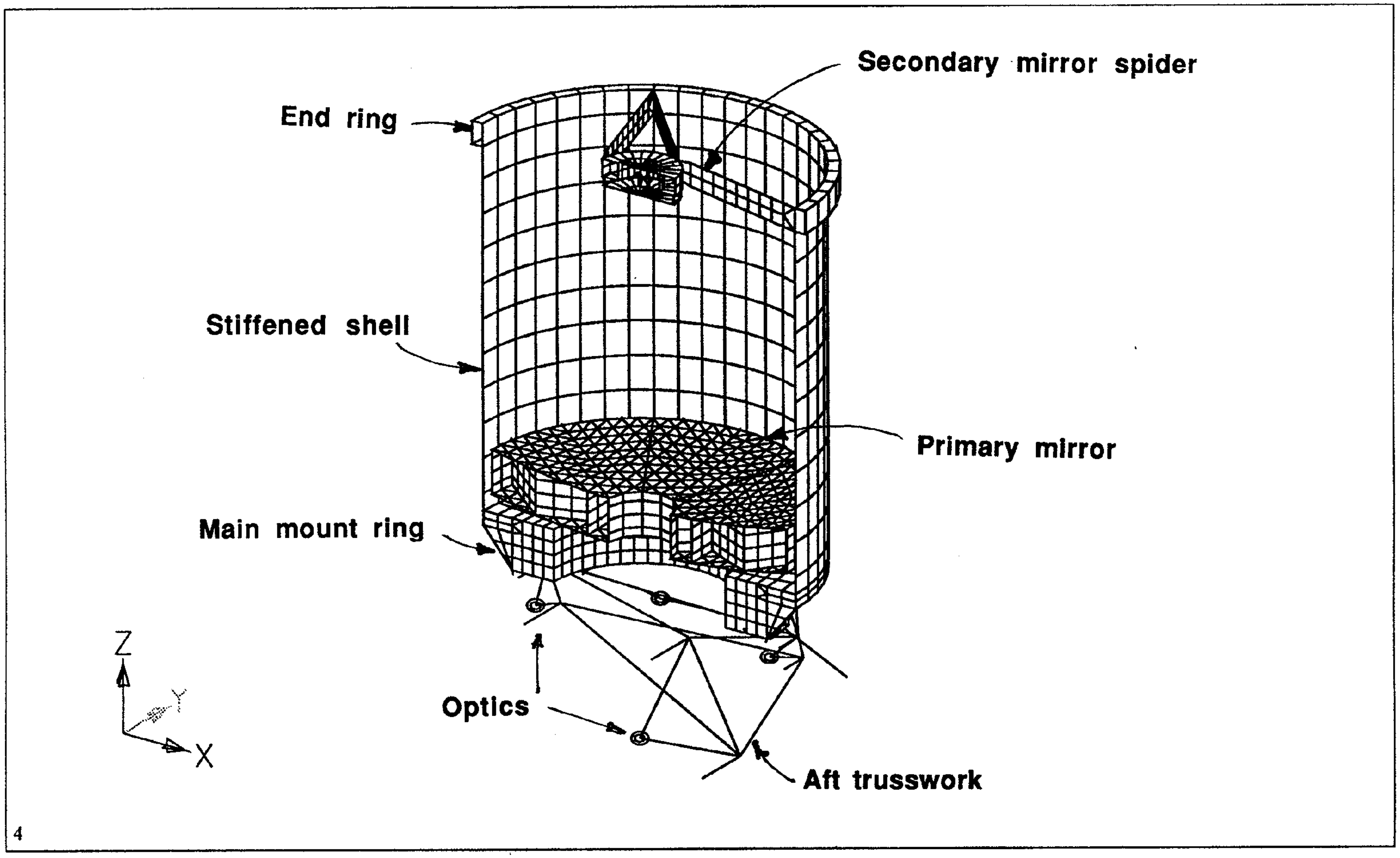

Figure 1. Lightweight telescope symmetric half-model

