A DESIGN STAGE NON-LINEARISATION OF STIFFENED-COMPRESSION PANELS FOR LINEAR MSC/NASTRAN/ARIES MODELLING OF DIAGONAL TENSION FIELD SHELLS

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Abstract

This paper extends applicability of the fundamental theory of compressed shells to the refinement of a linear finite element model.

In light of Von Karman's, Trefftz's, Cox's and Marguerre's interpretations of elementary elastic instabilities, the compression end-load member quad element thicknesses have been determined to take into account, with a reasonable degree of approximation, the main non-linear responses of a shell subjected to compression. Consequently, the Modal and the Static Finite Element Method Results will be improved.

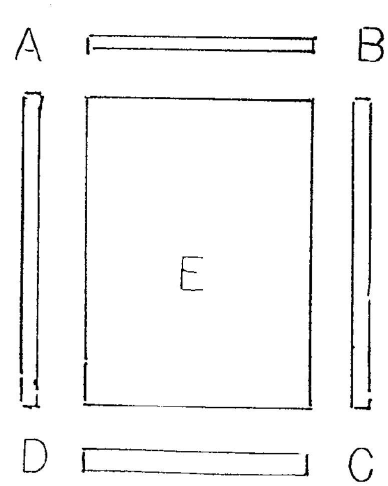
INTRODUCTION

A typical stiffened shell consists of flat plates, curved plates, fastened stiffeners, frames and end-load members, whose section-constants are least possible, to minimize design weight of the structure. Generally, the main objective of optimization of shells is determination of minimum principal dimensions to its main components. For example, an optimum wall thickness of the shells plate, likewise, an optimum aspect ratio of its array of thin-walled beams of open cross- section may be found for simultaneous onset of elastic instabilities at the beginning of permanent deformations, etc.

The critical natural frequency, or, the dynamic strength criterion will impose a set of additional stiffness and section requirements.

PROBLEM STATEMENT

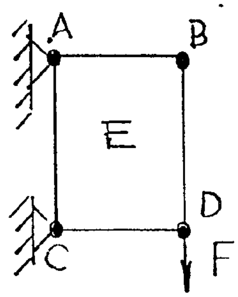
By the way of an example and not necessity, an elementary sub-assembly of a typical stiffened shell will be considered in light of the Maxwell's complementary strain energy theorem. The elementary panel bay consists of two horizontal end-load members AB ; DC, two vertical stiffeners AD ; BC (pin-pivoted, by definition, at the nodes A,B,C and D) and a fastened flat plate (web) E:



If AB=DC is the pitch of this elementary panel bay 'p' (in)
AD=BC is its height 'h' (in) and the cross-sectional areas of the edge members are:

	k	Ak (in*in)
AB	1	A1
BC	2	A2
CD	3	A3
AD	4	A4

t w : is the plate (web) thickness , the vertical displacement of the panel bay due to a vertically applied load;



is given by:

$$\Delta = \frac{q_1 \cdot q_f \times S}{G \ t_w} + \sum_{k=1}^{k=m} \int_0^L \frac{F_1^k \times F^k}{E \ A_k} \cdot dL$$

$m = 4$

where q1 and qf are the shear fluxes:
q1 = 1/h (lb/in) and qf = f/h (lb/in)
S; is the in plane area of the plate (web) = p*h (in*in)
E: is the flexural modulus (lb/in*in)
G: is the shear modulus (lb/in*in)
F; is the vertically applied load (lb)
F1: is an unit load that is collinear with 'F' (lb)

The term: $$\frac{q_1 . q_f \times S}{G \; t_w}$$

is the first component of the vertical displacement 'Δ'. This component is due to an elastic shear deformation of the plate E. Since the plate thickness 'tw' is located within denominator of this term, when 'tw' decreases towards zero value, the total vertical displacement of the panel increases towards infinity (i.e. this panel becomes a four-bar-mechanism that is hinged, by definition, at the nodes A,B,C and D).

The next term: $$\sum_{k=1}^{k=m} \int_0^L \frac{F_1^k \times F^k}{E \; A_k} . dL$$

