INNOVATIVE USES OF SYNTHETIC RESPONSES IN DESIGN OPTIMIZATION

by

Erwin H. Johnson
The MacNeal-Schwendler Corporation
Los Angeles, California

ABSTRACT

Synthetic responses in MSC/NASTRAN allow the user to combine responses, design variables, and grid locations to define new responses that can be incorporated into a structural design task. This paper indicates how this capability can be applied in a wide variety of applications. Four examples are presented showing the use of this capability to (1) minimize the maximum stress response, (2) create mean-square responses (3) include Johnson/Euler buckling conditions in the design task, and (4) perform topology optimization.

INTRODUCTION

The introduction of Design Optimization in Version 66 of MSC/NASTRAN contained a synthetic response feature that was a clear advance in the area of structural optimization techniques. Synthetic responses is a term used to designate responses that are constructed by the user out of existing MSC/NASTRAN responses, design variables, constants, and grid locations. The DRESP2 Bulk Data entry defines the parameters to be used in the synthetic response and invokes a DEQATN entry that specifies the equation used to compute the response. The DRESP2 response can be the objective of the design task, or it could be constrained and therefore represent one of the design conditions [see Moore (1994, Section 2.6)].

The development of the synthetic response in MSC/NASTRAN was a joint effort involving MSC and VMA Engineering (at the time, the latter company was EDO). The first MSC design document to mention the synthetic response was written by Wallerstein (1985). Version 67 of MSC/NASTRAN added the DNODE flag on the DRESP2 entry to indicate that the subsequent data referred to nodal locations [see Reymond and Miller (1992)]. For Version 68, the DRESP2 functionality was extended to allow for the generation of synthetic responses that span DRESP1 types and, for dynamic response, synthetic responses that span frequencies or time steps [see the Version 68 Release Notes (1994)].

The synthetic response feature is now in routine use, and its availability is considered a key asset in the overall design sensitivity and optimization capability. Two noteworthy examples of the application of the feature are in papers from the 1994 MSC World Users' Conference that applied synthetic responses to the matching of modal test results [Blakely (1994) and Herting (1994)]. The intent of this paper is to further publicize this capability by using it in ways that are considered innovative and that have been demonstrated by the author. This publication has the direct effect of documenting techniques that others may find useful in their design studies. An indirect, but perhaps more substantial, desired effect of the paper is to inspire others to find ways that the DRESP2 entry can be used to address design issues that are not supported directly by the standard structural responses.

EXAMPLES

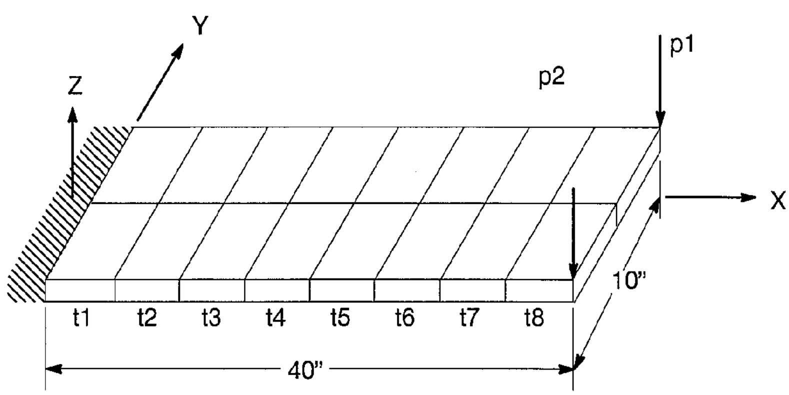
The paper contains four examples. The first one employs a penalty function concept to provide a design for which the maximum stress is minimized. The second example shows how the DRESP2 entry can be used to form a mean-square response that is useful in a dynamic response application. The concepts contained in these first two examples have been documented in Moore (1994, Sections 7.10 and 7.7, respectively). The third and fourth examples present concepts that are new with this paper. The third shows how DRESP2 entries can be used to construct local buckling constraints. It also indicates a work-around for the MSC/NASTRAN limitation that the synthetic responses cannot include IF–THEN–ELSE type constructions. The fourth example indicates how the existing MSC/NASTRAN design optimization capability can be extended to perform topology optimization. This final example uses examples that were addressed by other means at the 1994 MSC World User's Conference by Wang, Lu, and Yang (1994). The Appendix contains the input data files that illustrate the use of the synthetic responses.

Minimizing the Maximum Stress

Figure 1 shows a cantilevered plate that is designed in Section 7.3 of Moore (1994). In the *MSC/NASTRAN Design Sensitivity and Optimization User's Guide*, the design task is to minimize the weight while constraining the displacements and stresses. In the current problem, the task is to minimize the maximum stress while constraining the weight and the displacements. The design task is performed by introducing another design variable, β, and minimizing 10,000 β while constraining

$$\mathbf{10000\beta - \sigma \geq 10000} \tag{1}$$

Although this technique has been available in the literature for some time [Taylor and Bendsoe (1984)], it is not generally known. For this example, σ is a von Mises stress that is always positive, forcing β to also be positive. The minimum value of β occurs when the maximum σ is minimized. Figure A1 shows the input data file for this case that implements the objective and the constraints. The amount of Bulk Data entries required for the problem has been reduced significantly relative to the *Users Guide* through the use of data replication, direct input of basis vectors on DVPREL1 entries, and the use of continuations on the DRESP1 entries.



Analysis Model Description

2 x 8 array of CQUAD4 elements
Material: E = 1.0E+6 psi
Poisson ratio = 0.33
Weight density = 0.1 lbs/in^3
Two static load conditions:
Tip loading: Two 5,000 lb loads in -z direction
Pressure loading: Uniform at -6.44 lb/in^2

Figure 1. Cantilever Plate Model.

The results of the optimization are very similar to those of the *Users' Guide* in that the original design is grossly infeasible with respect to the displacement constraints. The weight increases to its constrained upper bound in this case while the stresses decrease significantly to a uniform value of 28 ksi across the span of the model. A smoothed representation of the optimal thickness distribution is given in Figure 2.

