EFFECTIVE MASS SENSITIVITY: A DMAP PROCEDURE.

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Abstract

To know how the effective modal mass changes, while the structure modifies in the design phase, is helpful in aerospace engineering. To this aim an MSC/NASTRAN DMAP procedure, for yielding the effective mass partial derivative with respect to the structural lumped mass, has been developed. To validate the procedure some outputs are compared with the standard Nastran ones for a spacecraft structural model. The results show a good agreement and qualify the procedure as very reliable.

Nomenclature

$\mathbf{I}$ identity matrix

L_i row vector of modal participation factor of mode i

m_i ith modal mass

$\mathbf{m}_{eff_i}$ effective modal mass vector of mode i

$\mathbf{m}_{eff_{i,k}}$ effective modal mass sensitivity vector of mode i and dof k

$\bar{\mathbf{m}}_{eff_{i,k}}$ approximation of $\mathbf{m}_{eff_{i,k}}$ (i.e., eigenvector variation neglected)

${}^c\mathbf{m}_{eff}$ cth component of effective mass vector of mode i

$\mathbf{M}$ mass matrix

$\mathbf{M}_{eff_i}$ effective modal mass matrix of mode i

$\mathbf{M}_{eff_{i,k}}$ effective modal mass sensitivity matrix of mode i and dof k

$\bar{\mathbf{M}}_{eff_{i,k}}$ approximation of $\mathbf{M}_{eff_{i,k}}$ (i.e., eigenvector variation neglected)

M_k kth diagonal element of the lumped mass matrix $\mathbf{M}$

$\mathcal{M}_h$ physical mass in grid h

$\mathcal{M}_{hj}$ scalar masses of grid h in direction x_j

N total number of modes (complete modal model)

N_{mod} number of modes retained in the calculation

$\bar{r}$ number of rigid body modes

$T_2 = \left(\boldsymbol{\phi}_i^T \frac{\partial \mathbf{M}}{\partial M_k} \boldsymbol{\Phi}_R\right)$

$T_3 = \left(\frac{\partial \boldsymbol{\phi}_i^T}{\partial M_k} \mathbf{M} \boldsymbol{\Phi}_R\right)$

γ_{ir} eigenvalue function defined in Eq. 14

$\boldsymbol{\Gamma}$ matrix whose generic element is γ_{ir} (Eq. 17)

δ_{ir} Kronecker delta

λ_i ith eigenvalue

$\boldsymbol{\Lambda}$ matrix defined in Eq. 18

$\boldsymbol{\Lambda}_{diag}$ eigenvalue matrix

$\boldsymbol{\phi}_i$ ith eigenvector

${}^k\boldsymbol{\phi}_i$ kth component of eigenvector $\boldsymbol{\phi}_i$

$\{{}^k\boldsymbol{\phi}_i\}$ matrix 1×1 containing ${}^k\boldsymbol{\phi}_i$

$\boldsymbol{\Phi}_R$ matrix of rigid body modes

$\boldsymbol{\Phi}$ eigenvector matrix

Other Symbols

$\oplus$ element by element division operator

$\otimes$ element by element multiplication operator

${}^k[\ ]$ kth row of matrix $[\ ]$

$([\ ])_k$ kth column of matrix $[\ ]$

$\star_{,k} = \partial \star / \partial M_k$

1 Introduction

Aerospace engineers make wide use of the effective modal mass concept in structural spacecraft design. The effective modal mass, as known, represents the partecipation of an elastic mode to the reaction at the junction and therefore the knowledge of such parameter is useful for the dynamic behaviour analysis of satellites and aerospace substructures coupled with the lancheur. For example the most popular methods to choose a set of target modes to build a truncated model of Space Station payload are based on the values of the effective modal mass with respect to the rigid mass [1].

In the preliminary phase of the spacecraft design, several modifications occur in the initial configuration (which usually, as in every engineering design, is similar to that successfully employed in a previous mission). So it becomes useful to have a first order prediction of the new values of the effective modal masses of the modified structure. One specific problem is to find a suitable location for the payloads of a new mission of a spacecraft of which the dynamic model has been previously built and valitated, i.e. to choose the payload distribution in such way that the effective modal masses are similary shared and the dynamc model is still valid [2].

To this end we have studied and developed an analytical procedure, coded using the DMAP MSC/NASTRAN language, for a general tridimensional FE model, that yields the sensitivities of the effective modal masses for the variation of a non structural mass lumped in the grids.

2 Analytical background

For the sake of clarity, we recall, following [2], the main mathematical relations involved in our procedure. The effective modal mass of the ith elastic mode is [3]:

$$\mathbf{M}_{eff_i} = L_i^T m_i^{-1} L_i \tag{1}$$

with (for modal mass normalization):

$$m_i = \boldsymbol{\phi}_i^T \mathbf{M} \boldsymbol{\phi}_i = 1 \tag{2}$$

and

$$L_i = \boldsymbol{\phi}_i^T \mathbf{M} \boldsymbol{\Phi}_R \tag{3}$$

where $\boldsymbol{\phi}_i$ is the ith elastic mode of the restrained structure, M is the mass matrix, $\boldsymbol{\Phi}_R$ is the matrix containing the rigid body modes (we consider the junction isostatic or perfectly rigid [3]) and consequently L_i is a row vector containing $\bar{r}$ modal participation coefficients to the rigid motion of the ith elastic mode. $\mathbf{M}_{eff_i}$ is a square matrix of $(\bar{r} \times \bar{r})$ elements where $\bar{r}$ is the number of rigid body modes interested in the motion.

Even if the independent elements of $\mathbf{M}_{eff_i}$ are only $\bar{r}$, which leads to consider $\mathbf{M}_{eff_i}$ as a vector, here we will maintain its matricial representation.