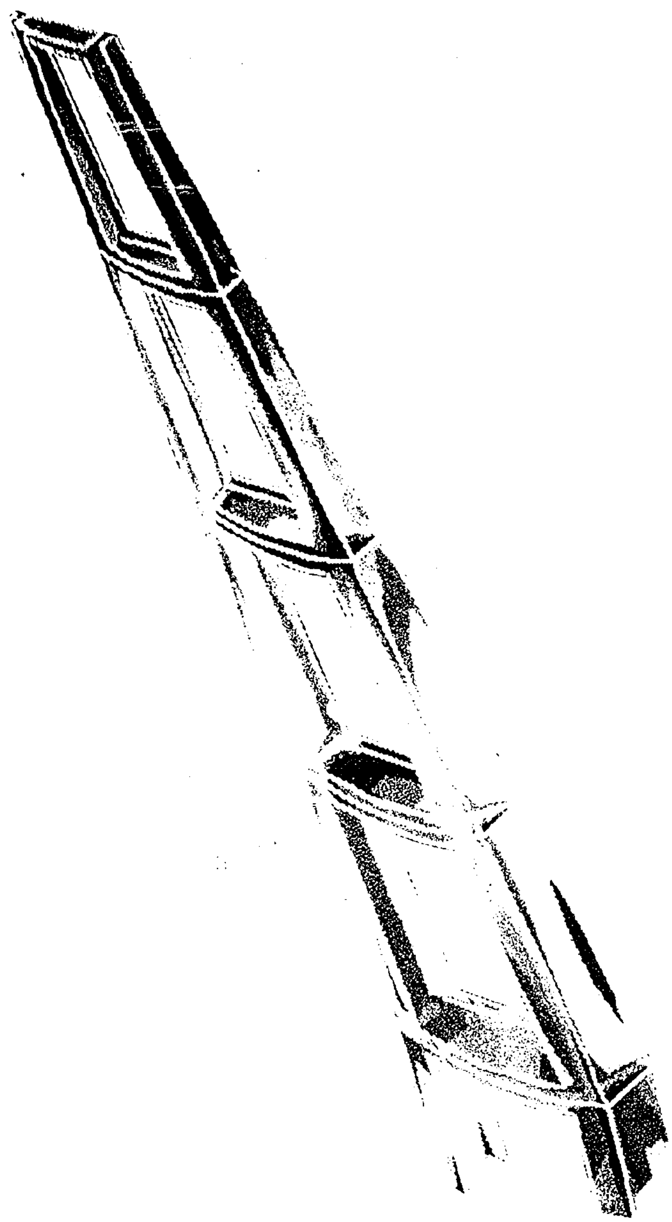
is the second component of the vertical displacement 'Δ'. This component is due to elastic axial deformation of two horizontal end-load members and two vertical stiffeners. Similarly, a reduction in cross sectional areas A1,A2,A3 and A4 increases the vertical displacement 'Δ'.

In the case of a balanced panel bay design, the first and the second component of the vertical displacement are of the same order:

$$\frac{q_1 . q_f \times S}{G \; t_w} \cong \sum_{k=1}^{k=m} \int_0^L \frac{F_1^k \times F^k}{E \; A_k} . dL$$

This is the main reason for exclusion of the rectangular cut-outs within the plate 'E' for modeling of the design stage non-linearisation.

On page 5 an aircraft wing cell shell linear model is shown incorporating a number of the plate cut-outs for modeling of non-linear responses. This technique has been found to be adequate for determination of flexural stresses and natural frequencies within the vertical-lift-plane only.



METHODOLOGY

The main objective of this analysis is to determine an expression for the compression end-load member and always compressed stiffener quad plate element thickness correction. The plate geometry remains intact (i.e. no rectangular cut-outs will be made into its unstable, inter-pitch-inter-height region, subjected to critical stress that does not change in a linear form relative to increments of the applied load).

It may be of some interest to note that according to the theory of shells (see reference 1), also in light of engineering applications of the theory, it would NOT be advisable to stiffen the panel plate with a stiffener whose wall thickness is smaller than that of the plate. Likewise, the wall thicknesses of the main end-load carrying members are always far greater than that of the plate. In addition to the cross-sectional areas of the end-load members and stiffeners, it is well known, in light of theory of shells, that the plate will remain partially effective even when its compressive stress approaches the crippling strength of compression elements (see reference 3). According to reference 2 the effective width of the plate, as established by the theory of Von Karman, is given by:

Effective plate width $= 0.94 * t_w * \sqrt{E/(\text{the crippling strength})}$

In the case of the aluminum alloy shells and conventional stiffening:

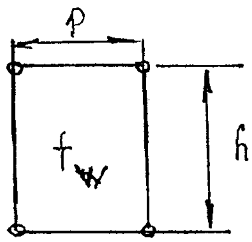
Effective plate width $= 15 * t_w$ (in) and the cross-sectional areas, that are additive to the cross-sectional areas of the end-load members and stiffeners, will be: Effective cross-sectional areas $= 15 * t_w^2$ (in*in)

hence, the cross-sectional areas that will remain unaffected by elastic instabilities will be:

$A = Ak + N * 15 * t_w^2$ (in*in)

where N = 1 in the case of an edge end load member fastened to a single plate, or, in the case of a stringer that is surrounded by plates at both sides N = 2.

In the case of an elementary panel:



the volume of the panel's plate E will be:

$V = h * p * t_w$ (in*in*in)

The circumference of the panel is given by:

$C = 2(h+p)$ (in)

The volume of the plate that is fastened to one single vertical stiffener:

$Vh = h * p * t_w * h/2(h+p) = h * p * t_w * 1/2(1+p/h)$ (in*in*in).

The volume of the plate that is fastened to one single end-load member:

$Vp = h * p * t_w * p/2(h+p) = h * p * t_w * 1/2(1+h/p)$ (in*in*in).

