Analytical Fatigue Life Assessment of Vibration Induced Fatigue Damage

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ABSTRACT

Vibration testing of components using accelerated test tracks or laboratory simulators is widely used in automotive design, as is fatigue testing for reliability. Furthermore, there are many common features between these two disciplines. However, problems often arise when engineers who are skilled in one field have to use techniques and concepts more generally used in the other. One example of such a situation concerns the use of frequency domain descriptions of structural response, which are commonplace in vibration testing for ruggedness, for computing the fatigue life or reliability of the same component. Many engineering applications, such as offshore engineering and wind turbine engineering, have already seen the benefits of using frequency domain fatigue tools for reliability assessment. The purpose of this paper is to assess the benefits of frequency domain fatigue analysis by comparison with a more conventional time series transient fatigue analysis. A typical automotive component has been analysed using MSC/NASTRAN and MSC/PATRAN FATIGUE in both the time domain (using a transient time history analysis) and in the frequency domain using spectral fatigue analysis techniques. Probability density functions and fatigue lives computed using the output from these two different approaches show good agreement.

INTRODUCTION

Most materials and particularly metals, are prone to a cyclic load degradation failure mechanism usually called fatigue. In metals this can be envisaged as the propagation of a crack through the material where the rate of growth of the crack is influenced primarily by the ranges of the applied stress (or strain). The larger the range of stress, the faster the crack grows and so fewer cycles are needed to reach some critical crack length. If the loading is in the form of a constant amplitude sine wave of stress against time as in Figure 1 then it has now been accepted, by comparison with experimental tests, that there is a relationship between S the stress range and N the number of cycles required to reach a 'failed' condition usually defined by a critical crack length. This is shown in Figure 2 where the parameters k and b are characteristics of the particular material being analysed.

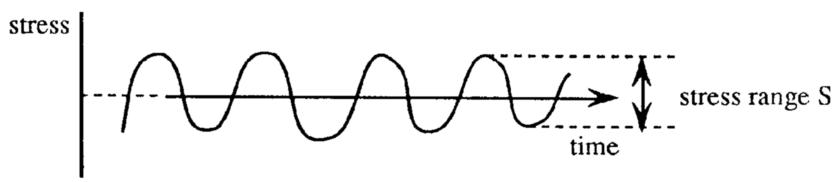


Figure 1: A typical constant amplitude stress-time history

It is usually assumed that this relationship is valid between 10^3 and 10^7 cycles. Below 10^3 cycles plastic deformation at the crack location means that a strain rather than stress based approach is more applicable, such as the Manson-Coffin strain-life curve. Above 10^7 cycles the stress level is so low that, for many materials, it is usually assumed that no fatigue damage occurs. For materials such as steel this is the so called endurance level. This approach is often referred to as a crack initiation method because it can be used to determine when a crack of a certain length has been 'initiated'. In fact, cracks of much shorter length can exist and the growth of these cracks can be analysed using Fracture Mechanics.

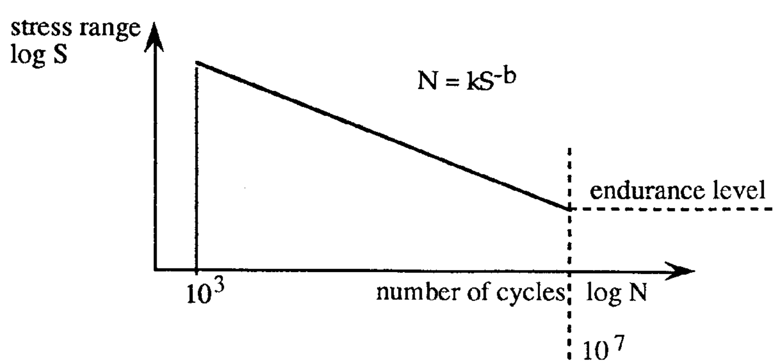


Figure 2: A typical stress range versus cycles to failure diagram.

If the loading is more irregular such as in Figure 3, as is usually the case for most engineering situations, then the irregular stress time sequence must be decomposed into appropriate fatigue causing cycles.

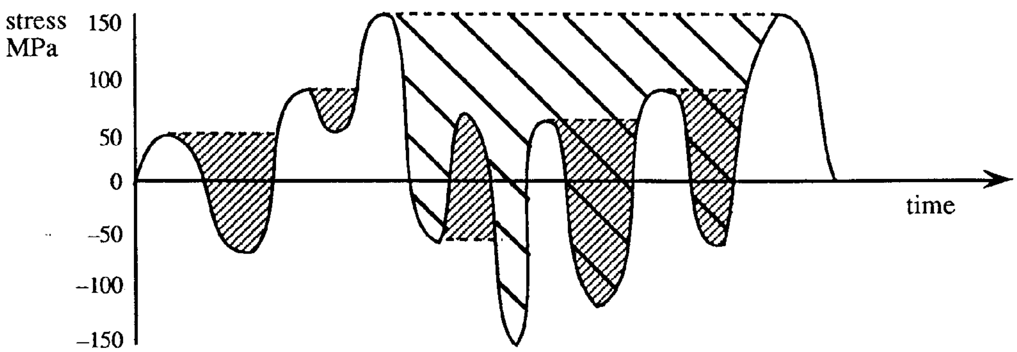


Figure 3: Rainflow counting an irregular stress-time history

Rainflow cycle counting is usually used for this. This is a technique whereby trends within the load history are identified and removed as individual cycles. Details of the rainflow cycle counting technique are given in Bishop and Sherratt, 1989. Using this approach the following 7 cycles are counted.

