## Applications of Finite Element Analysis for an Improved Musical Instrument Design

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# Abstract

Implementation of finite element analysis and proof of its power as a design to ol to an industry unfamiliar with this type of technology will be demonstrated through the use of MSC/NASTRAN and MSC/ARIES to develop a comprehensive guitar neck system. This system will provide a better playing, better sounding instrument by accounting for the following:

- -bending and twisting of the neck due to string forces
- -bending and twisting of the neck due to moisture content expansion forces -the cylindrical orthotropic nature of wood
- -individual musician's preferences (neck adjustment capability)
- -elimination of less resonant "dead spots" which occur for certain notes on the neck

Severalanalysis types will be utilized for different steps of the design phase a s follows:

- -linear static analysis for stability against string and moisture content loads
- -nonlinear slideline contact analysis for developing the adjustment capability
- -unstressed and prestressed modal analysis for tuning out "dead spots"

#### Introduction

The guitar industry is a unique place to be employed as a design engineer. It has thus far been successful at eluding even the slightest need to incorporate engineering analysis into the design of it's products in order to demonstrate continued and steep financial growth. The reasons for this are plenty and also cause obvious difficulties for the engineer attempting to prove that such analyses will generate a superior product. Guitar sales are heavily driven by trends in the music industry, aesthetically pleasing shapes and colors, and all the other marketing issues that are of little, if any, concern to an engineering analyst. It is doubtful an advertising campaign claiming Fender guitar's to be the best engineered guitars in the world would have a large, if any, effect on the guitar buying public, who nowadays believe a term like "vintage" (in other words old) to be the mark of excellence.

This paper demonstrates the first step to incorporating engineering analysis into guitar design by addressing an issue of vital importance to all players, the guitar's neck. The significance of neck behavior to the musician will be discussed shortly in the Problem Definition section and will gain clarity as the paper continues. The Problem Definition section will also define the forces that cause this complicated neck behavior, along with addressing wood as an engineering material. The Analysis section will then go on to show how MSC/NASTRAN and MSC/ARIES were used to systematically address the defined problems. The Solution section of the paper describes the comprehensive neck design developed using this technology.

#### Problem Definition

The ultimate goal in the design of a guitar neck is to obtain and maintain balance on the fine line of playability without sacrifice to sound quality. Playability can be defined as the distance, and thus the finger force, necessary to play, or fret, a note cleanly. Obviously the most desired case is the smallest possible string to fret distance at all frets along the entire span of the neck. In attempting to obtain this optimum distance, however, several challenges arise. Most prominant of these is the annoying fret buzz sound caused by the string slapping against a higher fret as it vibrates in its elliptical path as revealed in *figure 1*. Depending on the intensity and the direction in which the string is plucked, the amplitude of the string in the direction of the fret will vary. Since each musician's playing technique is different, an ability to adjust the neck, with what is called a truss rod, becomes a requirement in order to eliminate buzz.

The question may now be asked as to why fret buzz could not be eliminated in the manufacturing process by simply dressing, or sanding down, the deviant fret until the buzz subsided. This in fact is done, and is a necessary part of neck production, but unfortunately it is a temporary fix. This fix is merely temporary due to the ever changing forces acting on the neck and the subsequent displacements they incur.

In order to properly understand the complexity of the forces acting on the neck, wood, the primary constructive material of the neck, must first be addressed as an engineering material. The design of wooden structures contains several pitfalls inherent in working with a substance produced by nature. Besides the fact of wide variance in engineering properties from piece to piece, wood is an orthotropic material who's orthotropic properties are alligned in a cylindrical coordinate system.

This would not pose as much of an engineering problem if each of the neck

blanks (the rectangular form in which the wood is received before being cut into a neck) was cut from exactly the same location along the face of the log. Unfortunately this is not the case. *Figure 2* shows four of these possibilities and assigns the accepted names, flat sawn, rift sawn, and quarter sawn, to three of the grain patterns. There are obviously an infinite number of possiblities of grain angles and distances from the center of the tree which can combine to create odd warping behavior, as the fourth grain pattern in the figure shows. These four grain patterns were used for optimizing the final design, with interpolation between the four inferred.

