DMAP Alters to Add Differential Stiffness and Follower Force Matrices to MSC/NASTRAN Linear Solutions

by

David F. Bella
Senior Development Engineer
The MacNeal–Schwendler Corporation

ABSTRACT

This paper describes a DMAP procedure to add differential stiffness and follower force matrices to MSC/NASTRAN linear analysis solution sequences. Differential stiffness results from internal element forces due to applied loads. It is used in buckling analysis to determine buckling loads and also in geometric nonlinear analysis to more efficiently converge to correct solutions. Follower forces arise from loads which are dependent on a structures geometry. As a structure deforms, follower forces change in their magnitude or direction. This displacement–dependent change of loading can be characterized as a stiffness term in linear analysis. Inclusion of the differential stiffness and follower force matrices produce a corrected tangent stiffness matrices for linear analysis.

Introduction

The MSC/NASTRAN geometric nonlinear solutions converge to correct displacements, even though the tangent stiffness matrix does not include the follower force effect, because the solution algorithm updates the stiffness and loading based on the deformed geometry. If a structure is loaded by a constant force, but the deformations are assumed to be small, then a linear solution is possible. In this situation, a correct tangent stiffness matrix is required. This tangent stiffness not only contains the liner and differential stiffnesses, but also the effect of possible follower forces.

Differential Stiffness

To understand the source of differential stiffness, consider the standard static equilibrium equation for a single finite element.

$$K\delta u = P \qquad (1)$$

where K is the element stiffness in global coordinates
δu is the displacement in global coordinates
P is the load in global coordinates

The deformation, u, produces element reaction forces, F, which balance the applied load , P. The stiffness, K, can be viewed as the change in the element reaction forces with respect to a change in displacement.

$$K = \frac{\partial F}{\partial u} = \frac{\partial(TF_e)}{\partial u} \qquad (2)$$

$$K = T\left(\frac{\partial F_e}{\partial u}\right) + \left(\frac{\partial T}{\partial u}\right)F_e \qquad (3)$$

$$K = T\left(\frac{\partial F_e}{\partial u_e}\right)\left(\frac{\partial u_e}{\partial u}\right) + \left(\frac{\partial T}{\partial u}\right)F_e \qquad (4)$$

where F is the internal element force in global coordinates
T is a transformation from element to global coordinates
F_e is the internal element force in element coordinates
u_e is the displacement in element coordinates

For a linear material the term $\partial F_e / \partial u_e$ is constant, the element forces are linearly related to the element displacements. The terms T and $\partial u_e / \partial u$ are dependent on the orientation of the element coordinate system with respect to the global coordinate system and for small displacement problems are also assumed to be constant. In large displacement problems, these terms change as the element rotates with respect to the global coordinate system and must be updated as the structure deforms. The three terms, $\partial F_e/\partial u_e$, T, and $\partial u_e / \partial u$, are used to calculate the elemental *linear* stiffness matrix in global coordinates.

The second part of the stiffness calculation, $(\partial T / \partial u)F_e$, is usually ignored in standard linear analysis. It arises from small changes in the element orientation with respect to the applied load. For example, consider a bar under axial and transverse loading.

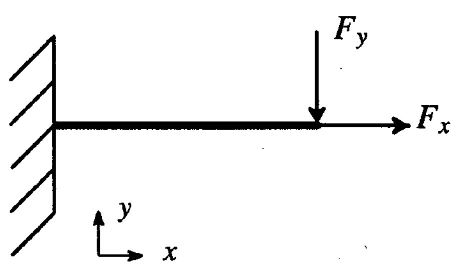

Figure 1. Cantilevered bar with axial and transverse loading

In standard linear analysis, the y–directed load is reacted by the bar bending and shear forces. The x–directed load is reacted by the bar axial forces. The calculated y displacement is independent of the x–directed load. Likewise, the calculated x displacement is independent of the y directed load. On closer inspection, one would find that as the free end of the bar rotates with respect to the applied loads, a component of the bar axial force reacts against the y–directed load. Likewise, a component of the bar bending and shear react against the x–directed load.