S (MPa)	50	100	150	200	250	300
n	1	2	3	0	0	1

Careful examination of the original sequence shows that counting segments ($^1/2$ cycles) between adjacent peaks and troughs gives the following results:

S (MPa)	50	100	150	200	250	300
n	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	0	0

Rainflow cycle counting therefore identifies the large (fatigue causing) trends in the signal which are missed by a segment counting approach. Once the cycles have been counted in this way the Palmgren-Miner (Miner, 1945) cumulative damage hypothesis is usually used which states that at failure:

$$\sum^{i} \frac{ni}{Ni} = 1.0 \tag{1}$$

where n_i is the counted number of cycles with stress range i (from the measured time history) and N_i is the allowable number of cycles (from the S-N diagram) with stress range S_i.

The reason for presenting these basic fatigue concepts is to show that fatigue analysis is generally thought of as a ***time domain*** approach. That is, all of the available tasks work on time based descriptions of the load function. It is the purpose of this paper to show that an alternative ***frequency domain*** fatigue approach can sometimes be more appropriate. The paper will concentrate on existing frequency domain fatigue tools. Research on new tools is currently ongoing as part of a collaborative arrangement between nCode and UCL. Before looking at these frequency domain tools it is necessary to discuss the role of vibration analysis in engineering. This is where general frequency domain techniques are already widely used.

A vibration analysis is usually carried out to ensure that potentially catastrophic structural natural frequencies or resonant modes are not excited by the frequencies present in the applied load. With an offshore platform, for instance, the objective is usually to make the structure so stiff that its first natural frequency is above the loading frequency. A typical example of this is shown in Figure 4. (see Bishop 1991 for more details)

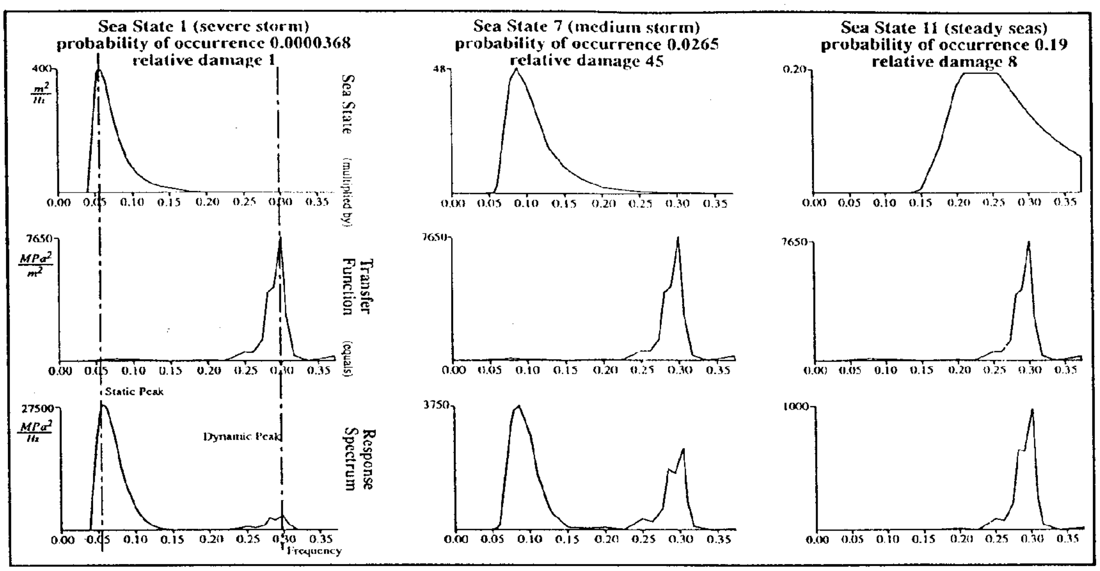
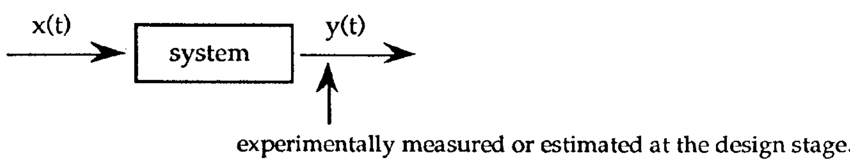


Figure 4: Sea state, transfer function and stress response PSD's for a typical offshore structure.