For all intents and purposes the guitar neck is a long cantilever beam with a large amount of force acting on the free end, as can be seen in *figure 4*. This force originates from the tension of the guitar's strings, which can vary dependent upon the preference of the musician. Since wood is orthotropic, a force in one direction does not transfer to displacement in only that direction. A twisting effect also occurs which again varies with the grain pattern of the wood. This is demonstrated in *figure 3*.

The second set of forces acting on the neck are caused by the moisture content expansion and contraction of wood with changing levels of relative humidity. The cells in the wood exchange moisture with the air until equalization occurs, expanding and contracting as necessary. This causes a neck to move significantly, particularly in the case of musicians who travel to varied climates over short periods of time. As would be expected wood also shrinks orthotropically in a cylindrical coordinate system as shown in *figure 2*, with a moisture content expansion coefficient approximately twice as great in the tangential direction as the radial direction. Their is little to no expansion in the longitudinal direction, and for modeling purposes it is neglected.

From the above discussion it should be apparent why neck stability and adjustability are of utmost importance for ultimate playability. The next issue to be addressed is sound quality. The primary detriment to sound quality which can be controlled by guitar neck design is a phenomena dubbed as "dead spots". As the name implies, these are a specific few notes on the fretboard that, when played, do not resonate like the rest. They die out quickly and lack the accompanying overtones which give a full, rich tone.

All of these factors must be considered in the design of a guitar neck. The engineering complexity becomes apparent upon consideriation of the high tolerances which must be kept under the varying dynamics of such a system. Finite element analysis techniques proved to be an essential tool in the creation of an improved musical instrument. These techniques are discussed in detail below.

## Analysis

Thissection is divided into five parts, each part discussing the finite element methods used to address the design problems mentioned above.

## The Cylindrical Orthotropic Nature of Wood

Modeling wood's orthotropic properties is a relatively easy, though cumbersome, process through the use of the MAT9 bulk data entry. The MAT9 entry generates an anisotropic material matrix for the element, but can be used for orthotropic materials if matrix entries are entered as described below. Since the material is alligned in a cylindrical coordinate system, the subscripts r, t, and I denote the radial, tangential, and longitudinal directions respectively. Thematerial behavior is defined by:

$$\left\{ \begin{matrix} \sigma_{\mathbf{r}} \\ \sigma_{\mathbf{t}} \\ \sigma_{\mathbf{l}} \\ \tau_{\mathbf{r}} \\ \tau_{\mathbf{t}} \\ \tau_{\mathbf{l}} \end{matrix} \right\} = \left[ \mathbf{G} \right] \left\{ \left\{ \begin{matrix} \epsilon_{\mathbf{r}} \\ \epsilon_{\mathbf{t}} \\ \epsilon_{\mathbf{l}} \\ \gamma_{\mathbf{r}} \\ \gamma_{\mathbf{r}} \\ \gamma_{\mathbf{t}} \\ \gamma_{\mathbf{l}} \end{matrix} \right\} - \left( \mathbf{T} - \mathbf{T_{ref}} \right) \left\{ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{matrix} \right\} \right\} equation (1)$$

where [G] is the anisotropic material matrix as defined by

$$\begin{bmatrix} G_{11} & symmetric \\ G_{12} & G_{22} \\ G_{13} & G_{23} & G_{33} \\ G_{14} & G_{24} & G_{34} & G_{44} \\ G_{15} & G_{25} & G_{35} & G_{45} & G_{55} \\ G_{16} & G_{26} & G_{36} & G_{46} & G_{56} & G_{66} \end{bmatrix}$$

for  $\ensuremath{\left[ \mathbf{G} \right]}$  to represent an orthotropic material , the following values for the matrix are applied