The volume to be subtracted from one single vertical stiffener's volume plus the effective volume of the plate will be expressed as the correction cross-sectional area Acv multiplied by the stiffener height h. This correction volume to be subtracted will be equal to the volume of the plate that is fastened to the stiffener Vh:

$Acv * h = Vh = h * p * t_w * 1/2(1+p/h)$.

hence;

$Acv = p * t_w * 1/2(1+p/h)$ (in*in)

It can be proven that

$Acv = p * t_w * 1/2(1+p/h) = Ach = h * t_w * 1/2(1+h/p)$

what simplifies the correction during design stages. With a sufficient degree of approximation the correction areas:

$Ac = N * p * t_w * 1/2(1+p/h)$

will be subtracted from the cross-sectional area of an end-load member plus the effective plate area. Likewise the same correction area will be subtracted from the cross sectional area of a stiffener plus the effective plate area.

The main reason for this subtraction of the correction areas 'Ac' from the cross-section areas of a linear Finite Element Method model, at the compressed region of a shell exclusively,

may be found within definition of the linear matrices. For example the stiffness matrix with its co-factors and determinants, takes in account, no doubt, all the web-plate areas, assuming them to be elastically stable quads.

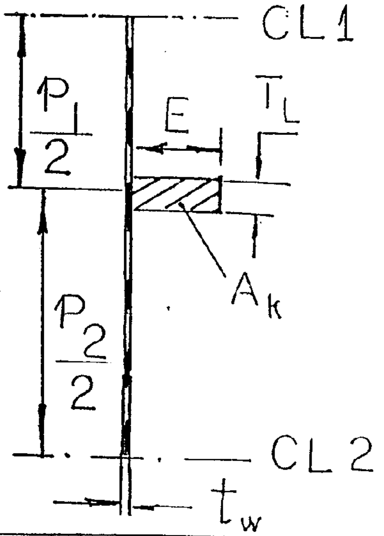
This matrix also incorporates all the cross sectional areas of the end-load members and stiffeners, as an analogous array of elastically stable elements.

A linear model of two adjacent panel bays is shown on the next page No 9 Fig.1. CL1 and CL2 are two central lines of two adjacent panels No1 and No2. 'Ak' is the cross-sectional area of an end-load member. 'e' is the height of the end load members and stiffeners as obtained from the theory of shells to prevent general instability. 'p_m': is the mean pitch of two adjacent panels that are symmetrical in respect to the central line of the end-load member.' T_L' is the quad plate element thickness of the linear model.

On the next page No 9 , Fig.2 a Non-Linear model is shown. The first term that is denoted by Ⓐ is the same linear model as shown in Fig.1. The second term that is denoted by Ⓑ is the main correction area. This correction area, in light of the presented method, eliminates the cross-sectional area of the web-plate: [a] when $h \to \infty$ the elimination is complete, [b] when $h \asymp p$, the elimination is substantial. The third term that is denoted by Ⓒ is the cross sectional area of the web-plate that remains, according to theory of shells, stable and effective at high stress levels at the onset of crippling.

The first term at the right hand side of this equation, denoted by Ⓓ is the cross sectional area of the web-plate that will be re-instated by the stiffness matrix: the web-plate mesh coordinates and its element-set quad plate thicknesses, in light of the presented method, will remain intact, to prevent a pre-processed transformation of the shell-model into an array of open, un-braced and un-triangulated frames.