Sometimes this is not possible and vibration engineers then have to estimate the maximum response at resonance caused by the loading. These analysis tasks are best performed in the frequency domain using Power Spectral Density's (PSD's) of input loading and stress response. The input and output are elegantly connected via the so called transfer function, which in this case is a linear frequency dependant function. To get the stress response at a point on the structure which results from an input loading at another point one simply multiplies the input loading by the relevant transfer function. Of course, the assumption of linearity should be carefully considered. Most structures are, or assumed to be approximately linear and so such vibration analyses are now standard techniques in most engineering fields. Nearly all FEA programs have the capability to perform such a frequency analysis. The main difficulty arises when the possibility of fatigue damage needs to be assessed using this frequency domain information. In other words, given a stress response PSD (in units of MPa^2/Hz), how can a fatigue life estimate be calculated? This is covered in a later section but first the various approaches for obtaining responses, in both the time and frequency domains are discussed.

ANALYSIS APPROACHES - TIME OR FREQUENCY DOMAIN

In order to decide what kind of analysis to perform when a fatigue life estimate is the required objective, consideration has to be given to both the form of the input loading and the system characteristics. By using the term system we mean the component or structure under consideration. In general we can envisage the system as an operator on x(t) which produces the output (stress, strain, displacement, acceleration or whatever response parameter is of interest) in the following way.



What the practical engineer needs to know is what analysis approach to use for a given type of system and form of x(t). Table 1 presents some guidance on this for given combinations. Generally, for spectral fatigue methods to be appropriate for the analysis of ***measured*** responses there is no restriction on the system as long as the measured response is ***stationary , random and Gaussian*** Fortunately this does not prove to be too restrictive because the vast majority of engineering components and structures exhibit response loads which do fit these required assumptions.

If, on the other hand, one wishes to estimate the response of a given system to a prescribed loading the system characteristics then become important. The most important requirement of ***linearity*** arises because conventional spectral analysis techniques use a transfer function to relate linearly the input and output. Strictly, spectral techniques should not be used for non-linear systems. However, in practice, weakly non-linear systems can be adequately dealt with using a linearisation technique. Also, neutral networks are now being proposed as a means to obtain a trained network to represent a non-linear system.

In summary, therefore, if the system is linear or approximately linear and the loading is random, stationery and Gaussian, or approximately Gaussian then spectral fatigue techniques are particularly elegant and very powerful.

SPECTRAL FATIGUE TOOLS

If spectral fatigue analysis techniques are used a Power Spectral Density (PSD) of stress or strain is usually the input from which a fatigue life estimate is required. The PSD is simply an alternative way of representing an equivalent time history of stress or strain. In fact there is a mathematical link or transformation which can be used to move from one domain (frequency or time) to the other as shown in Figure 5. The information which is extracted from the frequency domain directly, which is used to compute fatigue damage, are the moments of the PSD.

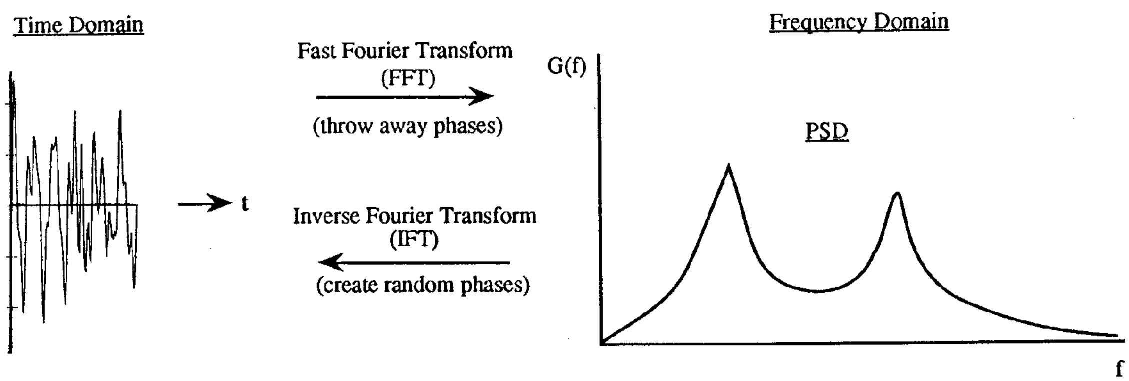


Figure 5 The transformation between time and frequency domains

These moments are used to compute all of the information required to estimate fatigue damage, in particular the probability density function (pdf) of stress ranges and the expected numbers of zero crossings and peaks per second. The nth moments of the PSD are computed using the following function

$$m_n = \int_0^\infty f^n \cdot G(f) \cdot df \qquad (2)$$

A method for computing these moments is highlighted in Figure 6. Some very important statistical parameters can be computed from these moments such as the expected number of zeros, E[0], and expected number of peaks, E[P], per second.

$$E[0] = \left[\frac{m_2}{m_0}\right]^{\frac{1}{2}} \qquad E[P] = \left[\frac{m_4}{m_2}\right]^{\frac{1}{2}} \qquad (3)$$

Using these the irregularity factor γ can be computed, which gives an indication of the spread of frequencies present in the signal;

$$\gamma = \frac{E[0]}{E[P]} \tag{4}$$

γ varies between 1.0 and 0.0. A value of 1.0 corresponds to a **narrow band** signal which, as the term implies, means that the signal contains only one predominant frequency. The value of γ tends to 0.0 when the signal contains a significant amount of energy at all frequencies.

