SELF-EXCITED OSCILLATION OF A 165 FOOT WATER TOWER

By

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ABSTRACT:

When a new 165 foot high water tank was being filled for the first time, a large amplitude oscillation of the entire structure occurred and forced an immediate shutdown of the filling process. Analysis of the problem, using a combination of math modeling and experimental measurements, revealed the cause to be an unstable coupled fluid/structural response that was being driven by the incoming water. Design modifications were needed to eliminate the coupling and allow the tank to be put into service.

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INTRODUCTION:

A water storage tower newly constructed on a chemical plant consisted of a large tank on a single support divided into three concentric compartments - each containing water intended for a different use. See Figures 1 and 2. All three compartments were connected by sequentially stepped weirs such that overflow water could flow radially from the inner compartment into the intermediate compartment, and then to the outer compartment, but not visa versa. The intermediate compartment was intended to be filled through the connecting weir by water from the inner compartment which was in turn filled by a pipe. The outer compartment was to be filled by its own pipe although it could also receive water from the intermediate compartment through the weir. During the initial filling, the procedure consisted of pumping water into the inner compartment and filling the intermediate and outer compartments via overflow through weirs. Normally, only the two inner compartments would be filled in this way. The outer compartment would be filled via its own pipe.

With the inner and intermediate compartments full and the outer compartment approximately three quarters full, the tower began to sway. The swaying became worse with time. The plant recognized this as a potentially dangerous situation and filling operations were suspended. The motion damped out when the filling process was halted. No measurement of the amplitude of the motion at the top of the tower was made and estimates of it varied from several inches to several feet. Personnel on the tower during this incident were alarmed and concerned for their safety. An attempt to resume filling resulted in more high amplitude swaying. It was then concluded that a serious problem existed and further efforts to fill were suspended.

PROBLEM DEFINITION:

The problem was tentatively identified as a sloshing phenomenon involving a self-excited vibration or dynamic instability. The first step in solving it was to devise an instability mechanism that explained the incident. A mechanism was postulated whereby sloshing taking place in the outer compartment could be coupled with vibration of the entire structure and an instability created. A back of the envelope calculation suggested the tower swaying natural frequency and the outer compartment sloshing natural frequency could have been approximately equal under the conditions existing when the swaying occurred. Based on this, a program of calculation and testing aimed at better defining the phenomenon and formulating recommendations for a fix was undertaken.

DISCUSSION:

INSTABILITY MECHANISMS:

The mechanism for the instability is illustrated in Figure 3. An unstable coupling between sloshing and structural vibration can occur when an outer compartment is receiving overflow from an inner compartment through the weirs connecting the two. If this flow has an oscillatory component due to normal swaying of the tower in response to wind gusts, it will produce a sloshing motion in the outer tank. If the sloshing natural frequency of the outer tank is close to the tower natural frequency, there will be a highly efficient energy transfer and large sloshing amplitudes can occur. The forces from this sloshing act on the tank walls and if properly phased, can increase the swaying of the tower. This, in turn, increases the surging through the weir and thus the sloshing in the outer compartment. Energy is made available by the filling process as it provides a continuous supply of water to surge over the weir. If the energy transferred to the liquid in the outer compartment from the flow surging over the weir exceeds the energy lost to damping, the process feeds on itself and the tower swaying amplitude gets higher with each cycle until something occurs to reduce the efficiency of the energy transfer. For example, slosh wave breakup at large amplitudes causes increased energy dissipation and frequency detuning. Also, the structural and sloshing natural frequencies shift as the liquid depth increases, which can eliminate the instability.

DEVELOPMENT OF AN EXPRESSION DEFINING A STABILITY CRITERION:

A crude mathematical representation of the process was developed to aid in understanding the conditions under which an instability might occur, and to gain insight into how the problem could be solved. Appendix 1 outlines the development. The final equation is:

$$\frac{W_{in}}{W_{dis}} = \frac{\beta \gamma \varnothing_{12} \sqrt{2gh}\ Sin\left(\omega\sqrt{\frac{2h}{g}}\right)}{\omega^2\left[\eta_1 M_{s1} \varnothing_{12}^2 \eta_2 M_{s2}\right]}$$

It shows the ratio of energy input to the energy dissipated in a cycle of vibration. If this ratio is greater than one, it implies that if the system is perturbed, energy will be input at a faster rate than it can be dissipated, and the amplitude will grow from one cycle to the next. If the energy ratio is less than one, energy will be dissipated faster than it is input; the system will then be stable and the vibration will decay with time.

It is difficult to assign numerical values to many of the terms in the equation. However, examining the terms still gives insight into how the system could be modified to produce the greatest benefit with respect to stability. For example:

- Increasing damping (η_1) or (η_2) in either tank by the addition of slosh baffles would be beneficial.

- Reducing the efficiency factor (γ) for converting the overflowing water into sloshing would be beneficial. This could be accomplished by leading the weir overflow through a pipe to the bottom of the lower tank, or cascading it down through a series of baffles, or diverting it back against the tank wall (much like beer is poured down the side of the glass to reduce the head).

- Reducing the free fall height (h) by cascading the overflow through baffles would be beneficial.