FIG. 1

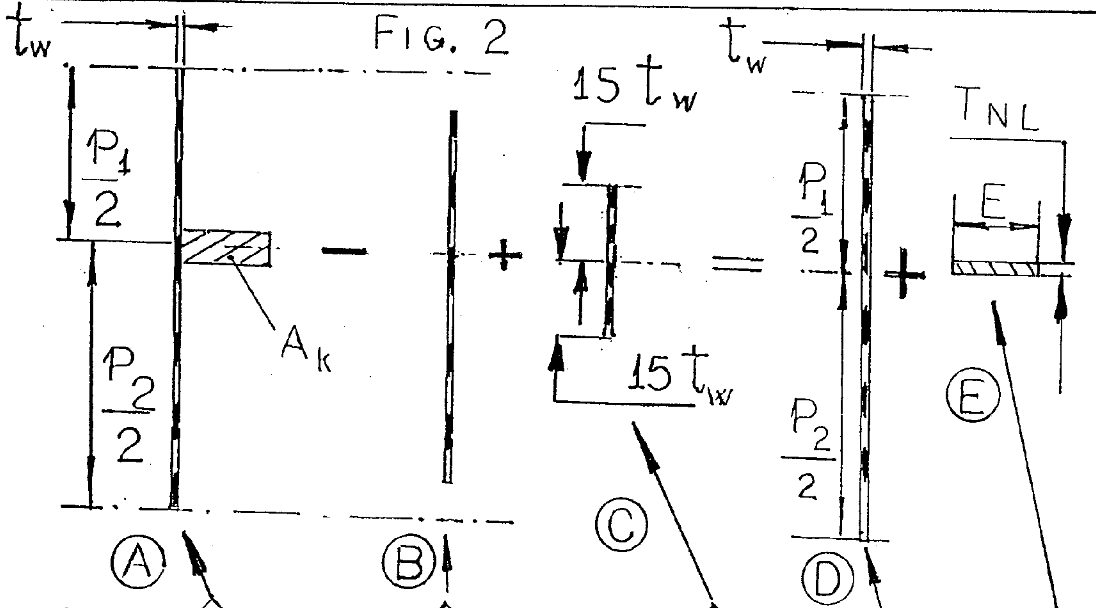


$$P_m = \frac{P_1 + P_2}{2}$$

$$A_K + N \frac{P_m}{2} t_w$$

$$T_L = \frac{A_K}{E}$$

FIG. 2



$$\left(A_K + N \cdot \frac{P_m}{2} \cdot t_w\right) - N\frac{P_m}{2} \cdot t_w \frac{1}{\left(1+\frac{P_m}{h}\right)} + N.15.t_w^2 = N\,\frac{P_m}{2}\,.t_w + E.T_{NL}$$

① $T_{NL} = \frac{1}{E}\left[A_K + N15\,t_w^2 - N\frac{P_m}{2}.t_w\frac{1}{\left(1+\frac{P_m}{h}\right)}\right]$

② WHEN $h \rightarrow \infty$

$$T_{NL\infty} = \left[T_{LINEAR} - \left(\frac{P_m}{2} - 15\,t_w\right)\frac{N\,t_w}{E}\right]$$

The second term at the right hand side is cross-sectional area of a Non-linearised Linear model. The end-load member quad plate element thickness was reduced to T_{NL} for equivalence:

$$(A) + (B) + (C) = (D) + (E)$$

However the magnitude at the left hand side will be equal to the cross sectional area of the end-load member Ak plus the cross-sectional area of effective lengths, when $h \to \infty$, and the left hand side will be substantially equal to Ak + $N\,15\,t_{w}^{2}$, when $h \asymp p$.

The first solution denoted by (1) was applied in the presented analysis of an Aluminum Alloy mass transit rail car shell.

From the second special case solution denoted by (2) ($h \to \infty$) it is possible to conclude that $p_m/2 = 15\,t_w$ the panels are shear resistant, therefore the linear model would be sufficient.
Consequently, upon reinstatement of the subtracted correction areas Ac, the remaining net cross-sectional areas, together with the reinstated adjacent quad element areas will be no greater than the cross-sectional area of the end-load member plus the effective plate (web) area and, also, no greater than the cross-sectional area of the stiffener plus the effective plate area.

ALUMINUM MASS TRANSIT RAIL CAR SHELL MODEL EXAMPLE

The developed design-stage non-linearisation method will be applied to the linear modeling of an aluminum mass transit rail car shell. The side view of the car and its quad plate model are shown on page 15.

On page 16 the main structural components are shown, from left to right: side sill, longitudinal omega stringer extrusions, longitudinal edge zee edge member extrusions, quadrant extrusion (aluminum alloy). On pages 16 and 18 the section constants of the main components are shown. The entire side-shell is shown on page 18. On page no 19 a view to the side shell model from inside is shown with all the longitudinal stringers and all the vertical frame plates exposed. At the right hand side of the same page a detail of the quad plate mesh is shown. On page 19 definition of the end-load members and stiffeners is shown at the left hand side. At the right hand side the plate quad elements of the linear model are marked. On page 12 calculation of the linear quad plate thicknesses is shown. On page 14 thicknesses of the compressed end load member and always compressed stiffener quad plate corrections are shown in light of the presented method of design-stage non-linearisation.

CONCLUSIONS

In light of the Finite Element Results obtained the presented Design-Stage Non-Linearisation enables users to analyze primary structures of lest weight and substantial complexity. While simultaneously increasing accuracy of Linear and Modal Finite Element Results, this method saves many months, or, even years of pre-processing and post-processing. Also the presented method improves correlation between: computed and strain-gauged critical stresses, calculated and ping-tested natural frequencies, predicted and observed elastic instabilities for purposes of diagnostic work and exclusion of liabilities.

It is also possible to conclude that without application of 'Super-Elements' , MSC Non-linear methods of analysis and without application of the presented Design-Stage Non-Linearisation, the error in computing critical stresses, relative to Linear analysis, may be as high as 20% , consequently, the error in computing natural frequencies may be up to 10%.