Taking the derivative of:

$$\mathbf{M}_{eff_i} = \left(\mathbf{\Phi}_R^T\mathbf{M}\boldsymbol{\phi}_i\right)\left(\boldsymbol{\phi}_i^T\mathbf{M}\mathbf{\Phi}_R\right) \tag{4}$$

with respect to the control variable M_k we have:

$$\begin{aligned}\frac{\partial \mathbf{M}_{eff_i}}{\partial M_k} &= \left(\mathbf{\Phi}_R^T\frac{\partial \mathbf{M}}{\partial M_k}\boldsymbol{\phi}_i\right)\left(\boldsymbol{\phi}_i^T\mathbf{M}\mathbf{\Phi}_R\right) + \left(\mathbf{\Phi}_R^T\mathbf{M}\frac{\partial \boldsymbol{\phi}_i}{\partial M_k}\right)\left(\boldsymbol{\phi}_i^T\mathbf{M}\mathbf{\Phi}_R\right) + \\ &+ \left(\mathbf{\Phi}_R^T\mathbf{M}\boldsymbol{\phi}_i\right)\left(\frac{\partial \boldsymbol{\phi}_i^T}{\partial M_k}\mathbf{M}\mathbf{\Phi}_R\right) + \left(\mathbf{\Phi}_R^T\mathbf{M}\boldsymbol{\phi}_i\right)\left(\boldsymbol{\phi}_i^T\frac{\partial \mathbf{M}}{\partial M_k}\mathbf{\Phi}_R\right)\end{aligned} \tag{5}$$

where M_k is the diagonal element of lumped mass matrix relevant to the kth dof. The first and second term is the transposed of the fourth and third one respectively. The classical position that the eigenvector sensitivities are linear combination of all the eigenvectors, yields:

$$\frac{\partial \boldsymbol{\phi}_i}{\partial M_k} = \sum_{r=1}^{N} b_{irk}\boldsymbol{\phi}_r \tag{6}$$

where, considering that no stiffness variation is considered, [4] :

$$b_{irk} = \frac{\boldsymbol{\phi}_r^T(\lambda_i\frac{\partial \mathbf{M}}{\partial M_k} + \frac{\partial \lambda_i}{\partial M_k}\mathbf{M})\boldsymbol{\phi}_i}{\lambda_r - \lambda_i} \quad \text{for} \quad r \neq i \tag{7}$$

$$b_{iik} = -\frac{1}{2}\boldsymbol{\phi}_i^T\frac{\partial \mathbf{M}}{\partial M_k}\boldsymbol{\phi}_i \quad \text{for} \quad r = i \tag{8}$$

and:

$$\frac{\partial \lambda_i}{\partial M_k} = -\lambda_i\boldsymbol{\phi}_i^T\frac{\partial \mathbf{M}}{\partial M_k}\boldsymbol{\phi}_i \tag{9}$$

Eqs. 9 are decoupled, i.e. each eigenvalue sensitivity is obtained by the corresponding eigensolution, while Eqs. 6 require the complete set of eigensolutions, but it can be approximated with a suitable set of eigensolutions.

Because $\frac{\partial \mathbf{M}}{\partial M_k}$ is the partial derivative of the mass matrix with respect to one of its diagonal elements M_k, we easily obtain:

$$\frac{\partial \mathbf{M}}{\partial M_k} = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & \ddots & & & \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & 0 \end{bmatrix} \tag{10}$$

Introducing the symbol ${}^{k}\phi_i$ for the kth component of mode i we have:

$$\frac{\partial \mathbf{M}}{\partial M_k}\phi_i = \left\{ \begin{array}{c} 0 \\ 0 \\ \vdots \\ {}^{k}\phi_i \\ \vdots \\ 0 \end{array} \right\} \tag{11}$$

Substituting Eq. 11 into Eqs. 9 and 6, keeping into account Eqs. 7 and 8 we obtain:

$$\frac{\partial \lambda_i}{\partial M_k} = -\lambda_i({}^{k}\phi_i)^2 \tag{12}$$

and:

$$\frac{\partial \phi_i}{\partial M_k} = \sum_{r=1}^{N} \gamma_{ir}({}^{k}\phi_r\, {}^{k}\phi_i)\phi_r \tag{13}$$

where:

$$\gamma_{ir} = \frac{\lambda_i}{\lambda_r - \lambda_i(1+2\delta_{ir})} \tag{14}$$

and δ_{ir} is the Kronecker delta.

3 Relations used in DMAP Sequence

Eq. 13 cannot be used directly in the DMAP language and therefore we need to transform it. By retaining N_{mod} modes out of N we have:

$$\frac{\partial \phi_i}{\partial M_k} = \sum_{r=1}^{N_{mod}} \gamma_{ir}({}^{k}\phi_r\, {}^{k}\phi_i)\phi_r = {}^{k}\phi_i[(\gamma_{i1}\, {}^{k}\phi_1)\phi_1 + (\gamma_{i2}\, {}^{k}\phi_2)\phi_2 + \cdots + (\gamma_{iN_{mod}}\, {}^{k}\phi_{N_{mod}})\phi_{N_{mod}}] \tag{15}$$

which rewritten in matricial form becomes:

$$\frac{\partial \phi_i}{\partial M_k} = [\ \phi_1 \quad \phi_2 \quad \cdots \quad \phi_{N_{mod}}\ ] \left\{ \begin{array}{c} (\gamma_{i1}\, {}^{k}\phi_1) \\ (\gamma_{i2}\, {}^{k}\phi_2) \\ \vdots \\ (\gamma_{iN_{mod}}\, {}^{k}\phi_{N_{mod}}) \end{array} \right\} \{ {}^{k}\phi_i \} =$$

$$= \mathbf{\Phi} \left\{ \begin{array}{c} \gamma_{i1} \\ \gamma_{i2} \\ \vdots \\ \gamma_{iN_{mod}} \end{array} \right\} \otimes \left\{ \begin{array}{c} {}^{k}\phi_1 \\ {}^{k}\phi_2 \\ \vdots \\ {}^{k}\phi_{N_{mod}} \end{array} \right\} \{ {}^{k}\phi_i \} =$$

$$= \mathbf{\Phi} \left\{ (\, ^i[\mathbf{\Gamma}])^T \otimes (\, ^k[\mathbf{\Phi}])^T \right\} \{ \, ^k\phi_i \} =$$
$$= \mathbf{\Phi} \left\{ ([\mathbf{\Gamma}]^T)_i \otimes ([\mathbf{\Phi}]^T)_k \right\} \{ \, ^k\phi_i \} \tag{16}$$