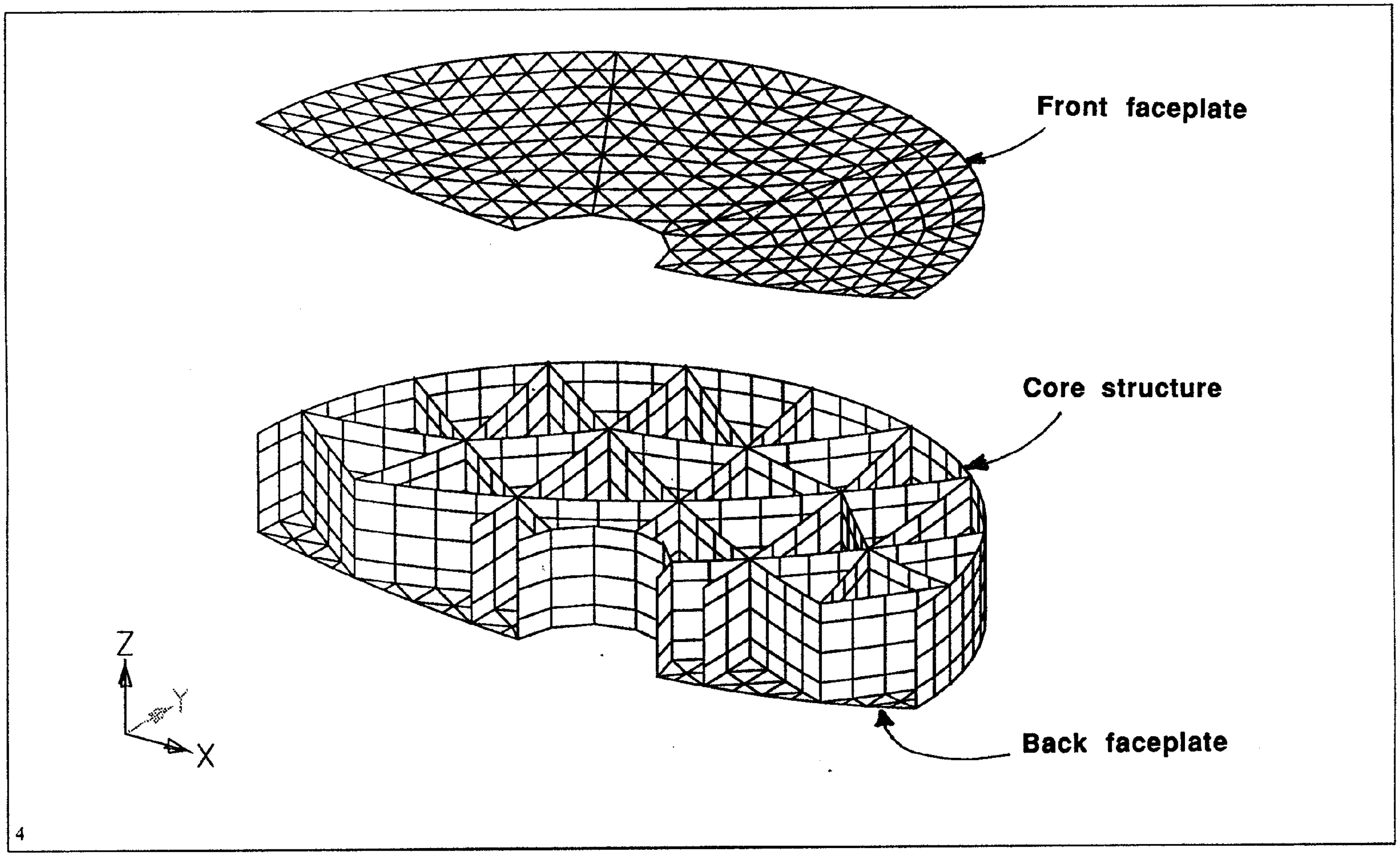

Figure 2. Exploded view of primary mirror

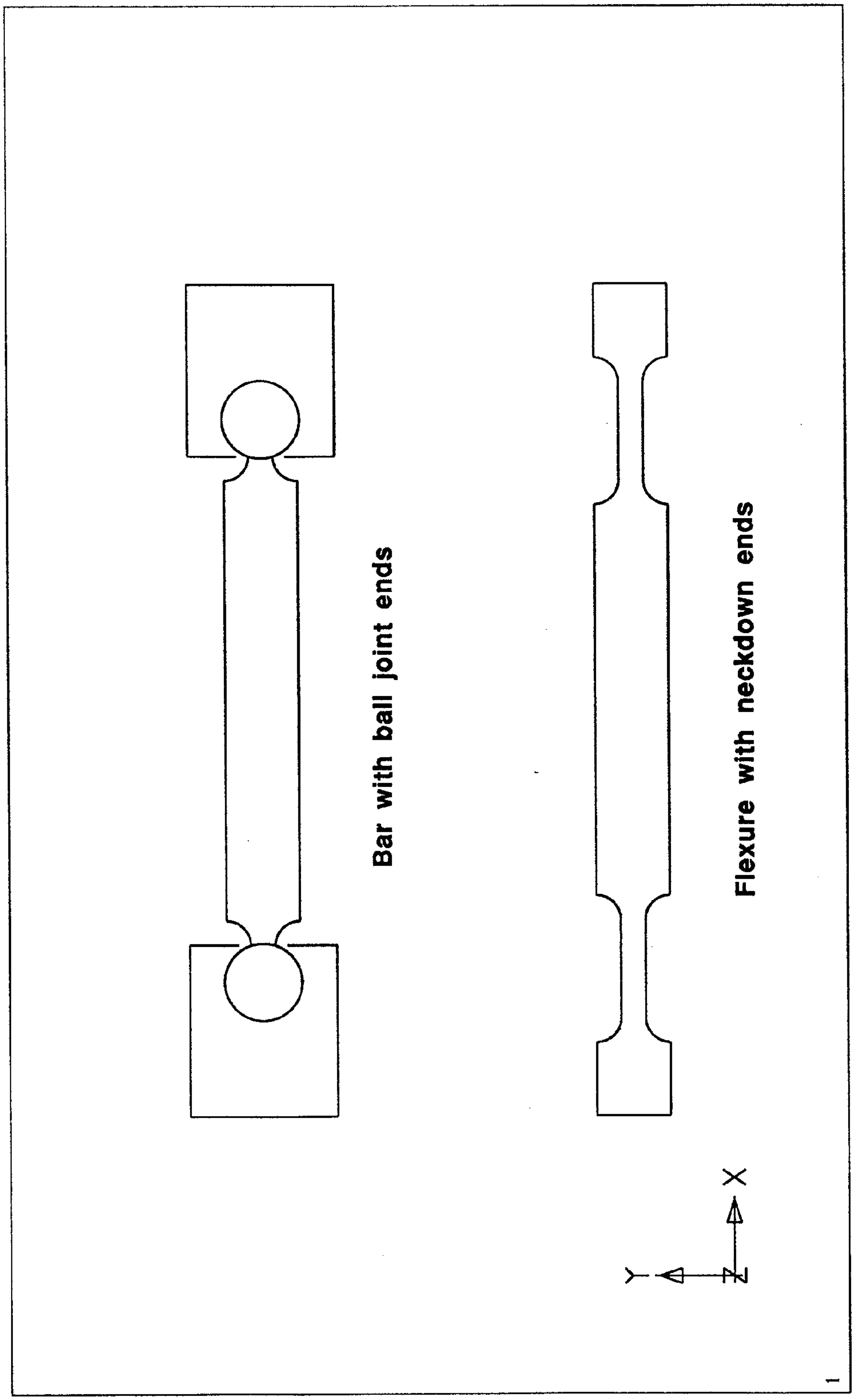