LINEAR ANALYSIS

(A) Compressed Panels

End Load Member	Description	Cross Section Area - All of the End-Load Member in^2	Quad Plate Element Thicknes T_L in
AB	QUADRANT	4.15	1.383
BE	QUANDRANT	4.15	1.383
DC	EDGE ZEE	0.77	0.256
CF	EDGE ZEE	0.77	0.256
AD	FRAME POST	1.68	0.56
EF	FRAME POST	1.68	0.56
BC	OMEGA STIFFENER	1.93	0.643
GH	EDGE ZEE	0.77	0.256
HI	EDGE ZEE	0.77	0.256
DG	FRAME POST	1.68	0.56
FI	FRAME POST	1.68	0.56
(B) Panels Subjected To Tensile Stresses			
JK	OMEGA STRINGER	1.93	0.643
KL	OMEGA STRINGER	1.93	0.643
MN	OMEGA STRINGER	1.93	0.643
NO	OMEGA STRINGER	1.93	0.643
PQ	SIDE SILL	9.03	3.01
QR	SIDE SILL	9.03	3.01
HK	OMEGA STIFFENER	1.93	0.643
KN	OMEGA STIFFNER	1.93	0.643
NQ	OMEGA STIFFENER	1.93	0.643
IL	FRAME POST	1.68	0.56
LO	FRAME POST	1.68	0.56
OR	FRAME POST	1.68	0.56
GJ	FRAME POST	1.68	0.56
JM	FRAME POST	1.68	0.56
MP	FRAME POST	1.68	0.56

Non-Linear compressed section of the shell

Corrections of the cross-sectional areas of end-load members, stringers and stiffeners, in light of the presented design-stage non-linearisation method.

There are four panel bay types subjected to compression.

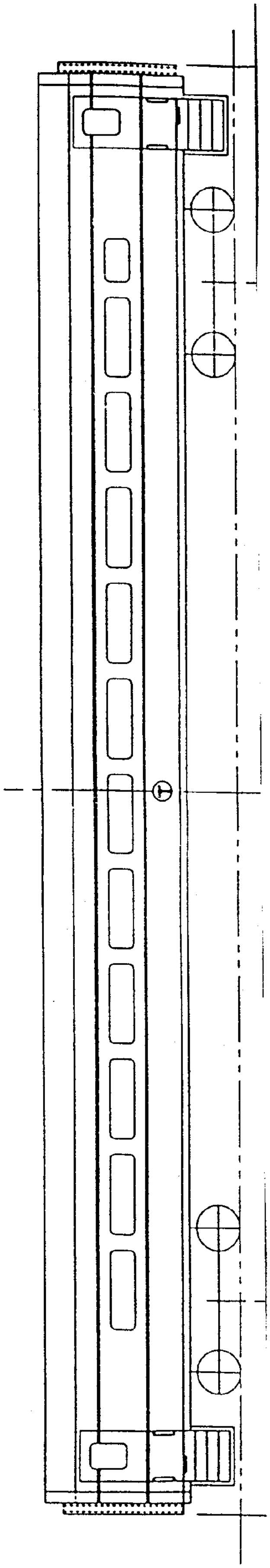
In the case of members surrounded by one single panel bay (hence subscript 1) the correction areas A_{c1} are given by:

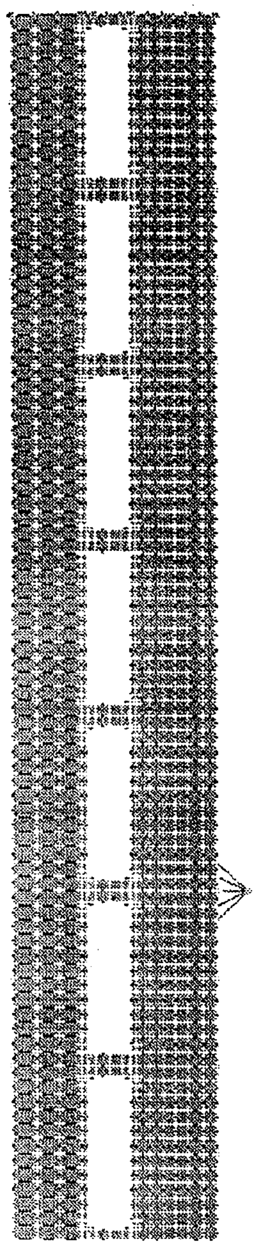
Panel Bay	Panel Pitch P (in)	Plate Height ht (in)	Plate Thickness t_w (in)	Correction Areas A_{c1} (in^2)
ABCD AND BCEF	24"	30.5"	0.125"	0.839
ADEF	6"	24"	0.125	0.300
DGFI	6"	22	0.125	0.295
GHJK AND HjLK	6"	12	0.125	0.25