Matrix $\mathbf{\Gamma}$ of which γ_{ir} is the generic element, can be obtained with suitable matricial operations from the vector of the eigenvalues as follows:

$$\mathbf{\Gamma} = \mathbf{\Lambda} \ominus (\mathbf{\Lambda}^T - \mathbf{\Lambda} - 2\mathbf{\Lambda}_{diag}) \tag{17}$$

where:

$$\mathbf{\Lambda} = \left\{ \begin{array}{c} \lambda_1 \\ \vdots \\ \lambda_i \\ \vdots \\ \lambda_{N_{mod}} \end{array} \right\} \{ \, 1 \;\; \cdots \;\; 1 \, \}_{(1 \times N_{mod})} = \left[\begin{array}{ccccc} \lambda_1 & \cdots & \lambda_1 & \cdots & \lambda_1 \\ \vdots & & \vdots & & \vdots \\ \lambda_i & \cdots & \lambda_i & \cdots & \lambda_i \\ \vdots & & \vdots & & \vdots \\ \lambda_{N_{mod}} & \cdots & \lambda_{N_{mod}} & \cdots & \lambda_{N_{mod}} \end{array} \right] \tag{18}$$

and:

$$\mathbf{\Lambda}_{diag} = \mathbf{\Lambda} \otimes \mathbf{I} = \left[\begin{array}{ccccc} \lambda_1 & & & & \\ & \ddots & & & \\ & & \lambda_i & & \\ & & & \ddots & \\ & & & & \lambda_{N_{mod}} \end{array} \right] \tag{19}$$

4 Observation

For the sake of generality the partial differentiation is performed with respect to scalar mass (CMASS2) in each dof, although the physical variables are the structural or non structural mass of the elements or the mass lumped in the grids. If for instance the problem requires the knowledge of the effective modal mass sensitivities with respect, to the lumped mass in a specified grid, indicating the scalar masses in the h grid with $\mathcal{M}_{h1}, \mathcal{M}_{h2}, \mathcal{M}_{h3}$, with $\mathcal{M}_h$ the physical mass in h and with k the number of the dof correponding to the jth dof of the hth grid we have:

$$\mathcal{M}_h = \mathcal{M}_{hj} = M_k \qquad j = 1, 2, 3 \tag{20}$$

$$\frac{\partial \mathbf{M}_{eff}}{\partial \mathcal{M}_h} = \frac{\partial \mathbf{M}_{eff}}{\partial \mathcal{M}_{h1}} \frac{d\mathcal{M}_{h1}}{d\mathcal{M}_h} + \frac{\partial \mathbf{M}_{eff}}{\partial \mathcal{M}_{h2}} \frac{d\mathcal{M}_{h2}}{d\mathcal{M}_h} + \frac{\partial \mathbf{M}_{eff}}{\partial \mathcal{M}_{h3}} \frac{d\mathcal{M}_{h2}}{d\mathcal{M}_h} \tag{21}$$

$$\frac{d\mathcal{M}_{hj}}{d\mathcal{M}_h} = 1 \tag{22}$$

$$\frac{\partial \mathbf{M}_{eff}}{\partial \mathcal{M}_h} = \frac{\partial \mathbf{M}_{eff}}{\partial \mathcal{M}_{h1}} + \frac{\partial \mathbf{M}_{eff}}{\partial \mathcal{M}_{h2}} + \frac{\partial \mathbf{M}_{eff}}{\partial \mathcal{M}_{h3}} \tag{23}$$

$$\frac{\partial \mathbf{M}_{eff}}{\partial \mathcal{M}_h} = \sum_{j=1}^{3} \frac{\partial \mathbf{M}_{eff}}{\partial \mathcal{M}_{hj}} \tag{24}$$

So the derivative of the effective modal mass matrix with respect to the physical mass equals the sum of the same partial derivatives w.r.t. the scalar masses along the three orthogonal directions. The property of Eq. 24 has been numerically verified for the case of the first component[1] of the effective mass relevant to the 6th mode, GRID 202. In fact the three derivatives $\frac{\partial ({}^1\mathbf{m}_{eff_6})}{\partial \mathcal{M}_{202\ j}}\ j=1,2,3$ obtained from the DMAP are respectively:

j		
1	+0.1013900	+
2	−0.00042986	+
3	−0.11851	=
Tot	−0.01754986	

The value of the same effective mass component (${}^1\mathbf{m}_{eff_6}$) after the variation of 0.010 (kg) of the lumped mass on the GRID 202 (CONM2) is: 4.345861463 (kg), while the reference effective mass is: 4.346037189 (kg). So the incremental ratio results:

$$\frac{4345861463 - 4.346037189}{0.010} = -0.0175726$$

The agreement with the analytical computed one is excellent.

5 DMAP Validation

In [2] an analogous procedure has been employed, with encouraging results, in a monodimensional structure, i.e. a shaft with lumped masses, in order to find a suitable distribution of the payloads. In the present paper we extend the procedure to a tridimensional aerospace structure [5] to calculate the effective mass sensitivities. In Fig. 1 is reported a view of the simplified model of the Satellite.