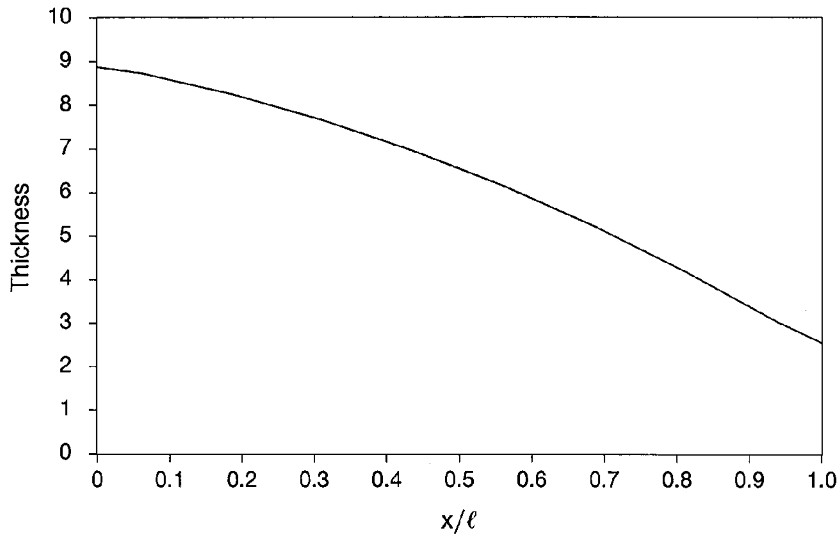


Figure 2. Optimal Thickness Values for the Cantilevered Plate.

Mean-Square Response

The second example deals with the minimization of the mean-square response of a structure while constraining the weight. Moore (1994) contains an example that applies the technique to a clamped-free plate acted upon by a pressure loading with a fixed magnitude over the range of 20.0 to 200.0 Hz. Figure 3 reproduces Figure 7-18 of Moore (1994) to demonstrate the utility of the approach. The mean-square response in this case was reduced from 230.13 to 133.33 in^2. Details of the example, including the input data file, can be found in the referenced *User's Guide.*

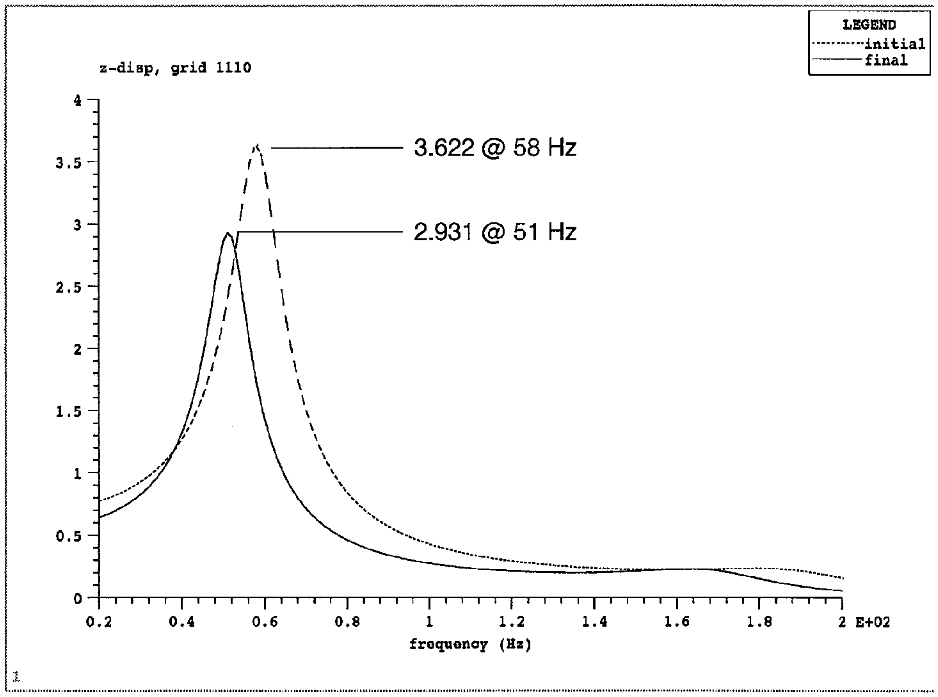


Figure 3. Results of Minimizing a Mean-Square Response.

Local Buckling Criteria

The third example was motivated by a paper by Basso, et al. (1993) that compared an optimization code developed for Aermaachi with MSC/NASTRAN. A perceived shortcoming of MSC/NASTRAN in this paper is that it does not address **"...crippling limits due to the difficulty of reproducing the Johnson/Euler curve by means of a unique user's defined equation."** The Johnson/Euler criteria is applicable to columns and can be characterized as saying that classical Euler buckling is applicable for long columns while short columns require an empirical formula. In terms of equations, the critical buckling stress is given by

$$\sigma_{cr} = \sigma_{co}\left[1 - \frac{\sigma_{co}(L'/\rho)^2}{4\pi^2 E}\right] \qquad L'/\rho \leq \pi\sqrt{\frac{2E}{\sigma_{co}}} \tag{2}$$

$$\sigma_{cr} = \frac{\pi^2 E}{(L'/\rho)^2} \qquad L'/\rho \geq \pi\sqrt{\frac{2E}{\sigma_{co}}} \tag{3}$$

Where σ_{cr} is the critical buckling stress, L' is the effective length, ρ is the radius of gyration, and σ_{co} is an empirically determined column yield stress [see Peery and Azar (1982, Chapter 11.6)]. The fact that there are separate ranges over which the formulas apply indicates the need for a branch in the DEQATN capability, a capability that is not supported. However, an alternative is at hand that allows the application of the above formulas. This entails recognizing that the Johnson formula governs only when the magnitude of the compressive stress is greater than $\sigma_{co}/2$. Therefore, if the stress magnitude is less than $\sigma_{co}/2$ the Johnson criterion need not be applied. This is done by calculating a factor

$$\boldsymbol{FAC = max(0.0, -\sigma - \sigma_{co}/2)} \tag{4}$$

and then multiplying the stress used in computing the Johnson criterion by this factor. In this way, the modified Johnson criterion will never be met whenever the stress magnitude is less than $\sigma_{co}/2$. A very simple example of a column loaded by an end load is used to illustrate the technique (Figure 4). The input file that implements this concept is shown in Appendix A2. The design variable is the radius of a solid cylindrical column so that the radius of gyration is the design variable divided by two. σ_{co} is set at 80,000 ksi. For this case, a length of 40 inches is sufficiently short so that the Johnson criteria will apply at the optimum design. When the length of the column is set at 60 inches, the optimal design buckles in the Euler mode and, in fact, it is possible to independently apply a constraint to the buckling eigenvalue and show that both the global buckling criteria and the local buckling criteria apply at the optimum (see Table 1). For a more complex built-up structure, the local buckling criteria would have to be applied in a large number of elements, each of which could have separate design parameters (e.g., length), but the basic concept is the same.