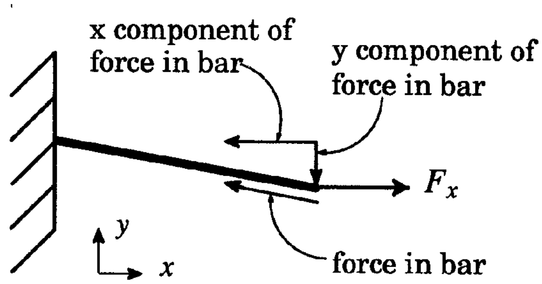

Figure 2. Bar forces due to axial loading

This phenomena results in a stiffness coupling which is not taken into account by standard linear analysis. This effect is due to the $F_e(\partial T / \partial u)$ term in Equation () and is referred to as the *differential* stiffness. Its calculation results knowledge of the element force F_e and the transformation matrix T. The sum of the linear and differential stiffness matrices is often referred to as the *tangent* stiffness.

$$K_{tangent} = K_{linear} + K_{differential} \qquad (5)$$

where

$$K_{linear} = T\left(\frac{\partial F_e}{\partial u_e}\right)\left(\frac{\partial u_e}{\partial u}\right) \qquad (6)$$

$$K_{differential} = \left(\frac{\partial T}{\partial u}\right)F_e \qquad (7)$$

Follower Forces

The previous section addressed changes in the standard linear stiffness matrix due to small deformations of the structure. In the analysis, the load, P, was assumed to be constant. If the loading is not constant but dependent on the shape or orientation of the element, then Equation (1) would contain a displacement–dependent load.

$$K\delta u = P(\delta u) \tag{8}$$

The displacement–dependent load can be represented by a Taylor series expansion.

$$P(\delta u) = P_0 + \frac{\partial P}{\partial u}\delta u + \frac{\partial^2 P}{\partial u^2}\delta u^2 + \cdots \tag{9}$$

If the above expansion is terminated at the linear term, then Equation (8) can be rewritten as

$$K\delta u = P_0 + \frac{\partial P}{\partial u}\delta u \tag{10}$$

or

$$\left(K - \frac{\partial P}{\partial u}\right)\delta u = P_0 \tag{11}$$

The term $(\partial P / \partial u)$ is the *follower force* term. This term, along with the differential stiffness term, is required for the analysis for any structure which is subjected to a load which is dependent on the shape or orientation of the structure.

$$K_{total} = K_{linear} + K_{differential} + K_{follower} \tag{12}$$

where K_{linear} and $K_{differential}$ are given be Equations (6) and (7) respectively and

$$K_{follower} = -\frac{\partial P}{\partial u} \tag{13}$$

DMAP Alters to Add Differential Stiffness and Follower Force Terms

The following alters were developed to add differential stiffness and follower force terms to standard MSC/NASTRAN linear analysis solution sequences. The differential stiffness used in the alters is the standard MSC/NASTRAN differential stiffness used for buckling and nonlinear analysis. The follower force, $-\delta P/\delta u$, is calculated by using a central difference calculation of the change in load due to a small change in model geometry. For each translation degree of freedom in the model, a follower force vector is calculated. These vectors are appended to form the follower force matrix.

These alters will be added to the sssalter library delivered with MSC/NASTRAN. The alters should only be used it there are true follower forces in the problem. Follower forces are: FORCE1, FORCE2, PLOADi, RFORCE, TEMP, and TEMPD.

follow_101.v68 This alter is for use in SOL 101 (linear statics) and will add the differential stiffness and follower force terms due to first specified loading to all superelement stiffnesses. This alter is intended to use with restarts into other solution sequences which would require the modified stiffness matrices, such as normal modes or frequency response analysis. Boundary conditions on restarts may be different than those used in the linear static analysis.

follow_105.v68 This alter is for use in SOL 105 (linear buckling) and will add the follower force terms to the differential stiffness for use in the buckling load calculation.

follow_106r.v68 This alter is for use when restarting from SOL 106 (nonlinear statics) with param nmloop. Use of param nmloop on a restart from SOL 106 causes the calculation of the residual structure linear and differential stiffnesses to use the deformation stored on the database for loopid nmloop. Use of this alter will also add the follower force terms to the stiffness matrix. The user is required to specify the load set corresponding to the nmloop load in the case control section. The load is specified with a LOAD case control command above any subcase entries.