- Reducing the coupling between the two slosh modes ($\varnothing_{12}$) by detuning the natural frequencies would be highly beneficial. However, this would be rather difficult to accomplish since changing the frequencies would require major dimensional or structural changes, such as the addition of guy wires to the tank structure.

STRATEGY:

The strategy of the analysis and testing phase was to determine if the proposed mechanism was correct. If so, an approach to an effective fix could be developed. The plan for the analysis was to determine the tower natural frequency and the sloshing natural frequencies corresponding to a range of liquid levels. Any conditions under which a structural natural frequency of the tower and the fundamental sloshing frequency of one of the compartments were close together would be situations when the tower was vulnerable to instability. Any such conditions analytically identified could then be verified by testing. This could be tricky since this kind of phenomenon must be "triggered" to occur. This is usually done by gusting wind loads. Consequently, filling the tower on a calm day when no measurable wind swaying occurs may be uneventful, while filling during gusting winds might be disastrous. Unfortunately, the precise amount of wind gusting and resulting swaying necessary to trigger the instability is unknown and very difficult to determine. If it fails to happen during one test, there is no guaranty it will not happen at a later time under different wind conditions. On the other hand, if the phenomenon could be demonstrated at predicted fill levels, we could assure ourselves that we did indeed understand the physical process. We could then decide on modifications to prevent the instability mechanism from working.

SLOSHING NATURAL FREQUENCY DETERMINATION:

The sloshing natural frequencies were determined from published equations and charts. Some inaccuracy resulted from the fact that the available methods were for right cylindrical shapes rather than the shapes having irregular conical bottoms in the tower. However, as the liquid depth in the tower increases, the shape of the bottoms of the compartments have less and less influence on the results. Also, the depth in the conical compartment bottoms rises comparatively fast during filling leaving little time for unstable motion to grow in amplitude. Accuracy of the available information in the range where the problem was experienced was judged to be adequate for our purposes. A series of calculations were made to predict the sloshing natural frequencies of the outer water compartment with varying depths in it. A similar series of calculations was made for the Fire Water compartment. Results are shown in Figure 4.

TOWER NATURAL FREQUENCY DETERMINATION:

The fundamental natural frequency of the water tower was calculated using MSC/PAL, and a relatively simple finite element model. The only difficulty with the model was in knowing the appropriate boundary condition at the base, where the support is built into the ground. To account for the base flexibility, a rotational spring was used in the model. The spring constant was to be determined from experimental measurements.

For the measurements program, two strain gages were installed at the thirty foot level in the tower's stem. They were located ninety degrees apart around the circumference of the stem and aligned with its centerline. This allowed measurement of axial strains in the stem due to axial (gravity) loads and bending (swaying) in any direction. Data from these gages was obtained on a relatively windy day when the tower was empty. The wind load caused the tower to sway at its fundamental natural frequency (empty) which was determined from the measured strains as .73 Hz. See Figure 5. A value for the rotational spring in the finite element model was selected that predicted the measured natural frequency for the case of no water in the tower. This spring stiffness was then used for a series of calculations having varying amounts of water in the tower.

PREDICTING VULNERABILITY:

The highest fundamental sloshing frequency to be expected in either outer compartment in the tower is approximately .25 Hz and occurs when the compartment is nearly full. The structural natural frequency of the tower only approaches this value when it is nearly full. Since the structural natural frequency reduces as water is added (increasing tower mass) and sloshing natural frequency increases as water is added (increased depth), it was clear that the conditions of concern were when two compartments were full and the third was filling. **Since only two of the three compartments could be filled by weirs, only these two conditions needed to be investigated for this mechanism.**

The results of the sloshing and mechanical natural frequency calculations for filling the intermediate and outer compartments when the other two compartments are full are plotted in Figures 6 and 7. Note that in Figure 7, the structural and sloshing natural frequencies converge as the water depth in the outer compartment increases. At a depth of 25 ft, they are within .02 Hz of one another. This is the approximate depth where the instability was originally experienced. Figure 6 shows the two frequency curves crossing at an approximate depth of 22 ft. in the intermediate compartment when the outer and inner compartments are full. Testing confirmed that **an instability does exist under those conditions** and can be triggered by wind loads. It is interesting to note that on a very calm day, the test was rerun searching for this instability and it was not detected during the filling process.

TESTING:

Testing was done by monitoring strain gage output during filling for several combinations of full and empty compartments. The presence of an instability was detected when the oscillations of the strain signals were observed to grow with each cycle. Motion was not allowed to increase to the point of significant stress in the tower before the test was stopped. Calculations had predicted instability when filling the outer compartment when the inner and intermediate compartments were already full and when filling the intermediate compartment when the outer and inner compartments were already full. The first had been experienced during initial filling without instrumentation installed. The second was verified by testing. A sample of the data is shown on Figure 8.

CONCLUSIONS:

The combined analysis and test program successfully identified the cause of the water tower oscillation as an instability due to water flow over the wiers, and it indicated the conditions under which the instability might occur. A number of modifications were identified to prevent recurrence of the problem. These consisted of changes in the filling procedures to avoid the critical conditions, and of modifications to the structure to close off some wiers, and add catch pans and downcomers to the others. The downcomers convey the overflow from one compartment to the bottom of the adjoining compartment, where it does not impact the surface and induce sloshing.