$$G_{11} = \frac{1 - v_{tl}v_{lt}}{E_t E_l \Delta}$$

$$G_{12} = \frac{v_{tr} - v_{lr}v_{tl}}{E_t E_l \Delta}$$

$$G_{22} = \frac{1 - v_{rl}v_{lr}}{E_r E_l \Delta}$$

$$G_{13} = \frac{v_{lr} - v_{tr}v_{lt}}{E_t E_l \Delta}$$

$$G_{23} = \frac{v_{lt} - v_{rt}v_{lr}}{E_r E_l \Delta}$$

$$G_{33} = \frac{1 - v_{rt}v_{tr}}{E_r E_t \Delta}$$

where

$$\Delta = \frac{1 - \nu_{rt}\nu_{tr} - \nu_{tl}\nu_{lt} - \nu_{rl}\nu_{lr} - 2\nu_{tr}\nu_{lt}\nu_{rl}}{E_r E_t E_l}$$

and

 $E_i$  = Young's modulus of elasticity in the i direction

 $\nu_{ij}$  = Poissons ratio in the ij plane with i being the direction of forced displacement

also

 $G_{44} = G_{rt}$  $G_{55} = G_{tl}$  $G_{66} = G_{lr}$ 

where

 $G_{ij}$  = modulus of ridgidity (shear modulus) in ij plane

Theremaining matrix entries are set equal to 0.0.

Due to the variety of wood species eventually to be analyzed and the fact that the engineering properties change with changing moisture content, a UNIX shell program was written to generate the appropriate MAT9 entries for insertion into the NASTRAN bulk data. This is also necessary because MSC/ARIES has not yet implemented orthotropic materials into it's input deck generation routine.

In order to specify a cylindrical coordinate system for the material, the fourth field of the PSOLID entry must specify the coordinate system identification number of a CORD1C or CORD2C bulk data entry. In this case a CORD1C entry was utilized by specifying unattached nodes with standard numbers for the three wood grain patterns, flat, rift or quarter sawn. This allowed for easy input deck modification when comparison between saw cuts was sought.

## Modeling Moisture Content Expansion of Wood

As can be seen from *equation (1)*, thermal expansion coefficients, the {A} vector, are considered in the solid elements formulation. As would be expected, thermal expansion and moisture content expansion are based on the same induced strain formula as demonstrated below.

From the Wood Engineering Handbook,

	where:
	$\Delta D_i$ = change in dimension in i direction
$\Delta D_i = D_0 [C_i (M_f - M_0)]$	D <sub>o</sub> = initial dimension
$\Delta D_{1} = D_{0} [C_{1} (101f - 1010)]$	C <sub>i</sub> = moisture expansion coefficient in i direction
	M <sub>f</sub> = final moisture content
	M <sub>o</sub> = initial moisture content

and by definition

$$\mathbf{E}_{i} = \Delta D_{i} / D_{O}$$

therefore

$$\mathbf{E}_{i} = (M_{f} - M_{O}) C_{i}$$

which is analogous to the last part of equation (1) with the following correspondences

$$T = M_f$$
$$T_{ref} = M_O$$
$$A_i = C_i$$

Thiseads to a slightl; y modified view of equation (1) for wood

$$\begin{cases} \sigma_{\mathrm{r}} \\ \sigma_{\mathrm{t}} \\ \sigma_{\mathrm{l}} \\ \tau_{\mathrm{r}} \\ \tau_{\mathrm{t}} \\ \tau_{\mathrm{l}} \end{cases} = [G] \begin{pmatrix} \epsilon_{\mathrm{r}} \\ \epsilon_{\mathrm{t}} \\ \epsilon_{\mathrm{l}} \\ \gamma_{\mathrm{r}} \\ \gamma_{\mathrm{r}} \\ \gamma_{\mathrm{t}} \\ \gamma_{\mathrm{l}} \end{pmatrix} - (M_{\mathrm{f}} - M_{\mathrm{o}}) \begin{pmatrix} C_{\mathrm{r}} \\ C_{\mathrm{t}} \\ C_{\mathrm{l}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

So in effect, moisture contents and moisture content expanson coefficients can be substituted for temperatures and thermal expansion coefficients, thus inducing moisture loads instead of thermal loads, and all this is done while maintaining the natural oddity of wood's cylindrical coordinate system. *Figure 2* illustrates the setup and results for this type of analysis.