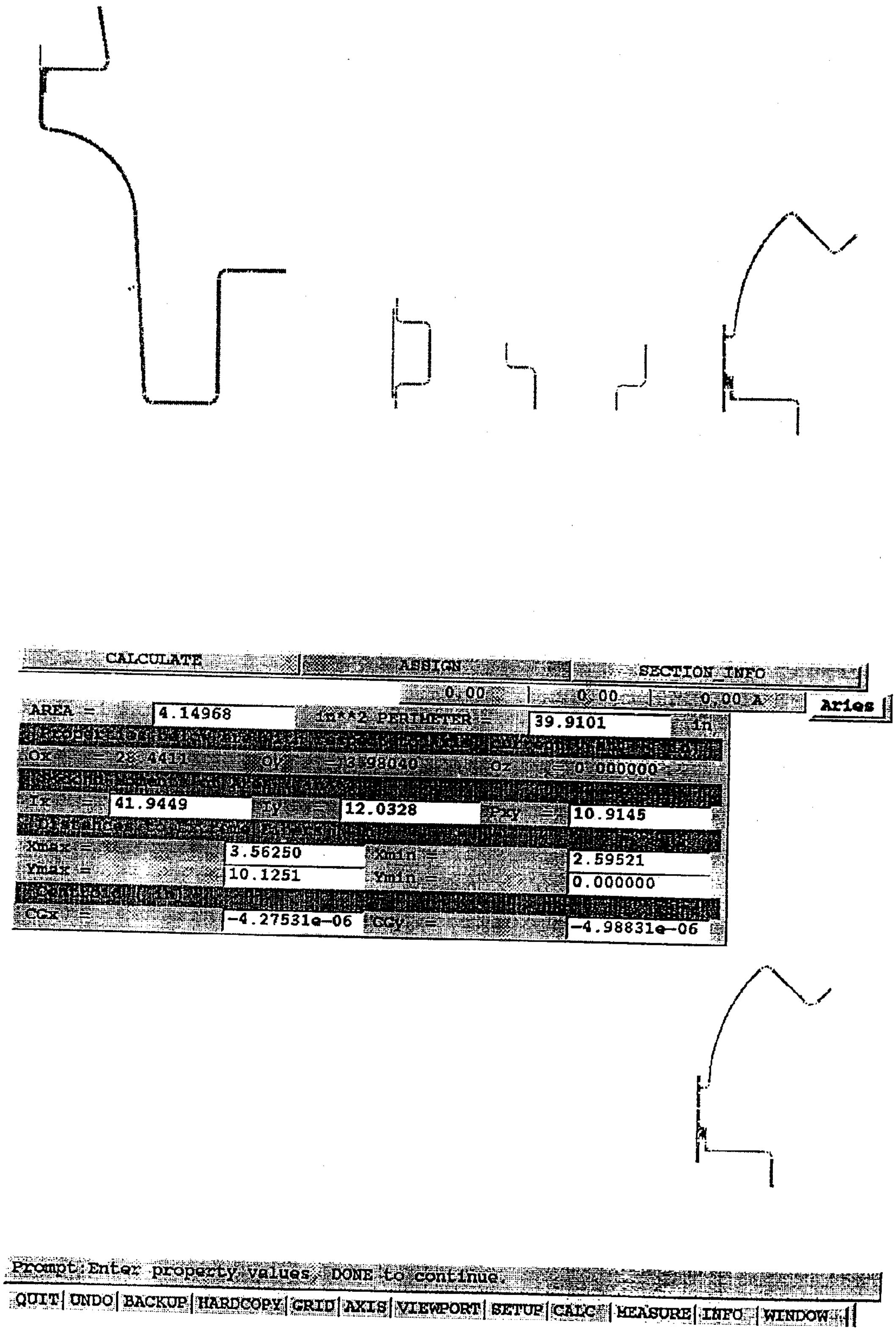
N_{xy} is the number of surrounding plates relative to the central line of an end-load member, stringer or stiffener. With reference to page No. , for example the Omega stiffener BC is surrounded with two plates ABCD and BCEF therefore $N_{BC} = 2$. However the quadrant AB is surrounded with one single plate ABCD therefore $N_{AB} = 1$.

n is the number of effective skin lengths. Relative to the central line. 'n' differs from N_{xy} only within surroundings of the window cut-outs because a narrow lip is wider than 15t but mid panel bay is a cut out.

End Load Member	N_{xy}	Cross Sectional Areas A_k in^2	n	Effective Plate Areas n x $15t^2$	$A_k + n15t^2$	A_{c1} in^2	Areas A_{NL}	Corrected Quad Element Thickness Non-Linear T_{NL} in
AB	NAB = 1	4.15	1	0.234	4.384	0.839	3.545	1.181
BE	NAE = 1	4.15	1	0.234	4.384	0.839	0.545	1.181
DC	NDC = 1	0.77	2	0.468	1.238	0.839	0.399	0.133
CF	NCF = 1	0.77	2	0.468	1.238	0.839	0.399	0.133
AD	NAD = 2	1.68	2	0.468	2.148	0.839	0.47	0.156
EF	NEF = 2	1.68	2	0.468	2.148	0.839	0.47	0.156
BC	NBC = 2	1.93	2	0.468	2.398	0.839	0.720	0.24
GH	NCH = 1	0.77	2	0.468	1.238	0.25	0.988	0.329
HI	NHI = 1	0.77	2	0.468	1.238	0.25	0.988	0.329
DG	NDG = 1	1.68	2	0.468	2.148	0.295	1.853	0.617
FI	NFI = 1	1.68	2	0.468	2.148	0.295	1.853	0.617



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[illegible]

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[illegible]

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[illegible]

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Ymax = 1.04435 Ymin = 0.768152

[illegible]