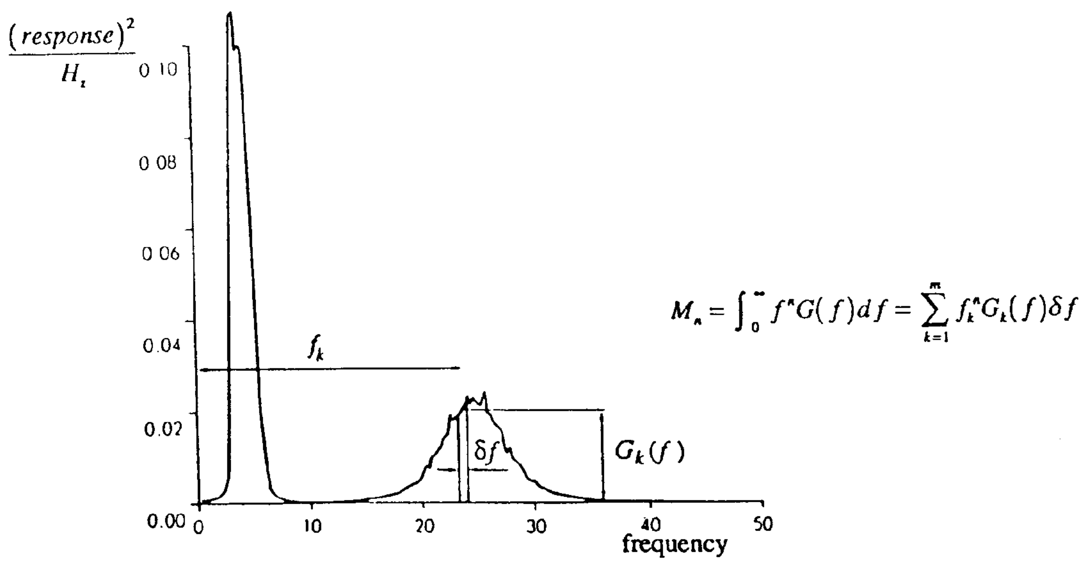


Figure 6 The method of calculating the moments of a typical PSD.

The first frequency domain method for predicting fatigue damage from PSD's made use of the so called narrow band approach [Bishop and Sherratt, 1989] mentioned above, which assumes that the pdf of peaks is equal to the pdf of stress amplitudes. The narrow band solution is then obtained by substituting a function of the Rayleigh pdf of peaks for the pdf of stress ranges. The problem with this solution is that by using the Rayleigh pdf, positive troughs and negative peaks are ignored and all positive peaks are matched with corresponding troughs of similar magnitude regardless of whether they actually form stress cycles. For wide band signals the method therefore overestimates the probability of large stress ranges and so any damage calculated will tend to be conservative. The narrow band formula is given below.

$$p(S) = \frac{S}{4m_0} e^{-\frac{S^2}{8m_0}} \tag{5}$$

Many expressions have been proposed to correct the conservatism associated with this solution. Most were developed with reference to offshore platform design where interest in the techniques has existed for many years. In general, they were produced by generating sample time histories from PSD's using Inverse Fourier Transform techniques, from which a conventional rainflow cycle count was then obtained. The correction factors of Wirsching [1980], Chaudhury and Dover [1985], and Hancock [Kam and Dover, 1988] were all derived using this approach.

Since these solutions assume that the pdf of rainflow ranges is the factor which controls fatigue life, a better approach is to estimate this directly from the PSD without using the narrow band approach as a starting point. Both empirical and theoretical expressions have been produced in this way.