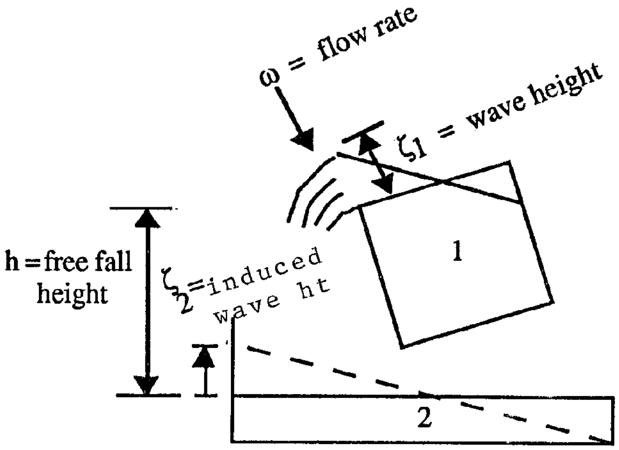
The tower has been commissioned and in operation now for approximately two years with no incidents.

ACKNOWLEDGMENT:

The assistance of J. G. Shukitt with the initial evaluation, and J. H. Nichols in making the measurements is gratefully acknowledged.

APPENDIX 1

MATH MODEL DEVELOPMENT



Consider two tanks with sloshing liquids arranged so that the overflow from the inner flows into the outer.

Assume the inner tank is sloshing at an amplitude ζ_1:

$$\zeta_1 = \zeta_{01} \text{ Sin } \omega t$$

and the outer tank is sloshing at an amplitude

$$\zeta_2 = \zeta_{02} \text{ Sin } \omega t$$

Assume the outflow w_1 from the inner tank is proportional to its slosh wave height:

$$\begin{aligned} w_1 &= \beta \zeta_1 = \beta \zeta_{01} \text{ Sin } \omega t \\ &= w_1 (t) \end{aligned}$$

The flow into the outer tank w_2 is the same as the outflow w_1, but delayed by the time to fall:

$$w_2 = w_1\,(t - \tau)$$

$$= \beta\,\zeta_{01}\,\text{Sin}\,\omega(t - \tau)$$

where τ is the time to fall the distance h between inner tank surface and outer tank surface.

The impact velocity V due to free fall with gravitational acceleration g is:

$$V = g\tau$$

The distance covered in time τ is

$$\frac{1}{2}V\tau = \frac{1}{2}\,g\tau^2 = h$$

Thus

$$\tau = \sqrt{\frac{2h}{g}}$$

The flow into the outer tank creates a force F on the liquid surface of the form

$$F = w_2\,V$$

Only a portion of this force goes into exciting slosh in the outer tank. Assume an efficiency factor (γ) to account for the loss. The exciting force is then

$$F = -\gamma\,w_2\,V$$

$$= -\gamma\,w_2\,\sqrt{2gh}$$

where the negative sign accounts for the downward direction of the force. Substituting for w_2:

$$F = -\gamma \beta \zeta_{01} \text{ Sin } \omega(t - \tau) \sqrt{2gh}$$

The work done by a sinusoidal force on an oscillating system is given by Den Hartog (Mechanical Vibrations). For a force

$$P = P_o \text{ Sin } (\omega t + \varnothing)$$

the work done per cycle is

$$W_{in} = \pi P_o X_o \text{ Sin } \varnothing$$

Therefore, for an exciting force of

$$F = [-\gamma \beta \zeta_{01} \sqrt{2gh}] \text{ Sin } (\omega t + (-\omega\tau))$$

the work done per cycle will be

$$W_{in} = \pi (-\gamma \beta \zeta_{01} \sqrt{2gh}) \zeta_{02} \text{ Sin } (-\omega\tau)$$

$$\boxed{W_{in} = \left(\pi \beta \sqrt{2g}\right) \gamma \sqrt{h}\, \zeta_{01}\, \zeta_{02}\ \text{Sin}\left(\omega \sqrt{\frac{2h}{g}}\right)}$$

Energy is put into an oscillating system only by the component of force that is in phase with velocity or 90 degrees out of phase with the displacement as shown by the sine term in the equation.

$$\text{Sin } \left(\omega\sqrt{\frac{2h}{g}}\right) \quad \text{reaches a maximum for } \omega\sqrt{\frac{2h}{g}} = \frac{\pi}{2}$$

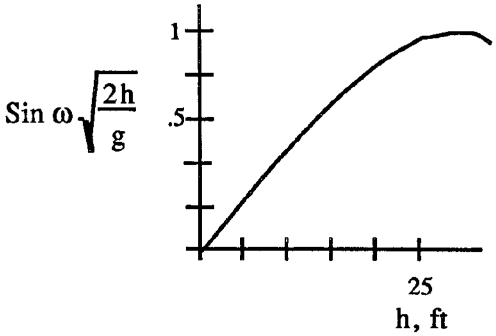
$$\text{or } h_{max} = \frac{\pi^2 g}{8\omega^2}$$

Thus if the height difference between the two tanks was equal to h_{max}, the conditions for pumping the maximum energy into the system would exist.