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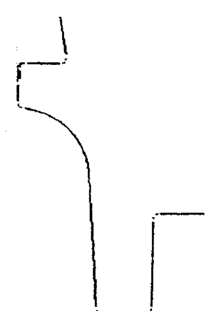

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Ymax =	1.62729	Ymin =	1.43521		
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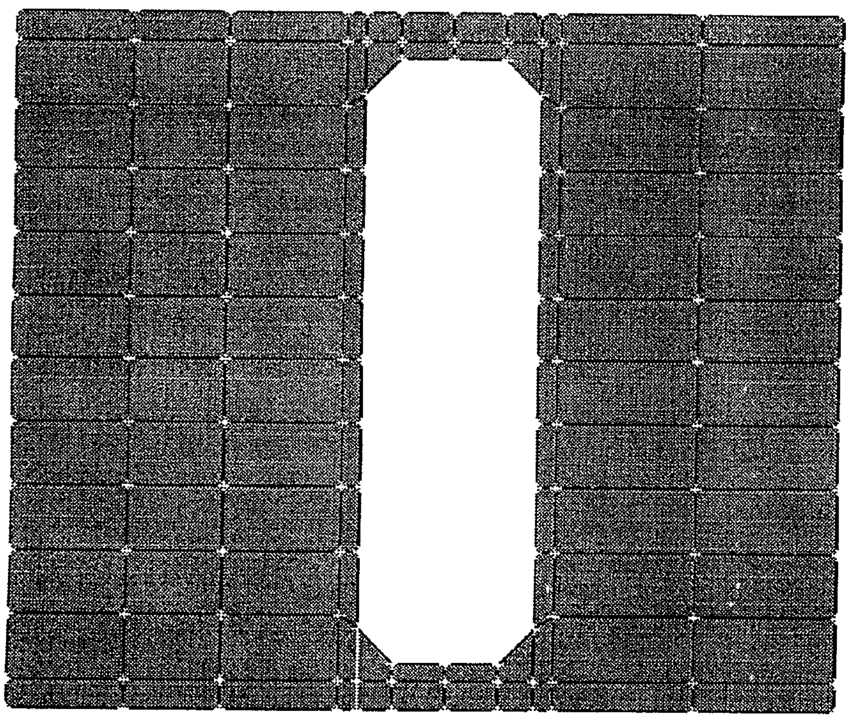