Figure 3. Mount struts

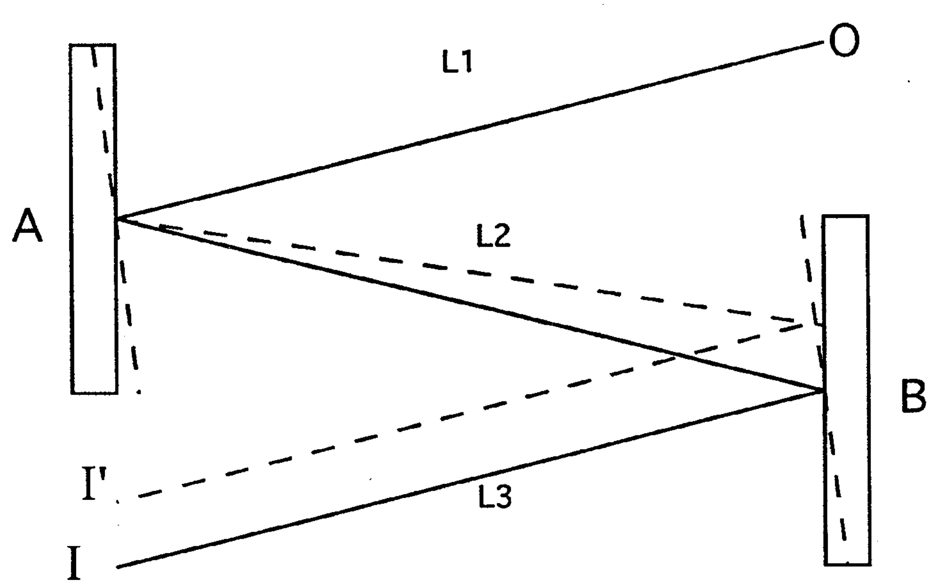

Figure 4. Image motion due to mirror rotations