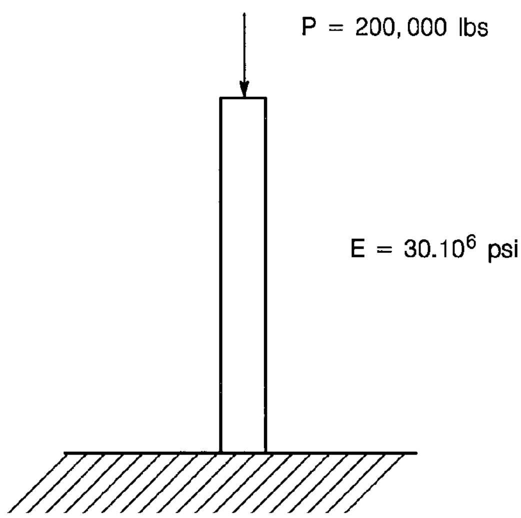


Figure 4. Structural Model for the Johnson/Euler Buckling Example. The Design Variable Is the Radius of the Uniform Circular Column.

Table 1. Results for the Johnson/Euler Buckling Design Task.

Length	Radius	Criteria			Comment
		Johnson	Euler	$1.0/\lambda_1$	
40	1.1072	1.003	0.916	0.916	Johnson is critical.
60	1.3265	1.011	1.000	1.000	Euler is critical.

Topology Optimization

The final examples of this paper were motivated by a paper by Wang, et al. (1994) that used MSC/NASTRAN to perform topology optimization. In that paper, the authors used design sensitivity data available from SOL 101 (static analysis) or SOL 103 (normal modes analysis), but did not use SOL 200 (design optimization). In this paper, it is demonstrated the the synthetic response option of SOL 200 can be used to replicate the results given in the earlier paper.

Two examples are shown here. The first is a cantilevered plate acted upon by a point load at its tip (Figure 5). The design task is to come up with the topology within the perimeter of the plate that will minimize the compliance of the structure (in this example, this is simply the product of the point load and the displacement at the point of application of the load and in the direction of the load) while constraining the weight to be equal to 25% of the weight of the structure that would be obtained from a solid plate that is one-unit thick.

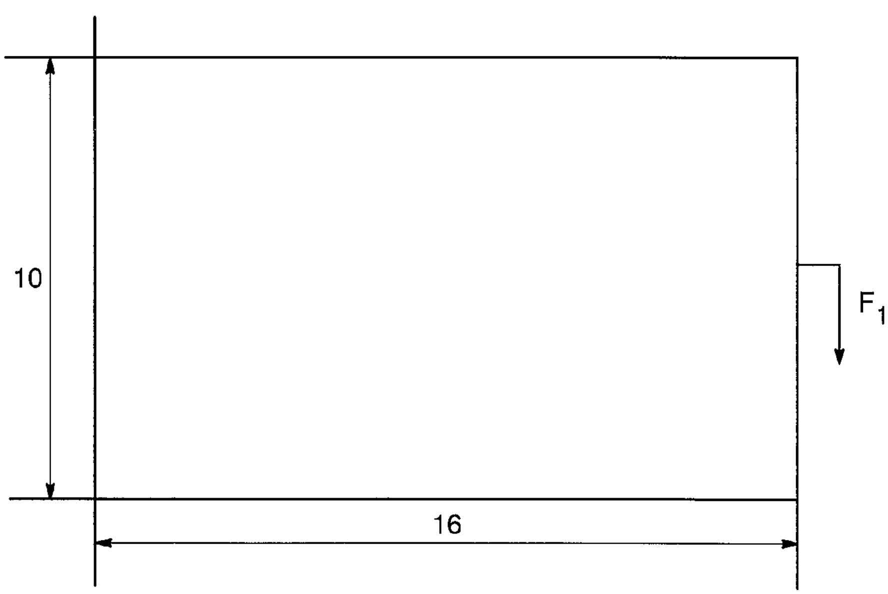


Figure 5. Cantilever Beam (from Wang, et. al., 1994).

The concept of topology optimization, as described in the Wang, et al., (1994) paper, is to divide the plate region into a large number of finite elements and to assign a density to each element ranging from zero to one. A density of zero represents a void while a value of one is a clear signal that structure is present for the element. Intermediate values are ambiguous, and the goal of the topology optimization task is to force the density values to either the zero or one limit. The earlier paper performed this by defining a non-physical relationship between the element Young's modulus and the element density of the form

$$\frac{E_i}{E_0} = \rho_i^n \tag{5}$$

and, quoting from the earlier paper, "**where n is an exponent, E_i and E_0 are intermediate and real material Young's moduli, respectively. The equation will penalize intermediate density and force the density to 0 or 1, when $n > 1$. In this paper, $n = 2$ is used for simplicity.**"

SOL 200 does not have the ability to perform material optimization, so Young's modulus cannot be specified in the form given above. However, for the current example, the stiffness is a linear product of the thickness and Young's modulus so that a thickness variable can be used equally as well. Therefore, the current paper holds Young's modulus at a fixed value, but has a relationship between density and thickness of the form

$$t_i = \rho_i^4 \tag{6}$$

This is a synthetic property relationship with ρ_i equal to the design variable and t_i the synthetic property value. The weight of the structure is computed using a synthetic response that adds all the densities (design variables)

$$WEIGHT = \sum_{i=1}^{nelem} \rho_i \tag{7}$$

Figure 6 shows the topology obtained using SOL 200 applied to a bottom half of the model of the structure of Figure 5 (symmetry conditions have been used to reduce the problem size). A truss-like structure is produced, and these results approximate those given in the earlier paper. The compliance value for the final design is 141.97.