The comparison of the analytical (DMAP) results with the ones calculated with incremental ratios relevant to two close mass distributions are quite satisfactory. The reference results have been computed changing one lumped mass at a time with the standard MSC/NASTRAN Rigid Format Alter checka1.v675 of Sol 103 which gives the effective masses [6].

The values are calculated adding a scalar mass in a few selected grids along each direction, one at a time. The amount of the mass increment has been 0.010 (kg), while the global mass of the spacecraft is 1790.131 (kg). The main feature of the F.E. model of the Italsat Satellite,

[1] As it is customary we will deal with an effective mass vector $\mathbf{m}_{eff}$ of $\bar{r}$ components, i.e., only with the diagonal terms of $\mathbf{M}_{eff}$ (the evaluation of the off-diagonal terms it is straight forward, see for instance [2])

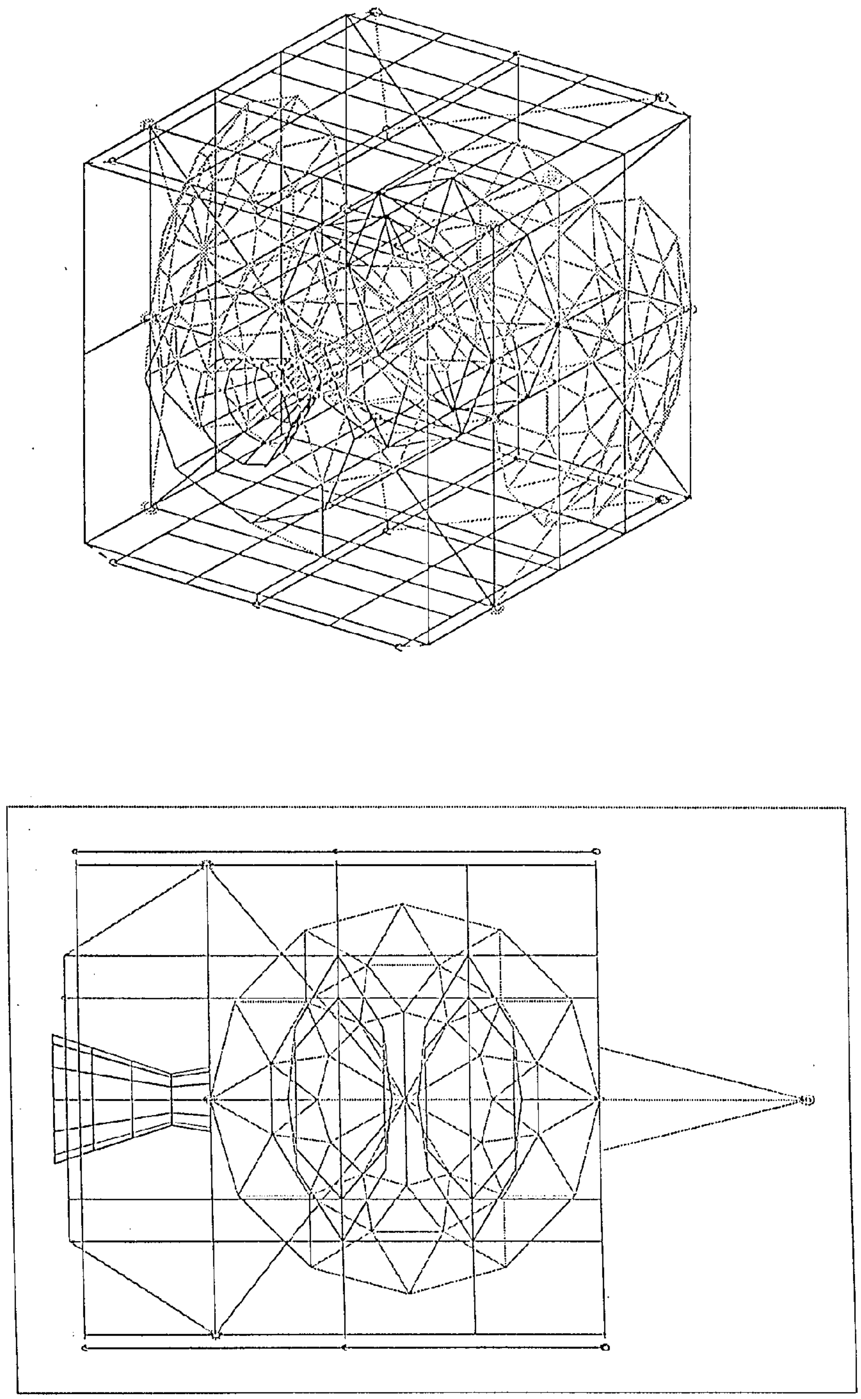

Figure 1: Simplified model of the Satellite

Structure	GRID	QUAD4	TRIA3	BAR
Bus	80	87	40	4
Two Main Antennas	90		120	8
Two Solar Arrays	224	156		12
Apogee Motor	64	48	12	
T.T.C.	17			18
Four Tanks	36	32	32	
Totals	511	323	204	42

Table 1: Main characteristics of Nastran model

used here as an example, are reported in Tab. 1. Modes with the bigger effective masses as well as some modes with negligible effective masses used in the validation are indicated with the relevant frequencies and effective mass components in Tab. 2. The number of dofs is 2582. The modal base has been truncated at one thousand modes, introducing negligible errors in all the cases here examined.

The values of the partial derivatives of the effective modal mass have been calculated for the selected control variables (lumped masses in chosen grids) for some modes below the 50th and for other modes between the 500th and the 520th. The selection is made through the DMIG input bulk data cards (see App. C). Some results are reported for comparison in Tab. 3. They are very satisfactory and generally the differences are due to numerical truncation.

6 Conclusions

The sensitivities of effective modal mass obtained analytically with the DMAP procedure are in good agreement with the ones calculated numerically as incremental ratio. With just one run such DMAP allow us to have them together with the effective modal mass, without too much time consuming.