The follower force terms used in the above alters are calculated by performing a finite difference at each translational degree of freedom in the G–set. This is done in a DMAP loop and is not very efficient.

Example Problems

follow_101.v68 This alter was used to determine the normal modes of free–free unit length cylinder under pressure loading. The cylinder properties are:

Elastic Modulus	3.0E+7
Poisson's Ratio	0.33
Cylinder Radius	5.0
Cylinder Thickness	0.1
Mass Density	4.28

The open ends of the cylinder were constrained such that there was no motion in the z–direction (U_z=0, θ_x=0, and θ_y=0) .

The following table displays the results of including no pressure effects, differential stiffness only, and differential stiffness and follower forces.

Mode Number	No Preload	Preload with Differential Stiffness	Preload with Differential Stiffness and Follower Force
1	2.447E–6	1.673E–6	3.718E–4
2	2.339E–6	2.542E–6	1.510E–2
3	1.809E–6	3.435E+0	1.625E–2
4	1.387E+0	4.804E+0	5.497E+0
5	1.390E+0	4.811E+0	5.503E+0
6	3.928E+0	9.489E+0	9.980E+0
7	3.928E+0	9.489E+0	9.980E+0
8	7.522E+0	1.444E+1	1.480E+1
9	7.522E+0	1.446E+1	1.482E+1
10	1.216E+1	1.995E+1	2.023E+1

Table 1. Natural Frequencies of a Pressurized Free–Free Unit Length Cylinder (Hz)

The results with no preload provide a baseline for comparison. The model exhibits the three expected rigid–body modes and repeated flexible modes. With the inclusion of differential stiffness, the frequencies of the flexible modes increase as expected. In addition, there appears to be a new flexible mode, while only two rigid–body modes are calculated. Inspection of the mode shapes reveals that the new flexible mode is actually the rotational rigid–body mode. This result makes all modes calculated with the differential stiffness suspect. Adding the follower force to the differential stiffness, the three rigid–body modes are again obtained. Comparison with the frequencies calculated without the follower force shows a 15% change in the lowest flexible mode, with smaller changes in the higher frequency modes.

follow_105.v68 This alter was used to determine the buckling load for a circular ring under uniform external pressure. The ring properties are:

Elastic Modulus	7.38E+10
Poisson's Ratio	0.33
Radius of Ring	100
Radius of Ring Cross Section	1
Bending Inertia	0.7854
Torsional Inertia	1.5708

The magnitude of the external pressure that will buckle the ring is

$$q_{cr} = \frac{3EI}{R^3} \tag{14}$$

Evaluation of the above equation for the ring in the analysis results in a buckling load of 1.74E+5. Using SOL 105 without the alter, a buckling load of 2.32E+5 is calculated. The error in the result is 25%. The same problem run with the alter calculates a load of 1.79E+5. The error in the results with the alter is reduced to 3%.

follow_106r.v68 This alter was used to analyze Beck's problem: Determine the natural frequencies of a column under a compressive load which always acts tangent to the deflection curve at the top of the column. The column properties are:

Elastic Modulus	10.3E+6
Poisson's Ratio	0.33
Mass Density	2.6E–4
Length of Column	100.0
Bending Inertia	1.0
Cross–Section Area	2.0

At a critical load given by

$$P_{cr} = 2.008\frac{\pi^2 EI}{l^2} = 20412.7 \tag{15}$$

The first and second natural frequencies coincide at a value of

$$f_{1,2} = \frac{Pi}{2l^2}\sqrt[2]{\frac{1}{q}} = 22.15 \tag{16}$$

where q is the mass per unit length of the column.

Analyzing this problem by applying the load in SOL 106 and restarting for eigenvalue analysis in SOL 107 (Complex Eigenvalue Analysis) produced eigenfrequencies of 8.63 and 106.7. The error in these results are 157% and 79.2% respectively. Analyzing the same problem, but using the alter to introduce the follower forces, produced eigenfrequencies of 22.7 and 26.7. The error with the alter is reduced to 2.4% and 17% respectively.