Dirlik [1985] has produced an empirical closed form expression for the pdf of rainflow ranges, which was obtained using extensive computer simulations to model the signals using the Monte Carlo technique. Dirlik's empirical formula for the pdf of rainflow ranges has been shown to be far superior, in terms of accuracy, than the previously available correction factors. However, the need for certification of the technique before its use meant that theoretical verification was required. Such a theoretical solution for predicting rainflow ranges from the moments of the PSD was produced by Bishop and Sherratt (1989, 1990). The Dirlik formula is given below.

$$p(S) = \frac{\frac{D_1}{Q} e^{\frac{-Z}{Q}} + \frac{D_2 Z}{R^2} e^{\frac{-Z^2}{2R^2}} + D_3 Z e^{\frac{-Z^2}{2}}}{2(m_0)^{\frac{1}{2}}} \tag{6}$$

$$\text{Where; } Z = \frac{S}{2(m_0)^{\frac{1}{2}}}\ ; \quad x_m = \frac{m_1}{m_0}\left[\frac{m_2}{m_4}\right]^{\frac{1}{2}}; \quad D_1 = \frac{2(x_m - \gamma^2)}{1+\gamma^2}\ ; \quad D_2 = \frac{(1-\gamma-D_1+D_1^2)}{1-R} \tag{7}$$

$$D_3 = 1 - D_1 - D_2; \quad Q = \frac{1.25(\gamma - D_3 - (D_2 R))}{D_1}\ ; \quad R = \frac{\gamma - x_m - D_1^2}{1-\gamma-D_1+D_1^2} \tag{8}$$

Dirlik's expression, in particular, has been found to be an accurate and computationally efficient means of computing the pdf of rainflow ranges once m_o, m_1, m_2 and m_4 are known.

In order to compute the fatigue damage we need values for Ni the number of allowable stress ranges from the S-N diagram. N_i is given by $N_i = kS_i^{-b}$. The value of n_i, for a given stress range interval S to S+ dS is given by $p(S_i)dS$. The total number of cycles in a given period of time T is equal to the number of peaks in the same period and this is given by

$E[P] \cdot T$. The total number of cycles in the stress range internal dS at S_i is therefore $E[P] \cdot T \cdot p(S_i)dS$. The total damage is then given by:

$$E[D] = \sum^{i} \frac{n_i}{N_i} = \sum^{i} \frac{E[P] \cdot T \cdot p(S_i)dS}{kS_i^{-b}} = \frac{T \cdot E[P]}{k} \int_0^{\infty} S^b p(S)dS \quad \ldots\ldots \quad (9)$$

m_0, m_1, m_2 and m_4 are sufficient to compute this expression if Dirlik's pdf is used; and m_0, m_1 if the narrow band approach is used. These techniques are now being incorporated into predictive tools for the development and design analyst by nCode International Ltd in conjunction with the MacNeal-Schwendler Corporation.

APPLICATION TO AUTOMOTIVE DESIGN

An important objective in terms of assessing the standard integrity of engine components is to be able to perform whole engine analysis, thereby simplifying the design modification cycle. An example of an FE mesh for such a complete engine is shown in Figure 7.

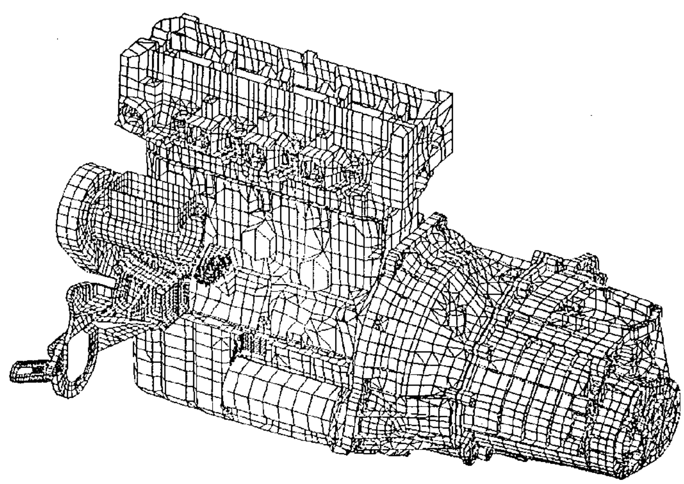


Figure 7: FE mesh of whole engine

Such a system is subjected to a combination of multi source random and cyclic load inputs. It would be extremely useful if PSD's of stress could be estimated at various critical locations on the model as caused by the multi source load inputs. The fatigue damage at these points could then be obtained using, for instance, the Dirlik damage expression. The influence of design modifications on damage could then easily be established by noting the change in the stress PSD's and hence fatigue damage. Such an approach is not yet fully developed. However, in order to demonstrate the technique, a smaller model of part of the whole system has been analysed. The model used is shown in Figure 8.

The inter-cooler model is comprised of 3628 solid elements (3402 CHEXA, 218 CPENTA and 8 CTETRA), 6767 shell elements (6477 CQUAD4 and 290 CTRIA3), 784 spring elements (CELAS2), 132 rigid bar elements (RBAR) and 25 multi-point control rigid bar elements (RBE2), and 12351 nodes. The spring elements were used to model the rubber isolators and the constraints of the rubber hose between the inter-cooler and compressor.