The results are very satisfactory even for a tridimensional complex structure like that analyzed for the DMAP validation. For the use and the generalization to all the structural parameters like stiffness, area, moments of inertia, material characteristics as well to both lumped and consistent models we refer to other papers [2]. The writing of a general program in our opinion should not present theoretical and numerical problems. Since the great interest in aerospace field of such dynamic parameter like effective modal mass we wish the proposed methodology will be fruitful and worth of further developments.

i mode number	Frequency (Hz)	c component of effective mass vector	${}^{c}\mathbf{m}_{eff_i}$
1	17.3924	1	1453.054887
		5	3161.469755
4	32.2667	2	1589.762535
		4	1963.969554
5	34.4878	2	10.60716446
		4	98.64733692
6	34.5901	3	7.140457229
7	35.5055	1	18.04375168
11	37.3903	6	799.2262197
13	42.6607	2	1.473228948
		4	299.5875573
15	45.3142	1	3.790288069
		6	46.82846306
16	46.2656	1	4.346037189
		2	0.0112934535
		3	662.112785
18	47.3468	1	283.1375862
		5	5.204555786
19	48.5945	3	516.6724509
20	51.4179	4	434.9118225
24	62.4771	1	3.271789483
		3	0.2119989356
29	70.0180	3	508.6929783
31	74.3956	2	7.128574895
		4	160.0391005
45	91.3089	2	59.97762476
		4	106.1396392
517	912.2159	1	1.143269397
518	914.2492	1	1.158747085E-2
		3	2.897842859E-4

Table 2: Modes of Spacecraft model

i mode number	c compon. of effect. mass vector	${}^c\mathbf{m}_{eff_i}$	Control variable: Grid ID h	Control variable: direct. j $1 \div 3$	Sensitivity w.r.t. scalar mass: Analytical DMAP		Sensitivity w.r.t. scalar mass: Numerical $\frac{\Delta^c\mathbf{m}_{eff_i}}{\Delta\mathcal{M}_{hj}}$	
1	1	1453.054887	202	1	6.5687	E-1	6.569	E-1
	5	3161.469755	202	1	4.2364		4.2364	
4	2	1589.762535	202	1	-8.8061	E-6	0.000	
	2	1589.762535	202	2	7.549	E-1	7.549	E-1
6	3	7.140457229	202	3	4.5259	E-2	4.52567	E-2
13	4	299.5875573	202	3	1.7782		1.77832	
15	1	3.790288069	202	1	-2.4547	E-1	-2.454367	E-1
	1	3.790288069	202	2	-7.8452	E-5	-7.85	E-5
	6	46.82846306	202	1	-3.72636	E-1	-3.72661	E-1
	6	46.82846306	202	2	-1.3551	E-4	-1.35	E-4
16	1	4.346037189	202	1	1.0139	E-1	1.014072	E-1
	2	0.0112934535	202	2	-1.0989	E-4	-1.09888	E-4
	3	662.112785	202	3	9.5015		9.50080	
24	1	3.271789483	202	1	5.7043	E-2	5.70464	E-2
	3	0.2119989356	202	3	1.5336	E-3	1.53397	E-3
1	1	1453.054887	3020	1	9.1674	E-1	9.167	E-1
7	1	18.04375168	3020	1	1.435	E-2	1.4053	E-2
15	1	3.790288069	3020	1	8.1302	E-2	7.15272	E-2
24	1	3.271789483	3020	1	4.4919	E-2	4.48900	E-2
	1	0.2119989356	3020	3	-1.9785	E-4	-1.9786	E-4
517	1	1.143269397	3020	3	-1.0780	E-2	-1.08765	E-2
518	1	1.158747085E-2	3020	1	4.0702	E-5	4.1142	E-5
	1	2.897842859E-4	3020	3	1.1760	E-3	1.1978513	E-3

Table 3: Comparison between analytical derivatives and incremental ratio sensitivity

APPENDICES

A DMAP Description

The DMAP sequence reported in App. B is included in the subdmap KEEFW which is in the RFAlter file checka1.v675 provided by MSC for kinetic energy and effective mass calculation. Since we need to read a DMIG bulk data card for the selection of control variables (dofs into which we are supposed to apply a scalar mass modification), data block MATPOOL has to be added in the TYPE DB statement. Also the LAMA table is added. In Tab. 4 are reported the data blocks used in the DMAP sequence.

At the beginning we read the trailer of the eigenvector matrix UGVS1 in order to get the number of modes NMOD and the number of dofs NDOF. With the module MTRXIN we read the matrix SELECT whose diagonal SELVEC contains 1 whenever the corresponding dof is a control variable. The datasets RBTM, GAMIR and their transposes MRB and GAMIRT are calculated outside the loops.

Starting from module LAMX, which transform the LAMA table into the matrix LMAT, up to the first DO WHILE statement we calculate matrix GAMIR according to Eq. 17. MATMOD LMAT extracts the eigenvalue column and put it in LVECT. The transposed of LVECT is used in the evaluation of λ derivative. The next four modules are used to generate a unit matrix of dimension 1. The next loop is then used to create a row vector of length NMOD containing ones. At this point we can generate LSING ($\mathbf{\Lambda}$) according to Eq. 18 through the MPYAD module. The next three ADD modules perform the matricial operation of Eq. 17.

The outer loop is relevant to the dof where the modification is possibly applied. PARAML extracts the value of SELVEC, if it is $\neq 1$ that means the dof is not of interest and therefore the value of K is incremented, otherwise we go to the inner loop which is performed for the first 50 modes. It is not difficult to understand the subsequent modules up to the end of the evaluation of λ derivative. At this point it is useful to rewrite Eq. 5 as follows:

$$\frac{\partial \mathbf{M}_{eff_i}}{\partial M_k} = (T_2^T L_i + T_3^T L_i) + (L_i^T T_3 + L_i^T T_2) \tag{25}$$

where T_2 accounts for mass matrix variation while T_3 for eigenvector variation. Since the two parenteses in Eq. 25 are transposed of each other we can rewrite Eq. 25 in DMAP variables as (DERAT + CORT)+(CORREA+DERA). T_2 is calculated right away by MPYAD FIITK. T_3 calculation is a little more laborious and is evaluated in the next six DMAP modules according to the last line of Eq. 16.