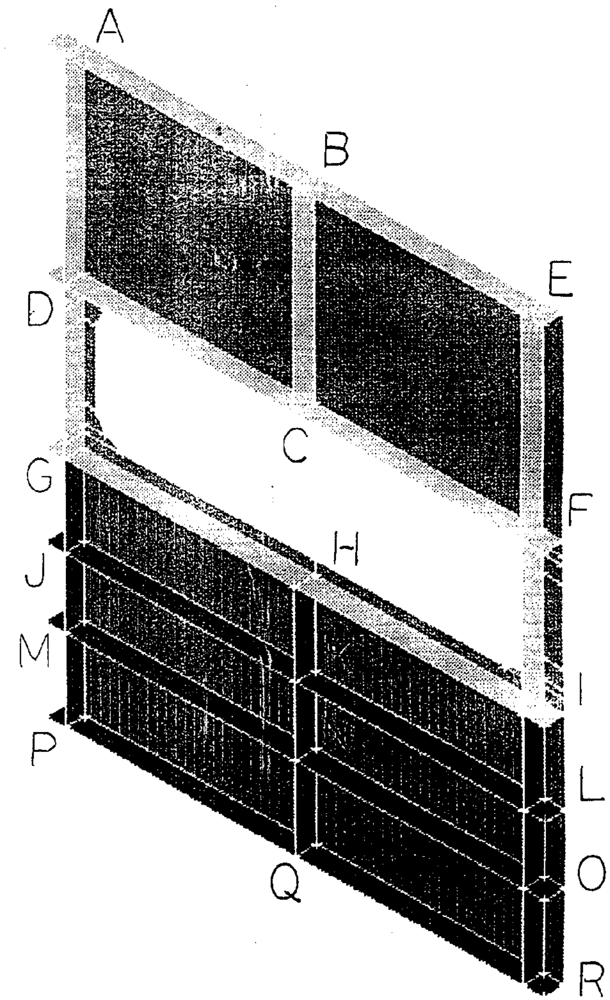
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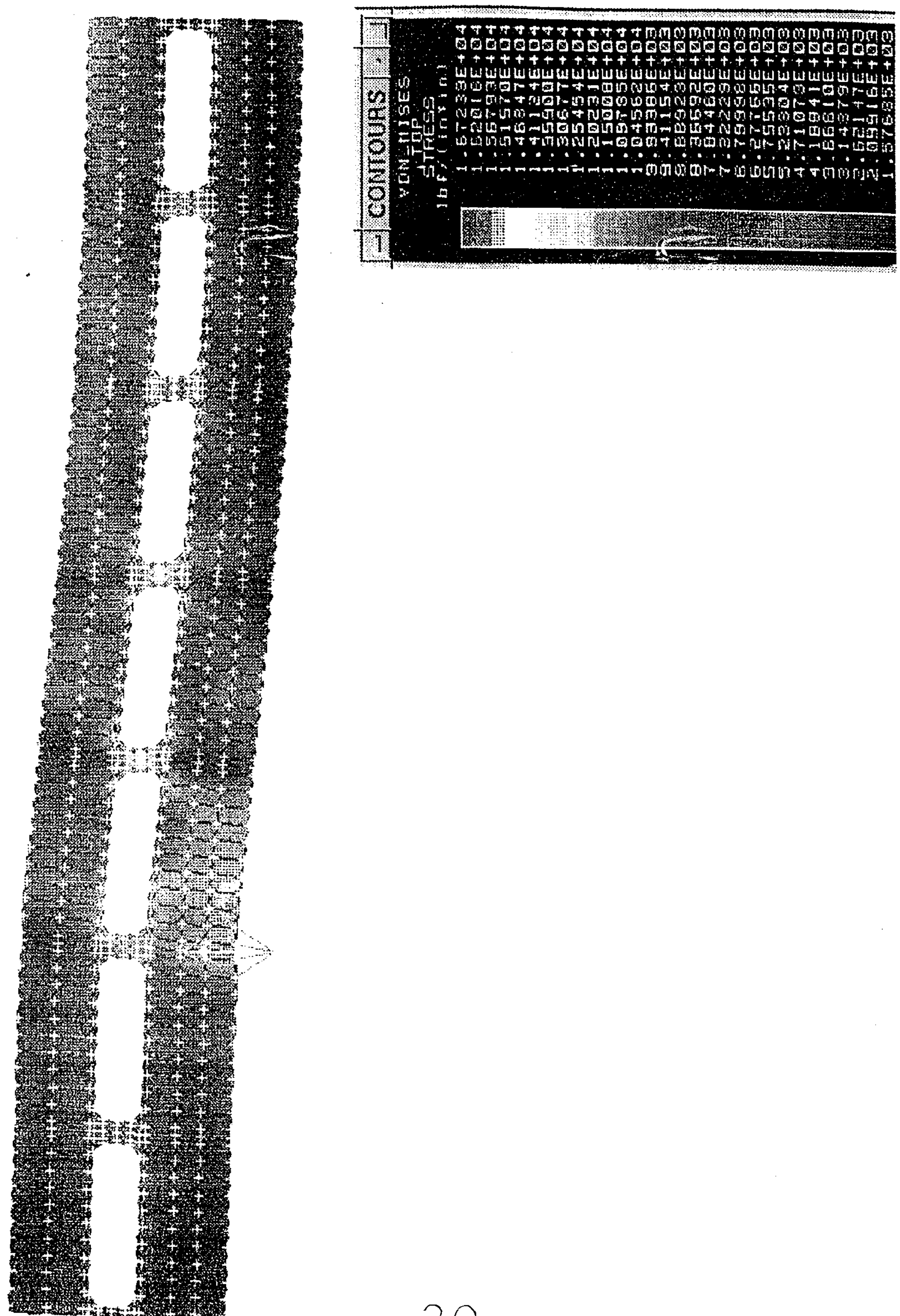
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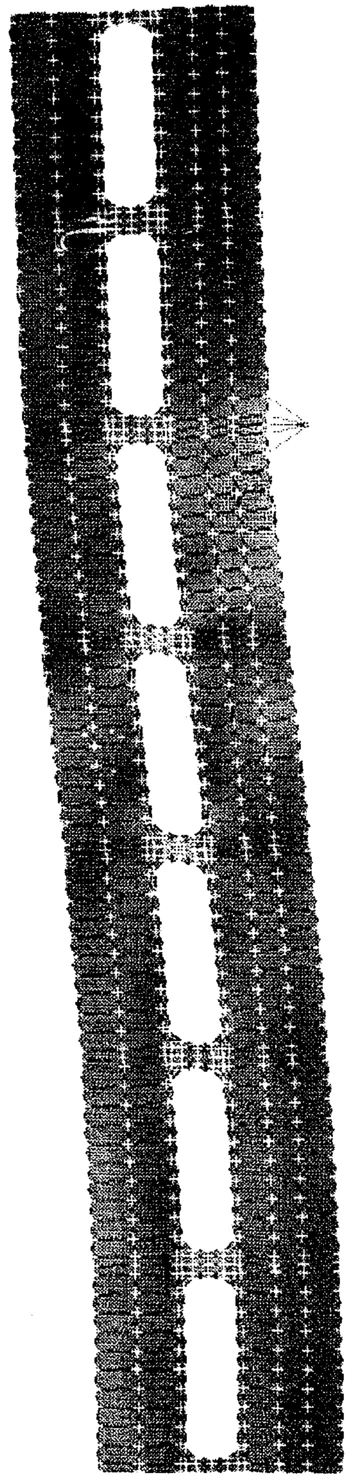

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