## String Forces

The modeling of the string forces and the restraints used to simulate the attachment of the neck to the body of the guitar are shown in *figure 4*. The guitar's strings vary in tension depending on pitch and also on the type of strings a player may prefer. In order to account for the worst case scenario, the highest tension strings which could be found were applied to the model. These were D'Addario set XL230 heavy gauge bass strings, and were rated as follows:

G-1st	58.2 lbs.	
D-2nd	59.9 lbs.	total= 206.8 lb s.
A-3rd	48.9 lbs.	10101-200.010 3.
E-4th	39.8 lbs.	

It should be noted that the main focus of this design analysis was on the bass guitar line, which have either four or five strings, instead of a standard guitar's six. More problems with neck unstability occur in basses due to the significantly longer length of the neck and the higher tension of the bass' strings. The significance of this can be seen by the cantilever beam equation

$$y = \frac{Wl^3}{3EI}$$

where y, the deflection at the free end, is proportional to the length of the beam, I, cubed, but only linearly proportional to the force, W, acting on the free end.

The string force and moisture content loads were used in conjunction with various wood grain patterns in linear static solution sequences in order to develop the stabilization portion of the neck system and enable it to work effectively with the adjustment capability. This is discussed fully in the Solution section of the paper.

## Truss Rod Adjustment Model

Several modeling techniques were used as means to gain better understanding of neck adjustment behavior. Adjustment is a crucial part of neck design due to the variety of forces acting on the neck. MSC/NASTRAN's nonlinear slideline contact ability was ideal for simulating this situation. The lack of a slideline interface between MSC/ARIES and MSC/NASTRAN made the building of slideline contact models a cumbersome process, but was made less so in this case by standardizing a practice of node numbering and creating a text file which could be easily inserted into the MSC/NASTRAN input deck with only slight need of modification.

*Figure 5* shows the basic configuration of the current standard truss rod system. This system was studied first in order to analyze it's behavior and see where improvements could be made. This configuration proved to demonstrate many ineffective, or even detrimental, actions on the neck upon adjustment.

Once the long solution time for contact models with many nodes was discovered, several methods of model reduction were developed for studying specific facets of rod and neck behavior. First of all, several simple linear static beam models were built in order to observe just the rod's behavior with changes in geometry configuration. An example of this is shown in *figure 6*.

The purpose of these geometry changes was to configure the rod for straight adjustment while maintaining the least amount of applied force necessary on the rod to do so. By observing the locations of maximum strain energy density in the wood structure of the neck obtained from the linear solution sequence results, the beam's geometry was modified until it's point of maximum displacement was at this point of maximum neck strain energy density.

A significant design idea resulted from a brief study of these simple linear static beam models and their behavior. This idea, which proved to be an essential part of the final neck system, is described in the Solution section.

It was next necessary to model and observe how and where the rod and neck were coming into contact. Coarse meshed solid models were used for this purpose. as shown in figure 7.

The BOUTPUT command proved greatly beneficial in studying rod contact. This command generates detailed output concerning the slideline contact regions for each load increment, disclosing information such as normal forces and shear forces on the slave nodes. A UNIX shell program was written to extract the normal force data from the .f06 file and put it in an MSC/ARIES table file format. From this information it was possible to plot where along the neck the rod was having contact effects. A plot such as this, along with an example of printed BOUTPUT information, also appears in *figure 7*.

Since it was desired to know how the neck adjusted while string forces were applied, a sequential loading condition was simulated in solution 106 with the use of two subcases. The first apllying previously demonstrated string forces, and the second maintaining those forces while applying a force on the end of the rod.

These slideline contact models were crucial for the development of the integrated neck system. The slot gemetry on which the rod slides was ultimately developed using these models. This geometry is discussed in the Solution section of this paper.