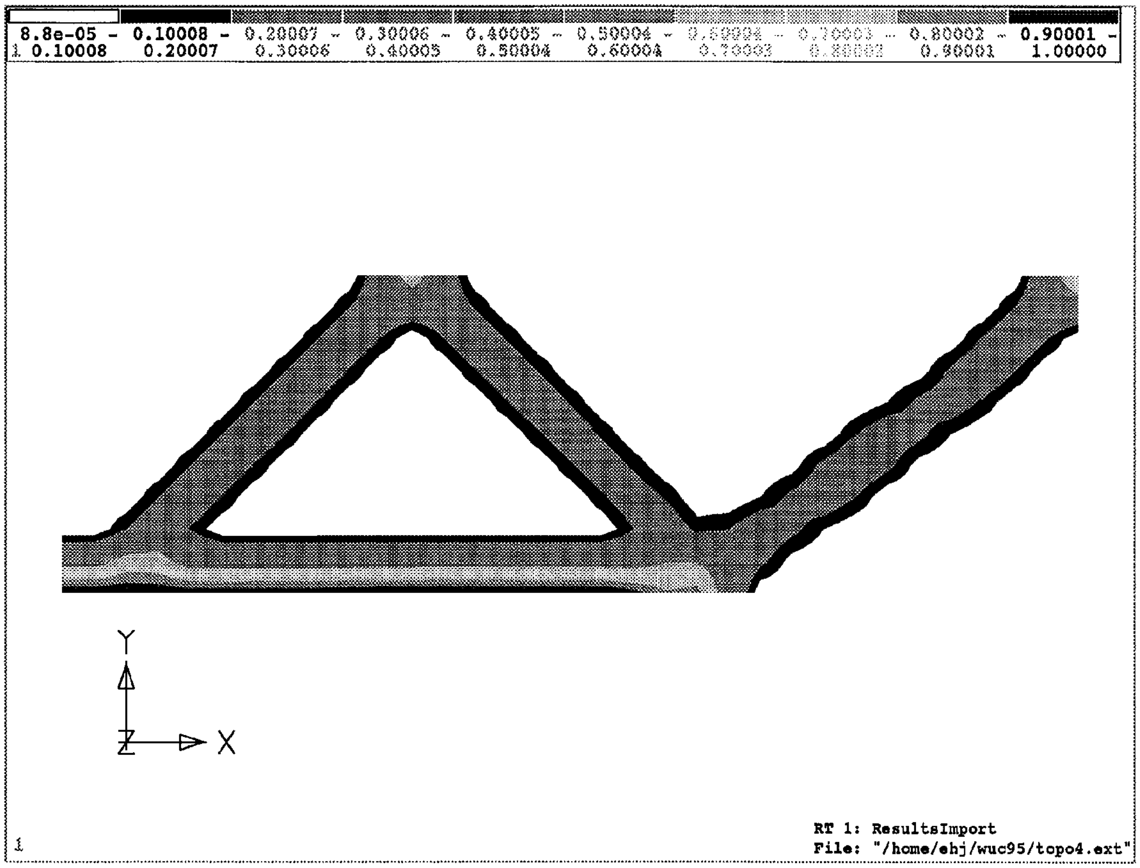


Figure 6. Optimal Topology for Cantilever Beam Example. Thicknesses Less than 0.1 Have Been Omitted for Clarity.

For a second example, the design task is to maximize the first natural frequency of a structure subject to an equality constraint on the structural weight. Figure 7 shows the dimensions of the initial structure, which is clamped at each end. A concentrated nonstructural mass with a magnitude of 350 is located at the center of the structure. This has the effect of making the structural mass negligible in the normal modes analysis. In this case, symmetry considerations permit the analysis of a quarter mode with the u and Θ_Z displacements constrained to be zero along the vertical centerline and u constrained to zero along the horizontal centerline. In this case, the final structure is constrained to have a nonstructural mass that is one half of the value that would pertain for a uniform structure with a unit thickness.

Figure 8 shows the final design of a quarter model for this case. The first frequency for the optimal design is 8.873 Hz.

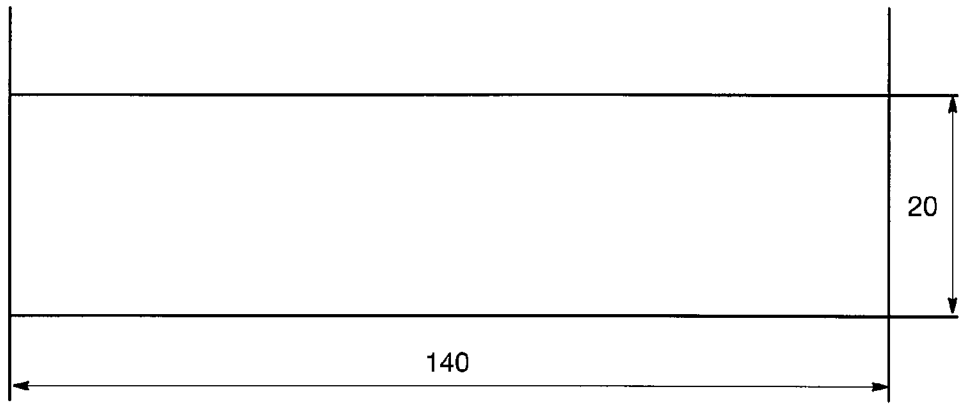


Figure 7. Clamped-Clamped Beam. A Mass with a Magnitude of 350 Is at the Center of the Beam (from Wang, et al., 1994).