Finally we compute DER1 which is the effective mass matrix sensitivity obtained neglecting T_3 and DER2 which is the correct one. In [2] it has been proven that only the diagonal terms of $\mathbf{M}_{eff_{i,k}}$ are independent and therefore only the diagonal terms $\mathbf{m}_{eff_{i,k}}$ are collected in DERIV1 and DERIV2 respectively. These are $6 \times (N_1 \cdot N_2)$ matrices where N_1 is the number of dofs selected and N_2 the number of modes for which the effective masses have to

MSC/NASTRAN Data blocks	Mathematical symbol
AUX12	$\bar{\mathbf{m}}_{eff_{i,k}}$
AUX14	$\mathbf{m}_{eff_{i,k}}$
BRIK	$\{\boldsymbol{\phi}^{(1)}_{max}\}_{(1\times1)}$
BVEC	$\left\{([\mathbf{\Gamma}]^T)_i \otimes ([\mathbf{\Phi}]^T)_k\right\}$
BVECT	BVEC$\cdot\{\ {}^k\boldsymbol{\phi}_i\}_{(1\times1)}$
DERL	$\{\lambda_i(\ {}^k\boldsymbol{\phi}_i)^2\}_{1\times1}$
DENOM	$(\mathbf{\Lambda}^T - \mathbf{\Lambda} - \mathbf{2\Lambda}_{diag})$
DERLAM	$\frac{\partial\lambda_i}{M_k}$ = -DERL
DERPHI	$\frac{\partial\boldsymbol{\phi}_i}{\partial M_k} = [\mathbf{\Phi}]\cdot$ BVECT
DER1	$\mathbf{M}_{eff_{i,k}}$
DER2	$\mathbf{M}_{eff_{i,k}}$
EFWWUP	$\mathbf{m}_{eff}$
FII	$\boldsymbol{\phi}_i$
FIIK2	$\{\ {}^k\boldsymbol{\phi}_i^2\}_{1\times1}$
FIIT	$\boldsymbol{\phi}_i^T$
FIITK	$\{\ {}^k\boldsymbol{\phi}_i^T\}_{1\times1}$
FIITKT	${}^k\boldsymbol{\phi}_i$
GAMI	$([\mathbf{\Gamma}]^T)_i$ ith column of $[\mathbf{\Gamma}]^T$
GAMIR	$\mathbf{\Gamma} = \mathbf{\Lambda} \ominus (\mathbf{\Lambda^T} - \mathbf{\Lambda} - \mathbf{2\Lambda}_{diag})$
GAMIRT	$[\mathbf{\Gamma}]^T$
LAMB	$\mathbf{\Lambda}_{diag}$
LAMBI	$\{\lambda_i\}_{(1\times1)}$
LIT	L_i^T
LSING	$\mathbf{\Lambda}$
LSINGT	$\mathbf{\Lambda}^T$
LVECT	$\{\lambda_1\cdots\lambda_{N_{mod}}\}^T$
MGG	$\mathbf{M}$
MRB	$\mathbf{M}\ \mathbf{\Phi}_R$
RBG1	$\mathbf{\Phi}_R$
RBT	$\mathbf{\Phi}_R^T$
RBTM	$\mathbf{\Phi}_R^T\mathbf{M}$
TERM2	T_2
TERM3	T_3
UGVS1	$\mathbf{\Phi}$
UGT	$[\mathbf{\Phi}]^T$
UGTK	$\left([\mathbf{\Phi}]^T\right)_k$
UN	$\{1\}_{(1\times1)}$
UNITV	$\{1\cdots1\}_{(1\times N_{mod})}$

Table 4: Data blocks used in DMAP sequence

be differentiated. These two matrices are then transposed and appended and the resulting matrix transposed again to obtain the final data block DERALT which is $12 \times (N_1 \cdot N_2)$. It is clear from those numbers that the contribution of the variation of the eigenvectors (term T_3) is not at all negligible. In App. C is reported a part of the input data deck. The dofs considered in the DMIG cards are 18 translations of 6 different grids. The number N_2 relevant to the effective masses to be differentiated is specified in the DMAP sequence in DO WHILE (IMOD=<50). Therefore in the example DERALT is $12 \times (18 \cdot 50)$. To be noted that the structure of matrix DERALT is made of 50 columns relevant to the first chosen dof of the grid with the lower ID number (in the example first dof of GRID 202) followed by other 50 columns of the next dof of the same grid and so on. Part of a page of the output is reported in App. D. The first six numbers in each column refer to the approximate derivatives while the last six numbers are the exact ones.