### **Dead Spot Elimination**

Throughout the design phase, modal analyses were done in order to gain insight into the vibratory, and thus sonic, characteristics of the guitar neck. As discussed earlier, the primary focus was to eliminate the occurrance of dead spotsin the neck. It is suspected from previous studies performed by other parties that twisting modes about the longitudinal axis, as shown in *figure 8*, areresponsible for this unwanted behavior. The next obvious step is to determine at what frequency these twisting modes occur and then proceed totune the neck design away from any notes in the tempered musical scale which will drive these modes into dominance. A list of the frequencies in the scale also appear in *figure 8*. As can be seen by the small difference in frequencies between notes, especially in the lower register, tuning the neck appropriately could prove a difficult endeavor. Upon considering the high variability wood's stiffness properties, one may even consider it a futile endeavor.

Two types of modal analysis were done. Linear, unstressed modes, solution sequence 103, were determined for reference. The more relevant prestressed, with string forces, modes were calculated using the PARAM,NMLOOP command in solution 106.

It is understood that physical testing and modal correlation must be implemented into a study such as this in order to truly understand this dynamic systems complicated behavior and more accurately refine the finite element model. All of this is planned for the future. The purpose here was to obtain information which could, at least somewhat, guide design parameters and shedsome light on a guitar neck's dynamic behavior.

#### <u>Solution</u>

This section describes the final solution developed with great help from the power of MSC/ARIES and MSC/NASTRAN. The major improvements to stability, adjustibility, and resonance, which work in concert to produce superior neck playability and sound, will be described seperately, each revealing it's importance and function within the whole scheme as the pieces are fit together.

#### <u>Stability</u>

The original design concept behind the stability phase was to create a load baring member which would take and control all of the forces, thus allowing the surrounding wood to be without essential stiffness properties. This was desired for two reasons. First of all, wood's high variance in elastic moduli make it a difficult material to design with confidently. Second, plans for the replacement of wood on future guitar model with an alternative material in which high stiffness is not essential dictate the need for such a load baring member.

Upon evaluation of several string force and moisture content expansion models with different grain patterns, one fact became clear. The stress center, and likewise the strain enrgy center, was distinct, but highly mobile dependent on the particular configuration and levels of the variables involved. This energy center, as it will henceforth be referred, must be controlled in order to maintain neck playability. The energy center, once controllable, should also residein the width wise center of the neck where the truss rod is located. This is forthe obvious reason of controlling the neck while it is being adjusted. The fundamental engineering stability of the arch was employed in the control of the energy center. For structural and manufacturing reasons it was desired that the load baring member be as close to the upper compression surface as possible. (Attempting to stabilize the lower, tension surface was unfeasible from a manufacturing standpoint.) Upon consideration of further manufacturing constraints and proper geometry for greatest stability, an optimumload baring member was designed with a constant radius and support base as shown in *figure 9* along with plots of strain energy density showing the location of the energy center which is properly placed and consistently maintained under ever changing loading variables, a new truss rod could be designed for ultimate effectiveness.

#### **Adjustability**

Adjustability depends on two factors, rod behavior and the slot geometry onwhich the rod comes in contact. The solution for the two are addressed here seperately first and then as a unit.

#### <u>The Rod</u>

As has been seen from previous deformed geometry plots, The current rod system bends in a very nonuniform fashion. This is the antithesis of what is sought in a device meant to provide straightness. With the idea of straightness in mind, a mechanism, working on the same principles as the standard rod, was derived.

This mechanism provides a straight platform with which the neck is adjusted. This platform was created by the simple addition of a hinge mechanism located in the proper position towards the butt of the neck and also at the far end. The rod displacement is shown in *figure 10*. With the rod functioning as desired, slot geometry then had to be considered.

#### Slot Shape

Coarse solid models, as discussed in the analysis section, were used to evaluate several scenarios of slot geometry. Two adjustment forces had to be considered in the design of the slot. Compensation for much of the string forces can be made by the tension on the rod and the opposite moment it createsabout the neck's inertial center. This adjustment capability is useful, but unfortunately does not adjust the neck as straight as necessary, especially when large adjustments are necessary.

The second factor to consider is cotact forces. The slot must be shaped in order to properly contact the rod in the optimum locations. These locations are demonstrated in *figure 10*, and were arrived at through several iterations forstraightness of adjustment.