As an aside, for a natural frequency of 0.2 Hz or 1.26 rad/sec, which corresponds to that of the oscillating tank,

$$h_{max} = \frac{\pi^2 (32)}{8 (1.26)^2} = 25 \text{ ft}$$

This is a height that is passed through early in the filling process. Note that although 25 ft represents the worst case, some energy would be pumped into the tank for any height up to 50 ft.



If energy input exceeds energy dissipated over a cycle, the amplitude will grow. Thus the next step is to estimate the energy dissipated.

For a system with a damping factor η, the energy dissipated over a cycle W_{dis} is related to the maximum stored energy, U, by:

$$W_{dis} = 2\pi \eta U$$

Since sloshing in both inner and outer tanks dissipates energy, the total energy dissipated in sloshing is

$$W_{dis} = 2\pi \eta_1 U_1 + 2\pi \eta_2 U_2$$

$$= 2\pi \left[\eta_1 \left(\frac{1}{2}\right) M_{s1} \zeta_{01}^2 + \eta_2 \left(\frac{1}{2}\right) M_{s2} \zeta_{02}^2 \right]$$

$$= \pi \left[\eta_1 M_{s1} \omega^2 \zeta_{01}^2 + \eta_2 M_{s2} \omega^2 \zeta_{02}^2 \right]$$

$$\boxed{W_{dis} = \pi \omega^2 \left[\eta_1 M_{s1} \zeta_{01}^2 + \eta_2 M_{s2} \zeta_{02}^2 \right]}$$

where M_{s1} and M_{s2} represent the slosh masses in the two tanks. (The energy dissipated in the structural components of the tank is ignored for this simplified analysis).

Consider the ratio of input energy to dissipated energy:

$$\frac{W_{in}}{W_{dis}} = \frac{\pi \beta \sqrt{2g}\, \gamma \sqrt{h}\, \zeta_{01}\, \zeta_{02}\, \mathrm{Sin}\left(\omega \sqrt{\frac{2h}{g}}\right)}{\pi \omega^2 \left[\eta_1 M_{s1} \zeta_{01}^2 + \eta_2 M_{s2} \zeta_{02}^2 \right]}$$

Since the vibration is occurring in a normal mode, the wave amplitudes in the two tanks are related by the model shape:

$$\zeta_{01} = \emptyset_{12}\, \zeta_{02}$$

Substituting:

$$\boxed{\frac{W_{in}}{W_{dis}} = \frac{\beta \gamma \emptyset_{12} \sqrt{2gh}\, Sin\left(\omega \sqrt{\frac{2h}{g}}\right)}{\omega^2 \left[\eta_1 M_{s1} \emptyset_{12}^2 + \eta_2 M_{s2} \right]}}$$

If W_{in}/W_{dis} is greater than one, the system will be taking in energy faster than it can dissipate energy, and the amplitude will increase with time. If the ratio is less than one, the vibration will decay with time.