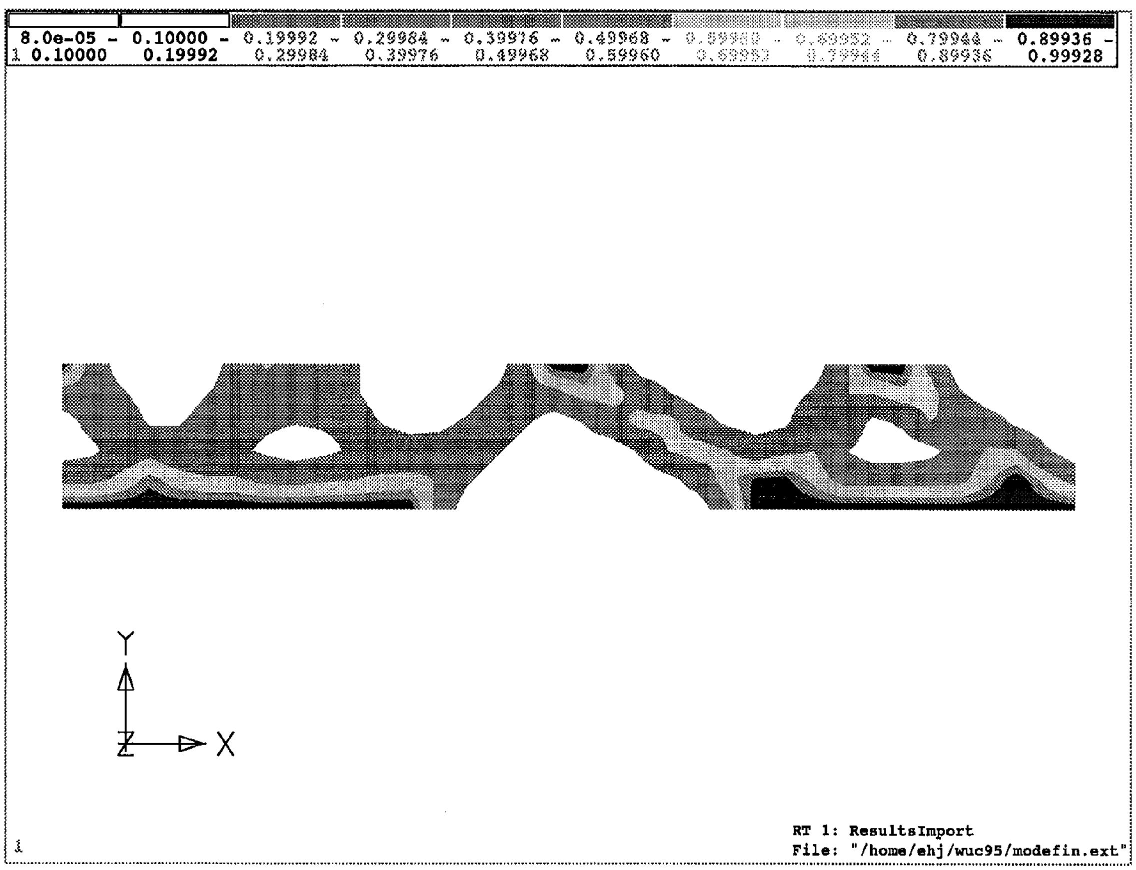


Figure 8. Optimal Topology for the Clamped-Clamped Beam Example. Thicknesses Less than 0.2 Have Been Omitted for Clarity.

DISCUSSION AND CONCLUSIONS

The intent of the paper is to demonstrate the versatility of the synthetic response in MSC/NASTRAN via nonstandard examples. It is recognized that presenting four examples dilutes the impact of each and that there may be a desire for added detail. Space limitations prohibit this, but this section offers a few additional comments and tips on each of the examples.

The example that minimized the maximum stress required the weighting of a β parameter. The parameter itself was initialized to be unity since the optimizer prefers design variables of this magnitude. For the stress example, the objective was obtained by weighting β by 10000. This was approximately the size of the expected maximum stress. In other applications, it might be desired to minimize the maximum displacement. In this case, a weighting equal to the expected magnitude of the displacement may be more appropriate.

In another scenario, it may be desired to minimize the magnitude of the maximum compressive stress. In this case, a constraint of the form

$$-10000\beta + \sigma < -10000 \tag{8}$$

would be appropriate.

The minimization of the mean square response example provides a solution for this special purpose requirement. The required user input required can be voluminous, and it would be preferred to have a special purpose DRESP1 type for this case. User interest and experience is solicited in this area.

The local buckling criteria example demonstrated a way of inserting an IF–THEN–ELSE type branch within MSC/NASTRAN's synthetic response capability. One caution here is that the technique has the most likelihood of being successful if the synthetic function is continuous and has a continuous first derivative across the branch point(s). Otherwise, the optimizer is likely to thrash across the branch and be unable to achieve a converged optimal design.

A number of comments could be made on the topology optimization examples. The key one is that the capability has to be characterized as being at the research level and is more a curiosity than a practical design tool. For the current paper, an exponent of four for the thickness/density relationship of Equation 6 was found to give the best performance, with values of two and three giving results that were qualitatively similar, but lacking in the clarity exhibited by the results shown. The designs consumed more design cycles than is customary, with 15 to 30 cycles a typical figure. This could be caused by the very flat nature of the optimum design space. Each design cycle would show a relatively small improvement in the objective, while the design variables and, more dramatically, the thicknesses would change significantly. The examples shown are meant to duplicate those given in an earlier paper. It should be pointed out however, that the technique is not restricted to the minimization of a compliance number or the maximization of a eigenvalue. The full mathematical programming features of MSC/NASTRAN can be applied to the topology optimization task.

A final, general comment is that the current synthetic response capability could be extended further if there was the possibility of bringing in external responses or allowing the user to do manipulations with the design variables and design responses that are currently beyond the DEQATN capability. A way of doing this would be to allow IPCs (Inter-Process Communication) between MSC/NASTRAN and a user-written software. IPCs are currently available in Version 68 of MSC/NASTRAN for the geometry definition of p-elements (Vaidyanathan, 1994). The application of this technology to design optimization could represent a major advance in the current capability.

ACKNOWLEDGMENTS

The use of a β function for minimizing the maximum stress was pointed out to me by Dr. Gary Vanderplaats of VMA Technology. Mr. Chris Porter assisted with the topology optimization examples during his tenure as a summer intern at MSC.