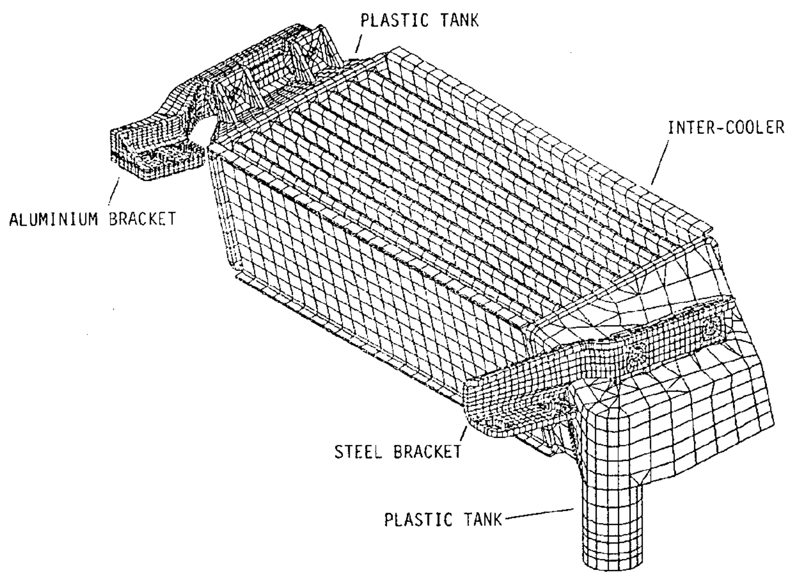


Figure 8: FEA of inter-cooler model

Both frequency and time domain approaches have been used to establish the response of the component to the acceleration load function shown in Figure 9. In both cases the load was applied, fully correlated with zero phase lag, to the brackets used to attach the inter-cooler to the engine.

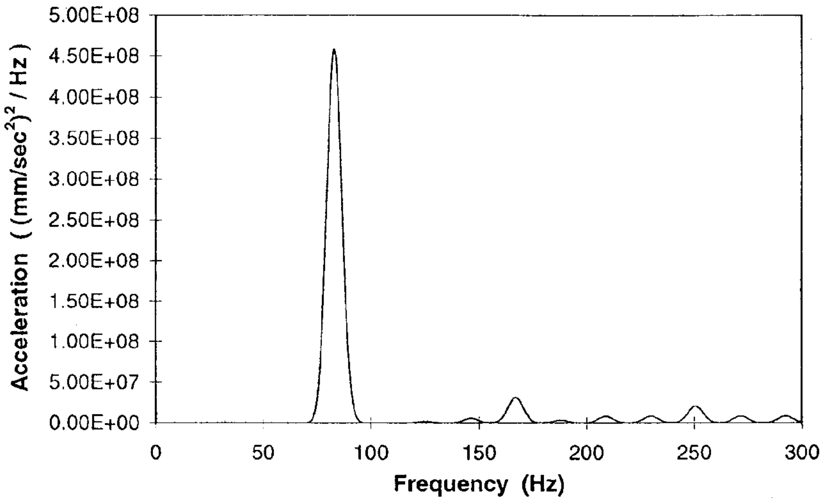


Figure 9: Acceleration PSD load function

This PSD load function is applicable for one engine speed only. If the engine speed varies then the load function varies. Since we are interested in finding the most damaging response conditions it is important to recognise that all engine speeds should be considered. This is where the flexibility of spectral techniques become particularly valuable since most of the computational work for a frequency domain analysis is done in computing the natural frequencies. Finding the response to many different inputs is then relatively straightforward and computationally efficient.

<u>Frequency analysis</u>

In order to perform a frequency domain analysis an eigenvalue analysis is firstly required to determine the frequencies to use as dynamic excitation. Once completed, a unit amplitude load function was applied, fully correlated with zero phase lag, to both fixing point brackets. The modal frequency response analysis (SOL 111) was then used to compute the transfer functions for the middle and end of the bracket, based on the excitation frequencies (FREQx). The actual PSD load function given by Figure 9 was then multiplied by the transfer functions to compute the required stress and acceleration responses.

A typical stress PSD of response, obtained from the middle of the aluminium bracket is shown in Figure 10. The Dirlik method described earlier was then used to evaluate the pdf of stress ranges using equation 9, from which a fatigue life estimate was then obtained. The complete frequency domain analysis took approximately 2 hours on a SUN Sparcstation 2.

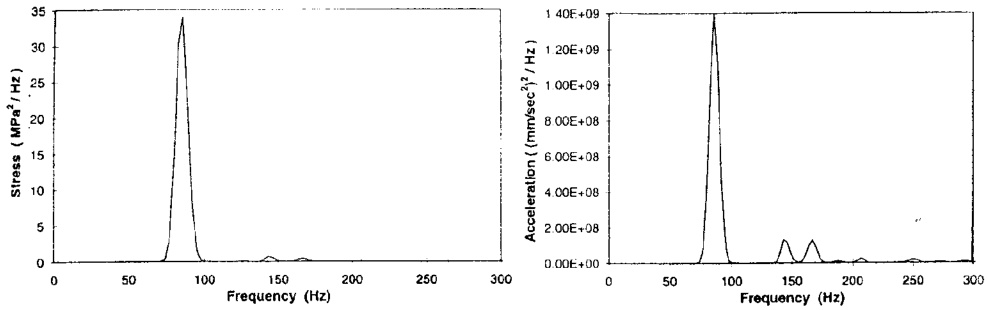


Figure 10: PSD's for (a) stress response at critical location and (b) acceleration at the end of the aluminium bracket.