Figure 1
Water Tower Configuration

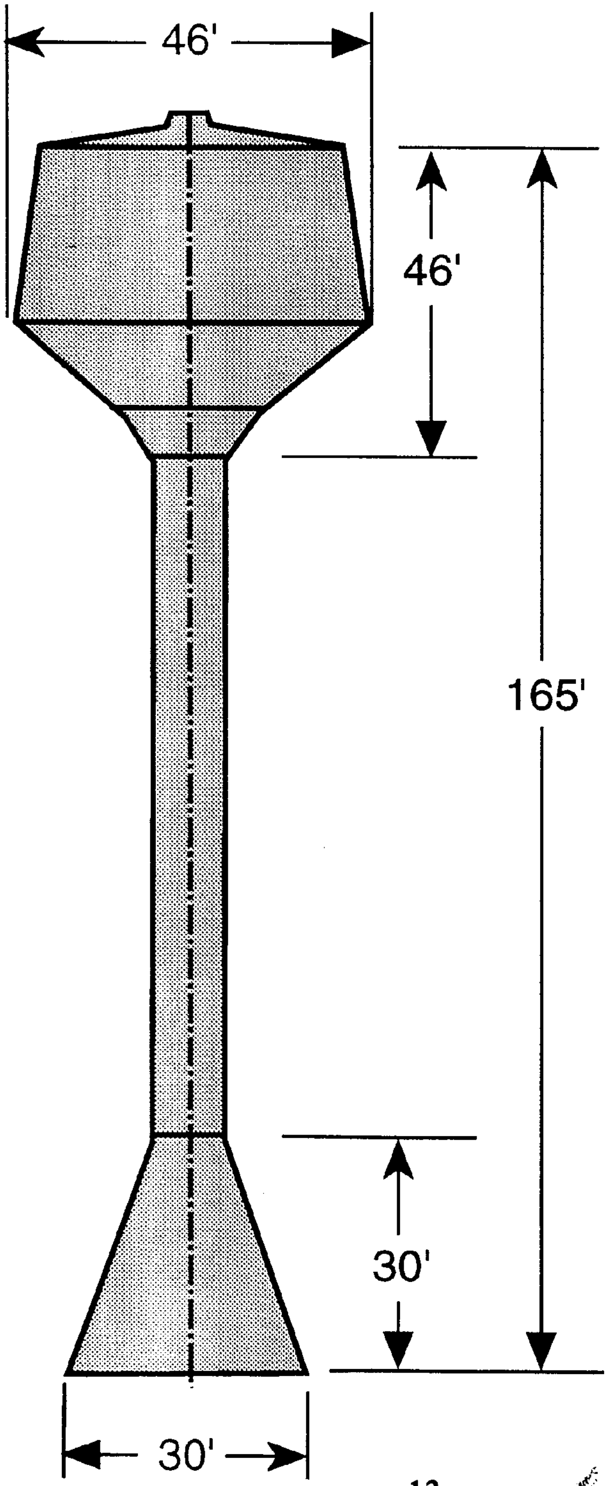


Figure 2
Tri-Compartment Tank

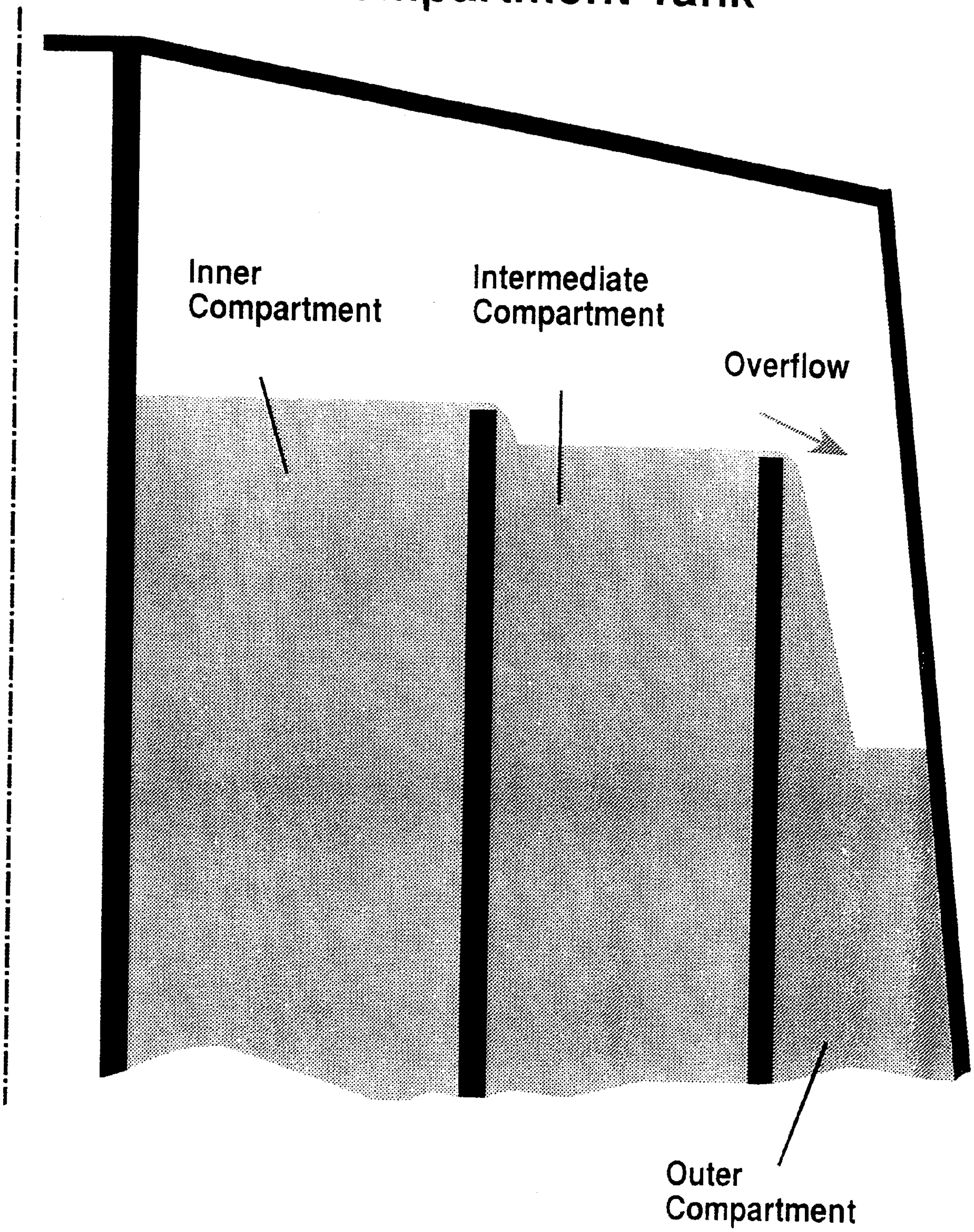
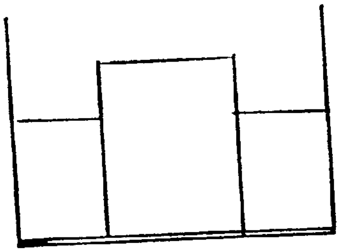
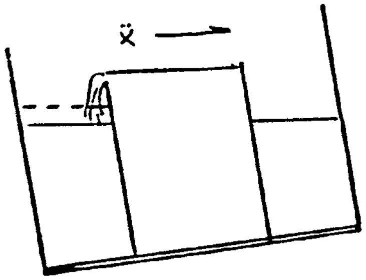