REFERENCES

1. **Basso,W., Pievani, S., Bindolino, G. and Ricci, S.** (1993), Optimisation Techniques in Aeronautical Structural Design, *Proceedings of the 20th MSC European Users' Conference*, Paper No. 37, September, 1993, Vienna, Austria.

2. **Blakely, K.D.** (1994), Matching Frequency Response Test Data with MSC/NASTRAN, *The MSC 1994 World Users' Conference Proceedings*, Paper No. 17, June 1994, Lake Buena Vista, Florida.

3. **Herting, D.N.** (1994), Parameter Estimation Using Frequency Response Tests, *The MSC 1994 World Users' Conference Proceedings*, Paper No. 18, June 1994, Lake Buena Vista, Florida.

4. **Moore, G.J.** (1994), *MSC/NASTRAN Design Sensitivity and Optimization User's Guide,* Version 68, The MacNeal-Schwendler Corporation.

5. **Peery, D.J, and Azar, J.J.** (1982), *Aircraft Structures,* McGraw-Hill Book Company.

6. **Reymond, M., and Miller, M.** (1992), *MSC/NASTRAN Quick Reference Guide*, The MacNeal-Schwendler Corporation.

7. **Taylor, J.E., and Bendsøe, M.P.** ***(1984),*** An Interpretation of Minmax Structural Design Problems Including a Method for Relaxing Constraints, *International Journal of Solids and Structures*, 20, 301–314.

8. **Wallerstein, D.V.** ***(1985),*** User Interface Document and Portions of EDO's ADS Into MSC/NASTRAN with an Outline of Necessary Other Modules to Interface, Internal Design Document DVW-10, The MacNeal-Schwendler Corporation.

9. **Vaidyanathan, Suresh** ***(1994),*** *MSC/NASTRAN Geometry Evaluator Developer's Guide*, The MacNeal-Schwendler Corporation.

10. **Wang, B.P., Lu, C.M. and Yang, R.J.** ***(1994),*** Topology Optimization Using MSC/NASTRAN, *The MSC 1994 World Users' Conference Proceedings*, Paper No. 12, June 1994, Lake Buena Vista, Florida.

11. *MSC/NASTRAN Release Notes for Version 68*, The MacNeal-Schwendler Corporation, 1994.

APPENDIX

```
ID MSC, $ MINIMIZATION OF THE MAXIMUM STRESS
TIME  130
SOL 200         $  OPTIMIZATION
CEND
TITLE           = MINIMIZATION OF THE MAXIMUM STRESS
SUBTITLE        = CANTILEVERED PLATE
ECHO            = BOTH
SPC             = 100
DISP(PLOT)      = ALL
STRESS(PLOT)    = ALL
ANALYSIS        = STATICS
DESOBJ          = 35
DESGLB          = 10
DESSUB          = 1
SUBCASE 1
        LABEL = CONCENTRATED LOADING
        LOAD  = 300
SUBCASE 2
        LABEL = PRESSURE LOADING
        LOAD  = 310
BEGIN BULK
PARAM   NASPRT  1
$
$ DESIGN DATA $
$
CQUAD4  1       1       1       2       12      11
=       *(1)    *(1)    *(1)    *(1)    *(1)    *(1)
=6
CQUAD4  11      1       11      12      22      21
=       *(1)    *(1)    *(1)    *(1)    *(1)    *(1)
=6
DCONSTR 1       33      -2.     2.
DESVAR  1       ALFA1   1.0     -1.0E10 1.0E10
DESVAR  2       ALFA2   1.0     -1.0E10 1.0E10
DESVAR  3       ALFA3   1.0     -1.0E10 1.0E10
DESVAR  12      BETA    1.0
DOPTPRM APRCOD  1       DESMAX  20      DELP    .5
        DPMIN   .01     CONV2   0.1     DELB    0.01    DELX    2.0
        P1      1       P2      15
DRESP1  2       S12     STRESS  PSHELL          9               1
        2       3       4       5       6       7       8
DRESP1  4       S14     STRESS  PSHELL          17              1
        2       3       4       5       6       7       8
DRESP1  33      D1      DISP                    3               9
        29
DRESP1  5       W       WEIGHT
DCONSTR 10      5       10.0    250.0
DRESP2  35      BETA    35
        DESVAR  12
DEQATN  35      OBJ(BETA) = 10000.0 * BETA
DRESP2  10      SIGBET  10
        DESVAR  12
        DRESP1  2
DRESP2  11      SIGBET  10
        DESVAR  12
        DRESP1  4
DEQATN  10       F(BETA,SIGMA) = 10000.0 * BETA - SIGMA + 100000.
DCONSTR 1       10      100000. 1.0E35
DCONSTR 1       11      100000. 1.0E35
DSCREEN DISP    -.5     10
DSCREEN EQUA    -.5     10
DVPREL1 1       PSHELL  1       4
        1       1.0     2       1.0     3       1.0
DVPREL1 2       PSHELL  2       4
        1       1.0     2       0.875   3       0.765625
DVPREL1 3       PSHELL  3       4
        1       1.0     2       0.750   3       0.5625
DVPREL1 4       PSHELL  4       4
        1       1.0     2       0.625   3       0.390625
DVPREL1 5       PSHELL  5       4
        1       1.0     2       0.5     3       0.2500
DVPREL1 6       PSHELL  6       4
        1       1.0     2       0.375   3       0.140625
DVPREL1 7       PSHELL  7       4
        1       1.0     2       0.250   3       0.0625
DVPREL1 8       PSHELL  8       4
        1       1.0     2       0.125   3       0.015625
FORCE   300     9               50000.  0.0     0.0     -1.0
FORCE   300     29              50000.  0.0     0.0     -1.0
GRID    1               0.      -5.     0.
=       *(1)    =       *(5.)   =       =
=7
GRID    11              0.      0.      0.
=       *(1)    =       *(5.)   =       =
=7
GRID    21              0.      5.      0.
=       *(1)    =       *(5.)   =       =
=7
MAT1    51      1.0E+7          0.33    0.1
        50000.  50000.  29000.
PLOAD2  310     -60.44  11      THRU    18
PLOAD2  310     -60.44  1       THRU    8
PSHELL  1       51      3.0     51              51
PSHELL  2       51      2.64062551
PSHELL  3       51      2.3125  51
PSHELL  4       51      2.01562551
PSHELL  5       51      1.75    51
PSHELL  6       51      1.51562551
PSHELL  7       51      1.3125  51
PSHELL  8       51      1.14062551
SPC1    100     123456  1       11      21
SPOINT  10      20
PARAM   POST    0
ENDDATA
```