Transient analysis

As a comparison, a transient time history analysis was carried out using, as an input function, a time history of acceleration obtained by Inverse Fourier Transforming the PSD given in Figure 9. A short sample from this input time history is shown in Figure 11. Forty seconds of this time history were used with a time interval between points of 0.0008 seconds, thereby giving 50000 time steps. The modal transient response analysis (SOL 112) was then used to compute the stress and acceleration responses based on the imposed time step interval. Responses were calculated for the same locations used in the frequency domain analysis.

A short sample of the computed time history of stress response is shown in Figure 12. This time history analysis took approximately 6 hours. The pdf of stress ranges, evaluated directly from the full stress time history, is shown in Figure 13 along with the equivalent pdf computed using Dirlik's method.

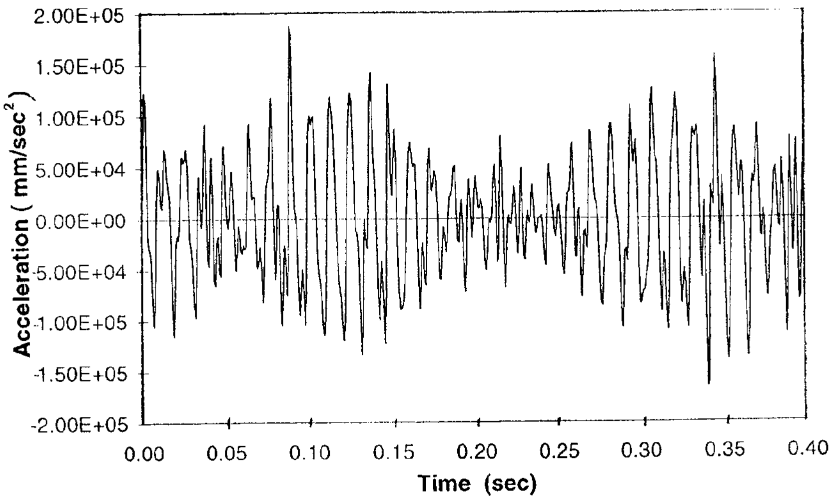


Figure 11: **Short sample from applied acceleration time history**

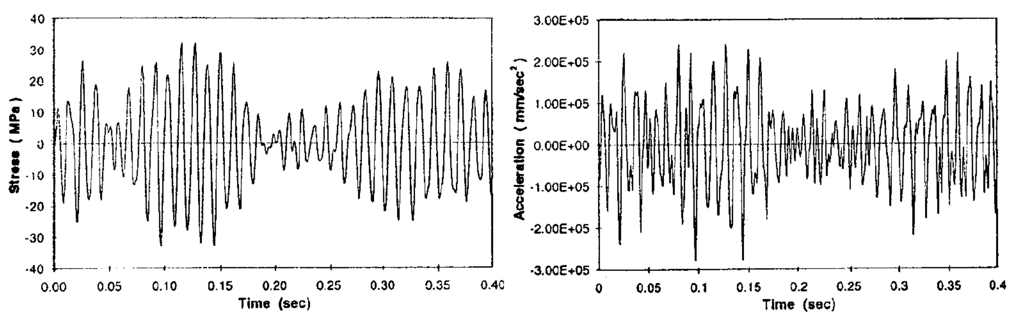


Figure 12: **Short sample from computed (a) stress time history at critical location and (b) acceleration time history at end of aluminium bracket.**

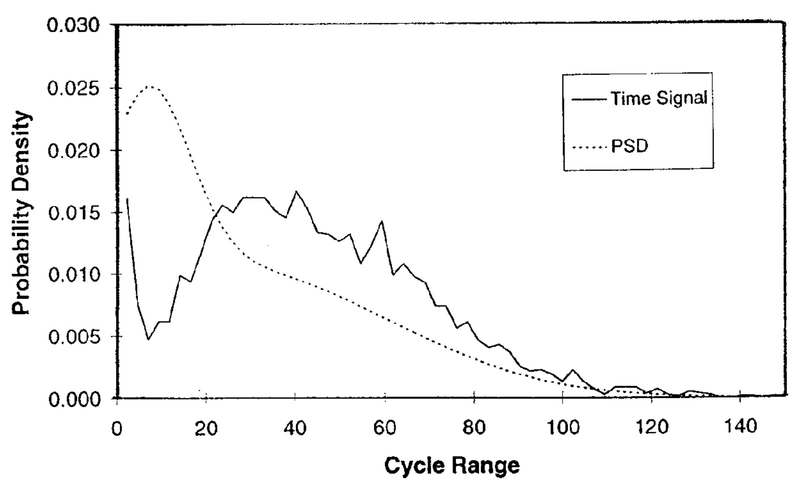


Figure 13: **pdf's of stress ranges computed from the output time signal and output PSD.**