B Listing of DMAP Sequence

```
$
$ SUBDMAP KEEFW - CALCULATE KE AND EFW IF REQUESTED
$
$------------------------------------------------------------------------
COMPILE KEEFW, noLIST, NOREF $
SUBDMAP KEEFW UGVS1/TEST99/SEID/PEID $
$
$ SUBDMAP KEEFW - CALCULATE KE AND EFW IF REQUESTED
$
TYPE DB,EQEXINS,GPLS,SILS,USET,MGG,MJJ,MATPOOL,LAMA $ MATPOOL, LAMA added
type db,bgpdts,cstms $ added for rbg if it doesn't exist
TYPE PARM,NDDL,I,N,SEID $
                    .
                    .
                    .
$
$ superelement has upstream superelements -
$ get assembly ke and efwgt
$
      MPYAD UGVS1,MGG,/PHITM/1//// $
      IF (EFWGT > -1)THEN $
         MPYAD PHITM,RBG1,/MER///// $
         TRNSP MER/MERT $
$
$------------------------------------------------------------------------
$ START MODIFICATION FOR EFFECTIVE MASS SENSITIVITY CALCULATION
$ ------------------------------------------------------------------------
$
         TYPE PARM,,I,N,NMOD,K,NDOF,IMOD,J,H,KRINT,HRINT
         TYPE PARM,,RS,N,KR,HR
         PARAML UGVS1//'TRAILER'/1/S,N,NMOD
         PARAML UGVS1//'TRAILER'/2/S,N,NDOF
         PRTPARM //0/'NMOD' $
         PRTPARM //0/'NDOF'
         MTRXIN, ,MATPOOL,EQEXINS,SILS,/SELECT,,/S,N,NDOF/S,N,NOM1 $
         MESSAGE //'NOM1='/NOM1 $
         MATPRN SELECT// $
         DIAGONAL SELECT/SELVEC/'COLUMN'/ $
         MATPRN SELVEC// $
$
         FILE UNITV=APPEND
         FILE DERIV1=APPEND
```

```
          FILE DERIV2=APPEND
          FILE DEPHAL=APPEND
          FILE DELAAL=APPEND
$
          TRNSP UGVS1/UGT $
          TRNSP RBG1/RBT $
          MPYAD RBT,MGG,/RBTM/ $
          TRNSP RBTM/MRB $
          LAMX,,LAMA/LMAT/-1 $
          MATMOD LMAT,,,,,/LVECT,/1/1 $ Extract eigenvector column
          TRNSP LVECT/LVECTT $
$
$ Start generation of row vector of NMOD components
$
          MATMOD UGVS1,,,,,/UG1,/1/1 $ Extract 1st eigenvector
          DIAGONAL UG1/UG1ABS/'WHOLE'/1. $ Convert to absolute value
          MATMOD UG1ABS,,,,,/BRIK,/7 $ Find maximum to be sure BRIK not =0
          DIAGONAL BRIK/UN/'SQUARE'/0. $ Unit matrix 1 x 1
          J = 1
          DO WHILE (J=<NMOD)
             APPEND UN,/UNITV/2
             J = J + 1
          ENDDO
$
$ End generation of unitary row vector
$
          MPYAD LVECT,UNITV,/LSING/ $ matrix Λ
          TRNSP LSING/LSINGT
          DIAGONAL LSING/LAMB/'SQUARE'/ $ Eigenvalue matrix Λ_diag
          ADD LSINGT,LSING/DEN/1./-1./ $
          ADD DEN,LAMB/DENOM/1./-2./ $
          ADD LSING,DENOM/GAMIR/1./1./2 $
          TRNSP GAMIR/GAMIRT $
$
$ Start loop on control variables (d.o.f. K)
$
          K = 1
          DO WHILE (K=<NDOF)
             PARAML SELVEC//'DMI'/1/K/S,N,KR $
             KRINT=INT(KR)
             IF (KRINT=1) THEN $ Evaluates M_eff_i,k only on selected d.o.f.
                MATMOD RBT,,,,,/RBTK,/1/K $
                MATMOD UGT,,,,,/UGTK,/1/K $
$
$ Start loop on modes
$
                IMOD=1
                DO WHILE (IMOD=<50)
                   MATMOD UGVS1,,,,,/FII,/1/IMOD $
                   MPYAD RBTM,FII,/LIT $
                   TRNSP FII/FIIT $
                   MATMOD FIIT,,,,,/FIITK,/1/K $
$
$ Start evaluation of eigenvalue derivatives
$
                   MATMOD LVECTT,,,,,/LAMBI,/1/IMOD $
                   TRNSP FIITK/FIITKT $
                   MPYAD FIITK,FIITKT,/FIIK2/ $
                   MPYAD LAMBI,FIIK2,/DERL/ $
                   ADD DERL,/DERLAM/-1./ $
$
$ End evaluation of eigenvalue derivatives
$
                   TRNSP RBTK/RBTKT $
                   MPYAD FIITK,RBTKT,/TERM2/ $
                   MATMOD GAMIRT,,,,,/GAMI,/1/IMOD $ Extract IMODth row of γ_ir
```

```
                ADD GAMI,UGTK/BVEC/1./1./1 $ product elem. by elem. of two vectors
                MPYAD BVEC,FIITK,/BVECT $
                MPYAD UGVS1,BVECT,/DERPHI $
                TRNSP DERPHI/DERPHT
                MPYAD DERPHT,MRB,/TERM3 $
                MPYAD LIT,TERM2,/DERA $ approx of derivative (i.e., no variat. of modes considered)
                TRNSP DERA/DERAT
                ADD DERA,DERAT/DER1
                MPYAD LIT,TERM3,/CORREA/ $ correction to be added to approx. to include mode variation
                TRNSP CORREA/CORT
                ADD CORREA,CORT/CORREC
                ADD DER1,CORREC/DER2/ $
                DIAGONAL DER1/AUX12/'COLUMN'/ $
                APPEND AUX12,/DERIV1/2 $
                DIAGONAL DER2/AUX14/'COLUMN'/ $
                APPEND AUX14,/DERIV2/2 $
                APPEND DERPHI,/DEPHAL/2 $
                APPEND DERLAM,/DELAAL/2 $
                IMOD = IMOD + 1
              ENDDO
            ENDIF $ KRINT=1
            K = K + 1
          ENDDO
          TRNSP DERIV1/DER1T $
          TRNSP DERIV2/DER2T $
          APPEND DER1T,DER2T/DERALL/ $
          TRNSP DERALL/DERALT $
          MATPRN DERALT // $
$
$------------------------------------------------------------------
$ END MODIFICATION FOR EFFECTIVE MASS SENSITIVITY CALCULATION
$------------------------------------------------------------------
$
                .
                .
       ENDIF $ EFWGT > -1
                .
                .
RETURN $
END $
```