FIGURE 3
INSTABILITY MECHANISM



- o tank at rest
- o center compt full
- o outer compt partially full



- o tank disturbed by wind gust or wier flow unbalance begins small oscillation at natural frequency
- o wier overflows causing liquid to be added to outer tank unbalancing the free surface
- o outer tank flow moves to restore equilibrium

- o tank moves in other direction
- o wier overflows on other side creating liquid unbalance in other direction

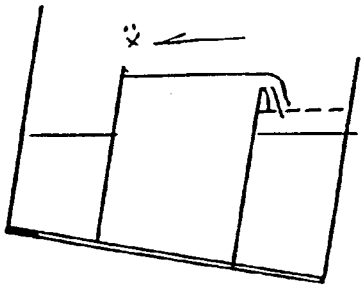


FIGURE 4

SLOSH NATURAL FREQUENCIES

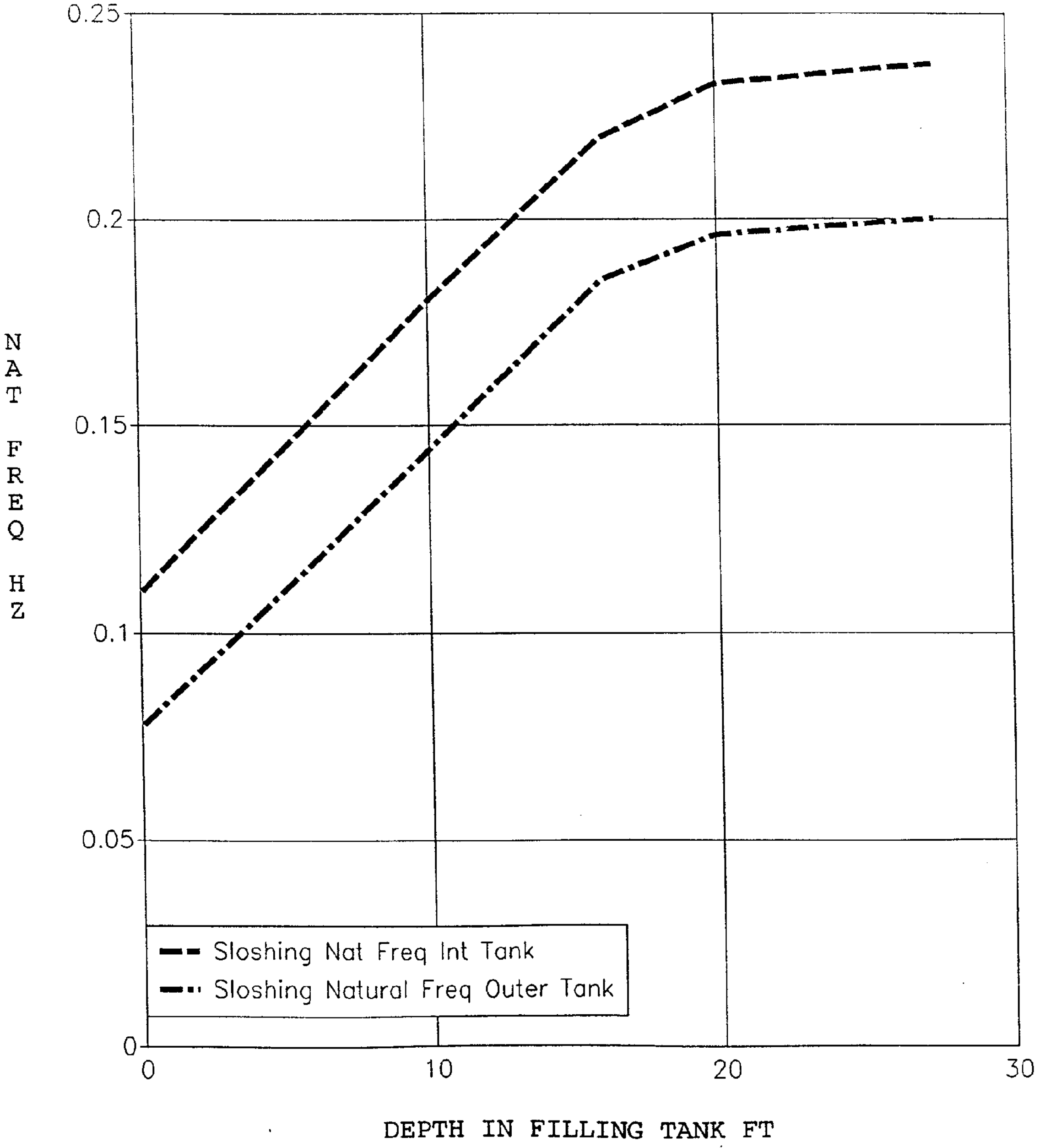


FIGURE 5

STRAIN RESPONSE TO WIND GUSTS

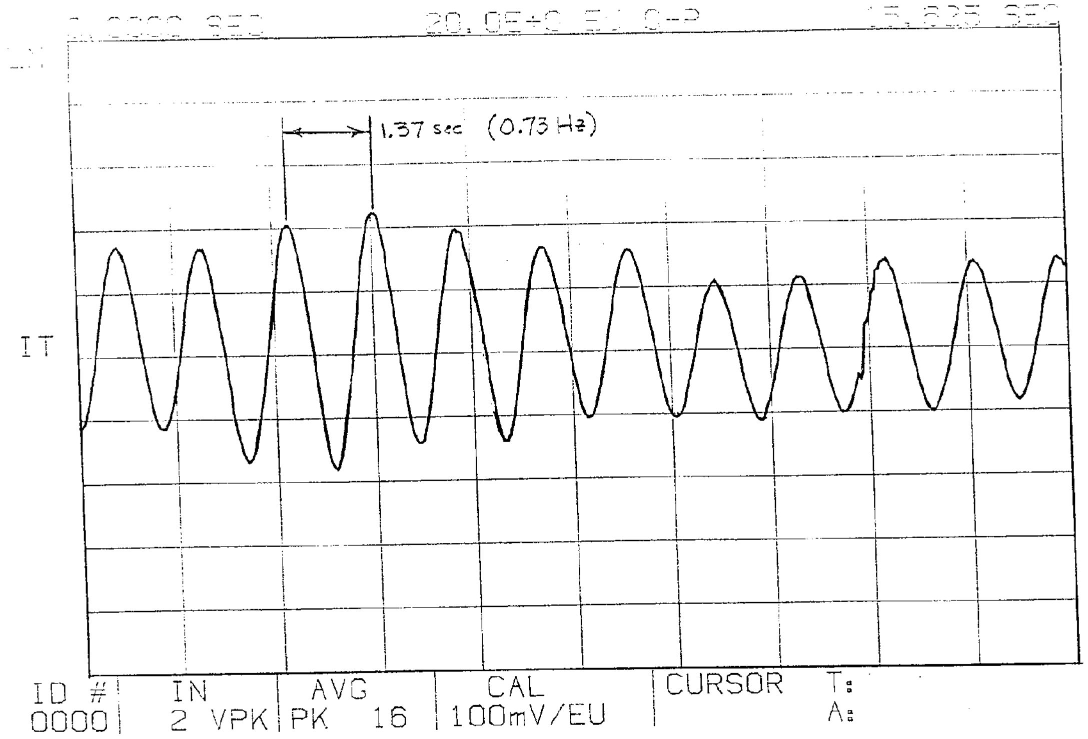


FIGURE 6

SLOSH AND STRUCTURAL NATURAL FREQUENCIES