Figure A1. Input Data File to Minimize the Maximum Stress.

```
ID MSC, JOHEUL
TIME  130       $
SOL 200         $  OPTIMIZATION
CEND
TITLE    = BUCKING TEST CASE
SUBTITLE = JOHNSON/EULER BUCKLING CASE
ECHO   = SORT
DISP   = ALL
SPC    = 100
STRESS = ALL
DESOBJ = 20
SUBCASE 1
        DESSUB = 1
        LABEL  = LOAD CONDITION 1
        LOAD   = 300
        ANALYSIS = STATICS
SUBCASE 2
   DESSUB = 2
   ANALYSIS = BUCK
   METHOD     = 1
   DISP       = ALL
   LABEL      = BUCKLING FACTORS
$
BEGIN BULK
EIGRL   1       .05             4
DRESP1  1       BUCK1   LAMA                            1
DCONSTR 2       1       1.0
CBAR    1       10      1       2       10
CBAR    2       10      2       3       10
CBAR    3       10      3       25      10
CBAR    25      10      25      4       10
CBAR    4       10      4       5       10
CBAR    5       10      5       6       10
DESVAR  1       RG      1.0     0.01    10.0
DOPTPRM APRCOD  2       P2      15      DESMAX  20      DELP    0.5
        DPMIN   0.1     PTOL    1.0E35  CONV1   0.002   CONV2   0.1
        GMAX    0.01    DELB    0.01    CONVDV  0.01    CONVPR  0.02
        P1      1
DRESP1  20      W       WEIGHT
DRESP1  23      S1      STRESS  PBAR            7               10
DRESP1  24      S1      STRESS  PBAR            8               10
DRESP1  25      S1      STRESS  PBAR            6               10
DRESP2  31      EUL     31
        DESVAR  1
        DTABLE  PI      L       E
        DRESP1  25
DEQATN  31      RGYRA(R,PI,L,E,SIGMA) = R / 2.0;
                EULER = - SIGMA * ( L / RGYRA ) **2 / (PI**2 * E)
DRESP2  32      JOHNSON 32
        DESVAR  1
        DTABLE  PI      L       E       SIGMAC
        DRESP1  25
DEQATN  32      LORHO2(R,PI,L,E,SIGMAC,SIGMA) = (2.0 * L / R ) ** 2;
                FAC = MAX((-SIGMA - SIGMAC/2.0), 0.0);
                JOHNSON = - FAC * SIGMA / ((-SIGMA - SIGMAC/2.0) *
                                SIGMAC * ( 1.0 - SIGMAC * LORHO2/
                                (4.0 * PI**2 * E) ) )
DCONSTR 1       23              100000.
DCONSTR 1       24      -80000.
DCONSTR 1       31              1.0
DCONSTR 1       32              1.0
DTABLE  PI      3.14159 E       30.0E6  L       60.0    SIGMAC  8.0E4
DVPREL2 10      PBAR    10      4                       10
        DESVAR  1
        DTABLE  PI
DVPREL2 20      PBAR    10      5                       20
        DESVAR  1
        DTABLE  PI
DVPREL2 30      PBAR    10      6                       20
        DESVAR  1
        DTABLE  PI
DEQATN  10      AREA(R,PI) = PI * R**2
DEQATN  20      I1(R,PI) = PI * R**4 / 4.0
$SCREEN EQUA    -100.
FORCE   300     6               -2.0E5  1.0
GRDSET                                                  345
GRID    1               0.0     0.0     0.0
GRID    2               12.0    0.0     0.0
GRID    3               24.0    0.0     0.0
GRID    25              30.0    0.0     0.0
GRID    4               36.0    0.0     0.0
GRID    5               48.0    0.0     0.0
GRID    6               60.0    0.0     0.0
GRID    10              0.0     0.0     100.0           123456
MAT1    1       3.0E7           0.33    0.1
PBAR    10      1       1.0     1.0     1.0
SPC1    100     1       1
SPC1    100     2       1       6
PARAM   DSNOKD  0.0
ENDDATA
```