The fatigue life was computed using typical material S-N data for aluminium The fatigue life obtained for the frequency domain approach was 8% higher than the time domain result in terms of equivalent stress, where the equivalent stress is defined by

$$S_{eq} = \left[\int_0^\infty S^b p(S) dS \right]^{\frac{1}{b}}$$

The close agreement between the results from the two approaches is as expected since the necessary assumptions that the structural model is linear, and that the loading is stationary, random and Gaussian, have been satisfied in the analysis. In order to get more close agreement a longer time history analysis would be needed. The results therefore demonstrate the computational elegance and efficiency which can be obtained by working in the frequency domain. An alternative comparison between the time and frequency domain analysis approaches was obtained by Inverse Fourier Transforming the time history output from the transient analysis. The result shown in Figure 14 confirms the close agreement with the frequency domain analysis (see Figure 10(a)).

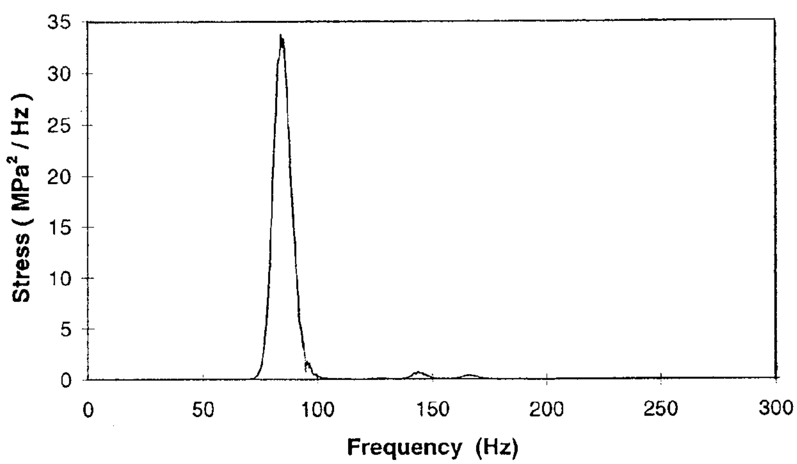


Figure 14: PSD for stress response at critical location computed from transient analysis output shown in Figure 12(a).

SUMMARY AND CONCLUSIONS

Frequency domain fatigue analysis tools have been applied to a typical automotive model in order to demonstrate the potential saving in computer effort which is obtained as well as the enhanced qualitative engineering understanding which is possible. As long as the necessary assumptions that the system is linear and the loading stationary, random and Gaussian are valid, fatigue life estimates can be computed far more rapidly using a frequency domain approach. Moreover, optimisation is feasible using this efficient analytical approach. A more thorough and complete analysis could easily be obtained in the frequency domain by simply including additional load cases for other rpm's, with no significant increase in run time. To undertake an extended time history analysis would result in a significant increase in run time. This makes such an analysis practically impossible. In general, therefore, the difficult task is to evaluate the PSD's of stress and for the type of whole engine analysis described earlier this is certainly possible although some development work is needed to fully achieve this goal. Careful consideration should also be given, by assessing experimental measured time records, to the validity of the loading assumptions mentioned above.

Input loading x(t)	SYSTEM					
	Linear Static	linear, dynamic single mode	non-linear static	non-linear, dynamic single mode	linear, dynamic multi mode	non-linear dynamic multi mode
Single sine wave						
Small number (say <10) of correlated sine waves						
Small number (say <10) of uncorrelated sine waves	✓✓✓	✓✓✓	✓✓	✓✓	✓✓✓	✓✓
Large number (say >10) of correlated sine waves	✓✓✓	✓✓✓	✓✓	✓✓	✓✓✓	✓✓
Large number (say >10) of uncorrelated sine waves	✓✓✓	✓✓✓	✓✓	✓✓	✓✓✓	✓✓
Purely random, stationary, Gaussian	✓✓✓	✓✓✓	✓✓	✓✓	✓✓✓	✓✓
Random, stationary, non-Gaussian	✓	✓	✓	✓	✓	✓
Random, non-stationary, Gaussian	R	R	R	R	R	R
Combined random with deterministic sine wave input	✓	✓	✓	✓	✓	✓
Combined random with a transient input	R	R	R	R	R	R
Multi-axial loading	R	R	R	R	R	R

✓✓✓ existing frequency domain fatigue tools generally applicable.

✓✓ frequency domain fatigue tools applicable as long as response satisfies stated conditions.

✓ advanced fatigue tools currently being developed

R research currently addressing these conditions

Table 1: Different combinations of load input conditions and system characteristics and the applicability of frequency domain fatigue tools.

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