INNER AND OUTER TANKS FULL

FILLING INTERMEDIATE TANK

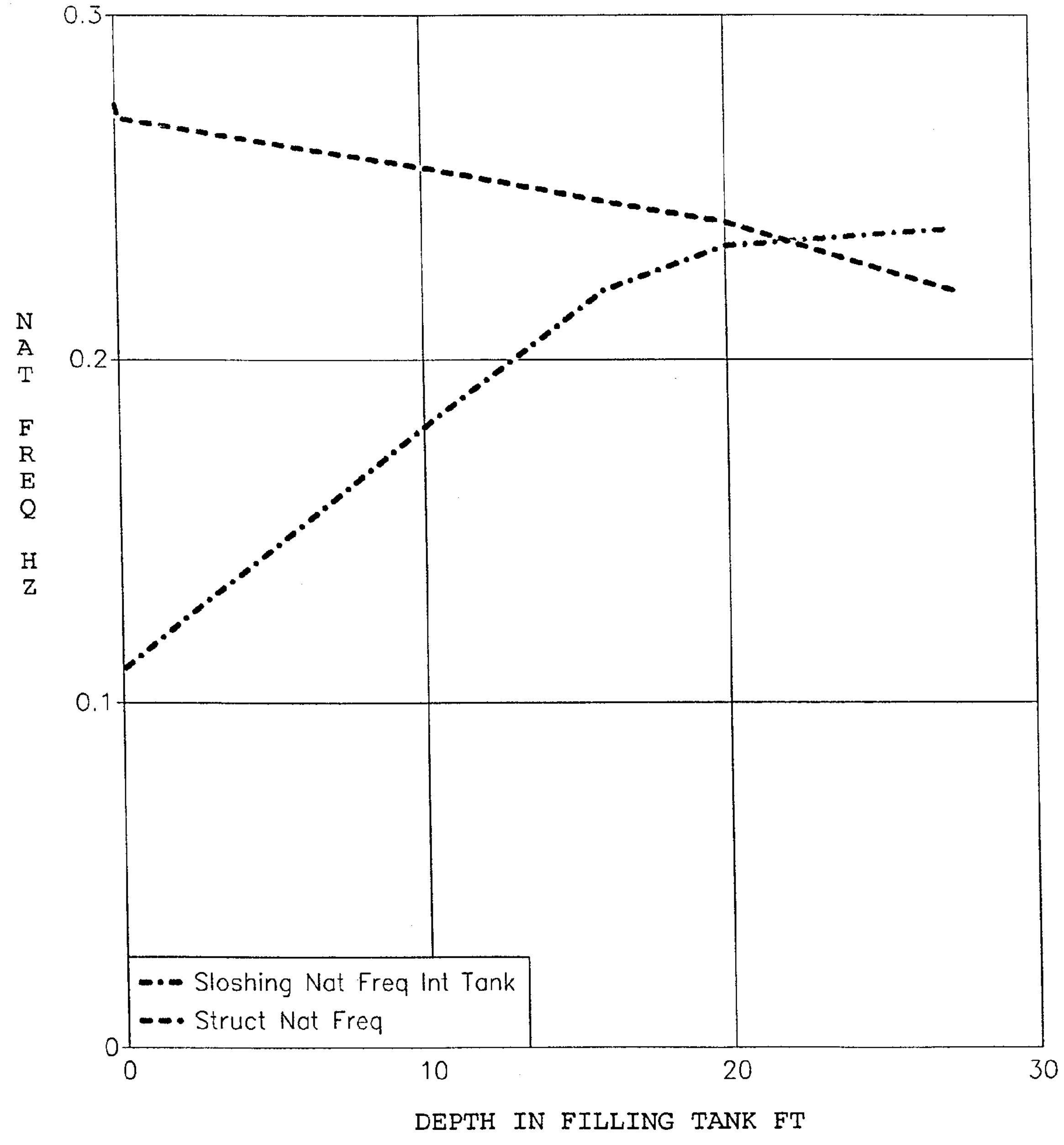


FIGURE 7

SLOSH AND STRUCTURAL NATURAL FREQUENCIES

INNER AND INTERMEDIATE TANKS FULL

FILLING OUTER TANK

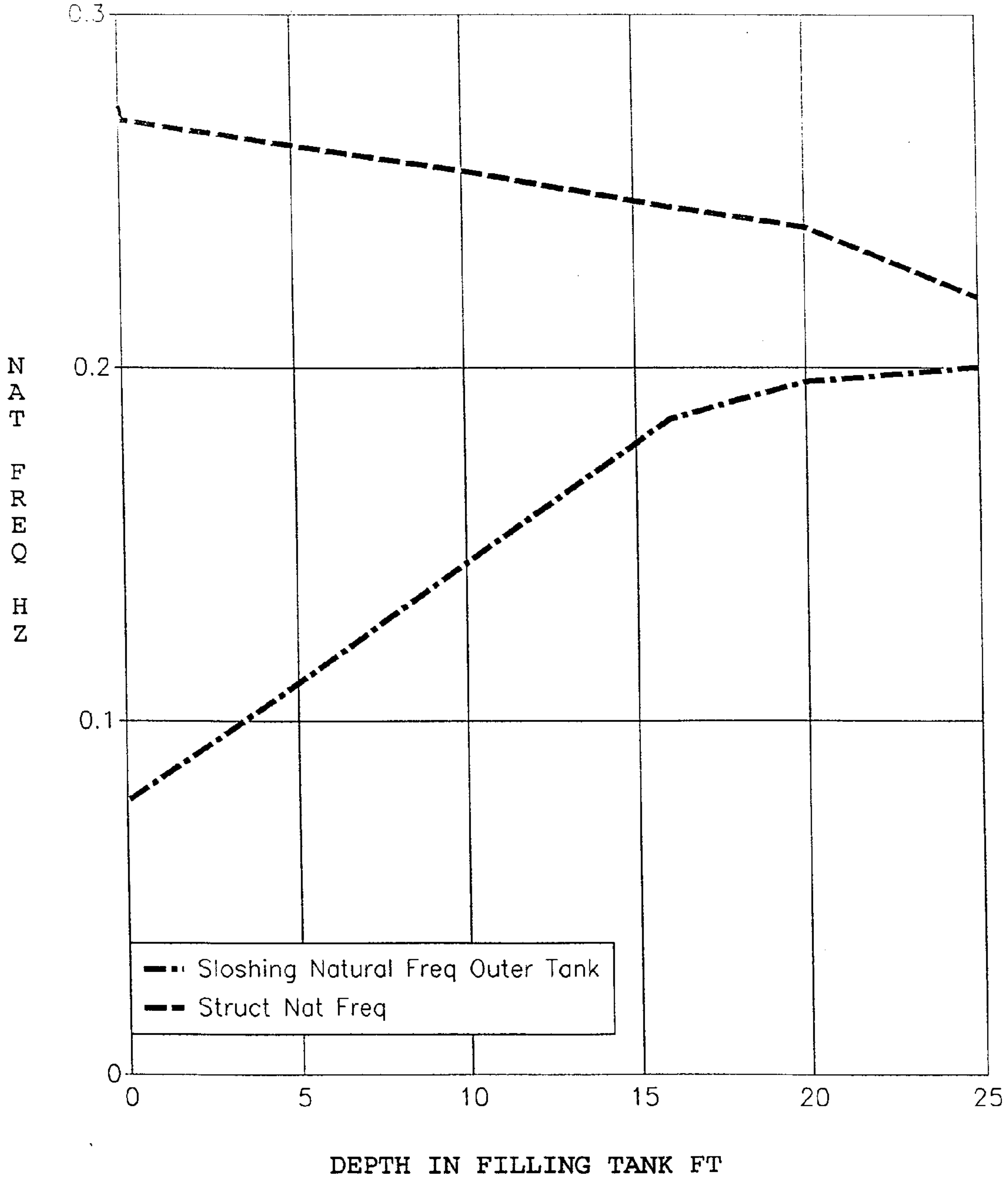


FIGURE 8

STRAIN RESPONSE DURING FILLING OF INTERMEDIATE TANK

OUTER & INNER TANKS FULL