Figure A2. Input Data File Applying a Johnson/Euler Cirterion.

```
ID MSC, TOPO
TIME 100
SOL 200
CEND
TITLE = SIMULATION OF TOPOLOGY WITH SIZING OPTIMIZATION
DISP(PLOT) = ALL
$STRESS = ALL
SUBCASE 1
SPC          = 100
DESGLB       = 10
ANALYSIS = MODES
METHOD = 10
DESOBJ(MAX) = 1
BEGIN BULK
CONM2   50100   501             87.5
PARAM, WTMASS, .002588
$-----------------------------------------------------------------------
$            ANALYSIS MODEL  III  EIGENVALUE PROBLEM
$-----------------------------------------------------------------------
$
$...GRID AND SPC DATA:
$
GRDSET, ,      ,      ,      ,      ,      ,      345
GRID,   1,     ,      0.,    0.,    0.
=,      *(1),  ,      *(2.), =,     =
=34
GRID,   101,   ,      0.,    2.,    0.
=,      *(1),  ,      *(2.), =,     =
=34
GRID,   201,   ,      0.,    4.0,   0.
=,      *(1),  ,      *(2.), =,     =
=34
GRID,   301,   ,      0.,    6.0,   0.
=,      *(1),  ,      *(2.), =,     =
=34
GRID,   401,   ,      0.,    8.0,   0.
=,      *(1),  ,      *(2.), =,     =
=34
GRID,   501,   ,      0.,    10.0,  0.
=,      *(1),  ,      *(2.), =,     =
=34
SPC1,   100,   16     1      101    201    301    401    501
SPC1,   100,   126    36     136    236    336    436    536
SPC1    100    1      502    THRU   535
$
$...ELEMENT DEFINITION AND PROPERTIES:
$   (ELEMENTS GROUPED BY PID SINCE THICKNESS OF ALL ELEMENTS IN A
GROUP
$    ARE TO BE AFFECTED BY A SINGLE DESIGN VARIABLE)
$
MAT1,   150,   2.07E5,  ,      0.3,   0.1
$...ELEMENT GROUP 1:
CQUADR,  1,    1,     1,    2,    102,    101
=,      *(1), *(1), *(1), *(1), *(1), *(1)
=33
CQUADR,  36,   36,    101,  102,   202,   201
=,      *(1), *(1), *(1), *(1), *(1), *(1)
=33
CQUADR,  71,   71,    201,  202,   302,   301
=,      *(1), *(1), *(1), *(1), *(1), *(1)
=33
CQUADR,  106,  106,   301,  302,   402,    401
=,      *(1), *(1), *(1), *(1), *(1), *(1)
=33
CQUADR,  141,  141,   401,  402,   502,    501
=,      *(1), *(1), *(1), *(1), *(1), *(1)
=33
PSHELL, 1,     150,    .50,    150
=,     *(1),   =,      =,      =,
=173
$-----------------------------------------------------------------------
$
$...SPECIFY DESIGN VARIABLES, THICKNESS = DESIGN VARIABLE TO THE
$   FOURTH POWER
DESVAR, 1,     T1,     .5,    .001,    1.0
=,     *(1),   =,      =,      =,  =
=173
$
$...RELATE DESIGN VARIABLES TO PLATE THICKNESSES
DVPREL2,101,    PSHELL, 1,      4,      .00008, ,     200,      ,
+00
=,      *(1),   =,      *(1),   =,      =,      =,     =,     =,
 *(1)
=173
+00,    DESVAR  1
*(1),   =   ,   *(1)
=173
$
DEQATN  200     THICK(X) =  X**4
$
DRESP1  201     VOLUME  VOLUME
DRESP1  1       G1033   EIGN                    1
$
EIGRL,     10,       ,        ,       10,      0
$
```

```
DEQATN  100      MASS(X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,
        X11,X12,X13,X14,X15,X16,X17,X18,X19,X20,
        X21,X22,X23,X24,X25,X26,X27,X28,X29,X30,
        X31,X32,X33,X34,X35,X36,X37,X38,X39,X40,
        X41,X42,X43,X44,X45,X46,X47,X48,X49,X50,
        X51,X52,X53,X54,X55,X56,X57,X58,X59,X60,
        X61,X62,X63,X64,X65,X66,X67,X68,X69,
        X70, X71,X72,X73,X74,X75,X76,X77,X78,X79,
        X80, X81,X82,X83,X84,X85,X86,X87,X88,X89,
        X90, X91,X92,X93,X94,X95,X96,X97,X98,X99,
        X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,
        X110,X111,X112,X113,X114,X115,X116,X117,X118,X119,
        X120,X121,X122,X123,X124,X125,X126,X127,X128,X129,
        X130,X131,X132,X133,X134,X135,X136,X137,X138,X139,
        X140,X141,X142,X143,X144,X145,X146,X147,X148,X149,
        X150,X151,X152,X153,X154,X155,X156,X157,X158,X159,
        X160,X161,X162,X163,X164,X165,X166,X167,X168,X169,
        X170,X171,X172,X173,X174,X175) =
              X1+X2+X3+X4+X5+X6+X7+X8+X9+X10+
        X11+X12+X13+X14+X15+X16+X17+X18+X19+X20+
        X21+X22+X23+X24+X25+X26+X27+X28+X29+X30+
        X31+X32+X33+X34+X35+X36+X37+X38+X39+X40+
        X41+X42+X43+X44+X45+X46+X47+X48+X49+X50+
        X51+X52+X53+X54+X55+X56+X57+X58+X59+X60+
        X61+X62+X63+X64+X65+X66+X67+X68+X69+
        X70+ X71+X72+X73+X74+X75+X76+X77+X78+X79+
        X80+ X81+X82+X83+X84+X85+X86+X87+X88+X89+
        X90+ X91+X92+X93+X94+X95+X96+X97+X98+X99+
        X100+X101+X102+X103+X104+X105+X106+X107+X108+X109+
        X110+X111+X112+X113+X114+X115+X116+X117+X118+X119+
        X120+X121+X122+X123+X124+X125+X126+X127+X128+X129+
        X130+X131+X132+X133+X134+X135+X136+X137+X138+X139+
        X140+X141+X142+X143+X144+X145+X146+X147+X148+X149+
        X150+X151+X152+X153+X154+X155+X156+X157+X158+X159+
        X160+X161+X162+X163+X164+X165+X166+X167+X168+X169+
        X170+X171+X172+X173+X174+X175
DRESP2  100     MASS    100
        DESVAR  1       2       3       4       5       6       7
                8       9       10      11      12      13      14
                15      16      17      18      19      20      21
                22      23      24      25      26      27      28
                29      30      31      32      33      34      35
                36      37      38      39      40      41      42
                43      44      45      46      47      48      49
                50      51      52      53      54      55      56
                57      58      59      60      61      62      63
                64      65      66      67      68      69      70
                71      72      73      74      75      76      77
                78      79      80      81      82      83      84
                85      86      87      88      89      90      91
                92      93      94      95      96      97      98
                99      100     101     102     103     104     105
                106     107     108     109     110     111     112
                113     114     115     116     117     118     119
                120     121     122     123     124     125     126
                127     128     129     130     131     132     133
                134     135     136     137     138     139     140
                141     142     143     144     145     146     147
                148     149     150     151     152     153     154
                155     156     157     158     159     160     161
                162     163     164     165     166     167     168
                169     170     171     172     173     174     175
$
DCONSTR 10      100     87.0    88.0
DOPTPRM DESMAX  10      P1      1       P2      15      DELP    0.5
        DELX    0.4     CONV1   1.0E-6  CONV2   1.0E-4  METHOD  2
$
$PARAM, OPTEXIT, 4
PARAM   POST    0
PARAM   NASPRT  1
ENDDATA
```

Figure A3. Input Data File to Find the Topology That Maximizes the First Natural Frequency.