C Input Data Deck

```
ID SPACE,CRAFT
TIME 2000
SOL 103
INCLUDE 'checka1.cor'
CEND
TITLE=SPACECRAFT IN CONFIGURAZIONE DI LANCIO
METHOD=36
DISP(PLOT)=ALL
BEGIN BULK
DMIG,SELECT,0,1,1,0 $
DMIG,SELECT,202,1,,202,1,1. $
DMIG,SELECT,202,2,,202,2,1. $
DMIG,SELECT,202,3,,202,3,1. $
DMIG,SELECT,1110,1,,1110,1,1. $
DMIG,SELECT,1110,2,,1110,2,1. $
DMIG,SELECT,1110,3,,1110,3,1. $
DMIG,SELECT,3020,1,,3020,1,1. $
DMIG,SELECT,3020,2,,3020,2,1. $
DMIG,SELECT,3020,3,,3020,3,1. $
DMIG,SELECT,4020,1,,4020,1,1. $
DMIG,SELECT,4020,2,,4020,2,1. $
DMIG,SELECT,4020,3,,4020,3,1. $
DMIG,SELECT,2091,1,,2091,1,1. $
DMIG,SELECT,2091,2,,2091,2,1. $
DMIG,SELECT,2091,3,,2091,3,1. $
```

```
DMIG,SELECT,7091,1,,7091,1,1. $
DMIG,SELECT,7091,2,,7091,2,1. $
DMIG,SELECT,7091,3,,7091,3,1. $
$
PARAM,GRDPNT,68
PARAM,POST,0
PARAM,KEPRT,1
PARAM,EFWGT,2
PARAM,CHKSTIF,1
PARAM,CHKMASS,1
PARAM,NEWSEQ,-1
      .
      .
      .
EIGRL,36,,,1000
ENDDATA
```

D Output of Matrix DERALT

```
0       MATRIX DERALT    (GINO NAME 101 ) IS A DB  PREC          900 COLUMN X          12 ROW RECTANG  MATRIX.
0COLUMN      1       ROWS       1 THRU      12      ----------------------------------------------
    ROW
      1)     2.7701D+00  0.0000D+00  0.0000D+00  0.0000D+00  8.3763D+00  4.0304D-04  6.5687D-01 -4.6455D-08 -5.2417D-07 -1.7071D-07
     11)     4.2364D+00  5.2271D-04
0COLUMN      2       ROWS       1 THRU      12      ----------------------------------------------
    ROW
      1)    -1.0477D-03  0.0000D+00  0.0000D+00  0.0000D+00  2.3738D-03  2.6447D-05  5.6708D-03 -8.7990D-02 -8.9158D-08 -4.4900D-02
     11)     4.0179D-03  7.9591D-05
0COLUMN      3       ROWS       1 THRU      12      ----------------------------------------------
    ROW
      1)    -3.4382D-03  0.0000D+00  0.0000D+00  0.0000D+00  7.7748D-03 -1.2072D-05  5.9614D-04  8.7990D-02  7.4473D-09  4.4902D-02
     11)    -8.7663D-03 -3.6111D-05
0COLUMN      4       ROWS       1 THRU      12      ----------------------------------------------
    ROW
      1)    -4.8856D-08  0.0000D+00  0.0000D+00  0.0000D+00 -1.6526D-07  1.6735D-06  6.7111D-08 -8.8061D-06  9.8023D-10 -4.2478D-05
     11)     6.4847D-08  6.7784D-06
0COLUMN      5       ROWS       1 THRU      12      ----------------------------------------------
    ROW
      1)    -1.9137D-06  0.0000D+00  0.0000D+00  0.0000D+00  3.9519D-08  8.8758D-06  3.0415D-06  6.8580D-06  5.1249D-08  6.9163D-05
     11)    -1.2053D-08  6.1178D-05
0COLUMN      6       ROWS       1 THRU      12      ----------------------------------------------
    ROW
      1)     3.5288D-07  0.0000D+00  0.0000D+00  0.0000D+00 -4.5679D-06 -9.7349D-08 -5.7237D-07  2.4958D-08  7.7409D-08  1.2689D-07
     11)     1.3730D-08 -6.9421D-07
0COLUMN      7       ROWS       1 THRU      12      ----------------------------------------------
    ROW
      1)    -3.0188D-02  0.0000D+00  0.0000D+00  0.0000D+00  5.5304D-03  1.1578D-03  4.1489D-02 -5.6963D-07 -1.1102D-06 -7.1393D-06
     11)    -1.5852D-03  1.2030D-02
0COLUMN      8       ROWS       1 THRU      12      ----------------------------------------------
    ROW
      1)     2.4756D-06  0.0000D+00  0.0000D+00  0.0000D+00  6.0148D-08  1.3301D-06 -2.9231D-06  1.5935D-08 -1.7806D-06  7.6625D-08
     11)    -1.6420D-08  2.7908D-05
0COLUMN      9       ROWS       1 THRU      12      ----------------------------------------------
    ROW
      1)    -1.8194D-05  0.0000D+00  0.0000D+00  0.0000D+00  4.5083D-05  1.1076D-06  2.2205D-05  1.6687D-07  1.2394D-07  8.5827D-07
     11)    -1.2291D-05  2.6424D-05
0COLUMN     10       ROWS       1 THRU      12      ----------------------------------------------
    ROW
      1)     7.4875D-07  0.0000D+00  0.0000D+00  0.0000D+00 -1.7835D-06  1.3465D-06 -8.3419D-07  1.2332D-07  8.2589D-09  1.1024D-06
     11)     5.9873D-07  3.2155D-05
1    SPACECRAFT IN CONFIGURAZIONE DI LANCIO                          DECEMBER  10, 1994  MSC/NASTRAN   6/21/93   PAGE   173
```

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