The pivot point on the bottom surface of the slot is to provide for the rare caseof a neck which experiences extreme moisture warping and actually overcomes the string forces and bows backwards against them. In this case theneck becomes unplayable due to the strings lying on the frets. When the rod is put into compression instead of tension, this point becomes the pivot andpushes the neck into under bow as shown in *figure 10. Figure 11* demonstrates the greatly improved capability for straight neck adjustment with the new rod and slot system.

## **Conclusion**

Finite element modeling and analysis, through MSC/NASTRAN and MSC/ARIES, were used to develop an improved, integrated guitar neck system. The versatility of these programs allowed for adaptations and applications perfectly suited for a musical instrument company (e.g. the modelingof wood). The demonstration of this technology to upper management allowed for greater acceptance and persual of pure research as integral part of the guitar design process. This impact is especially impressive upon considering Fender's lack of need for advanced engineering technology in order to display continued growth.

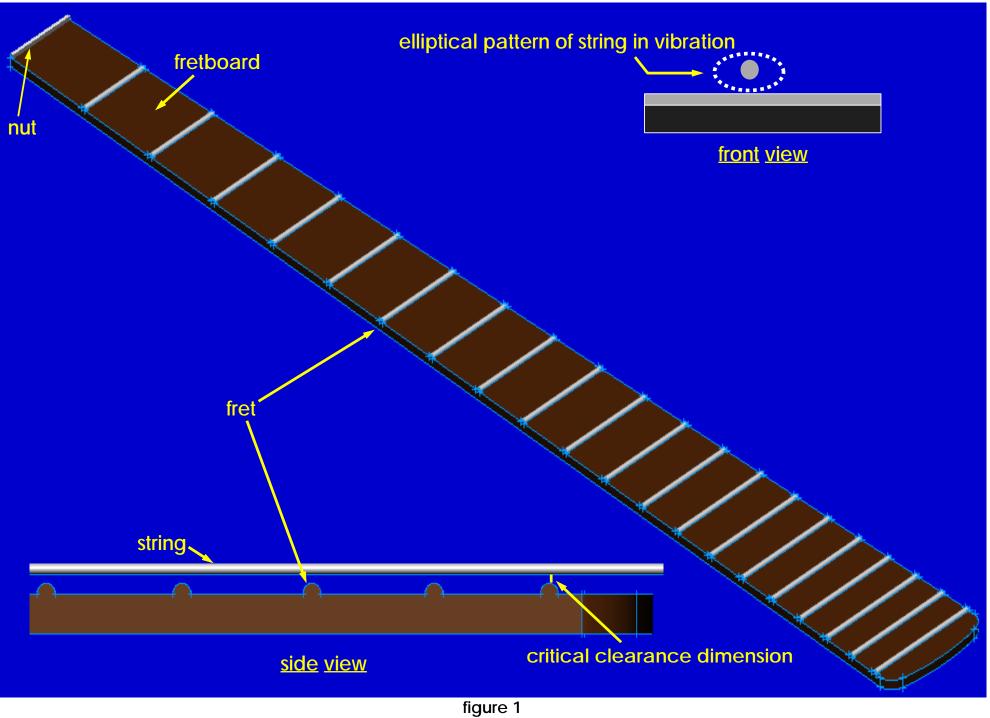
Several problems had to be addressed in order to design this comprehensive neck system. These include the problems of stability, adjustability, and sonics. Different analysis types were implemented in the design. Linear static analysis was used for creating stability under string forces and the moisture content expansion forces of wood. Nonlinear statics was used for developing the truss rod and slot for optimum adjustability. Finally, modal analysis was used to minimize dead spots and thus improve the neck's soniccharacteristics.

In conclusion, this design would not have been possible without the insight gainedfrom finite element analysis and MSC/NASTRAN and MSC/ARIES in particular. Future plans for research include physical testing for material properties along with highly increased dynamic studies incorporating modal testresults with modal finite element analysis results through the use of modal assurance criteria software packages. The first year of applying this technology to guitars has been a success and has opened the door for future scientific intensive ventures into the world of sound.

## **References**

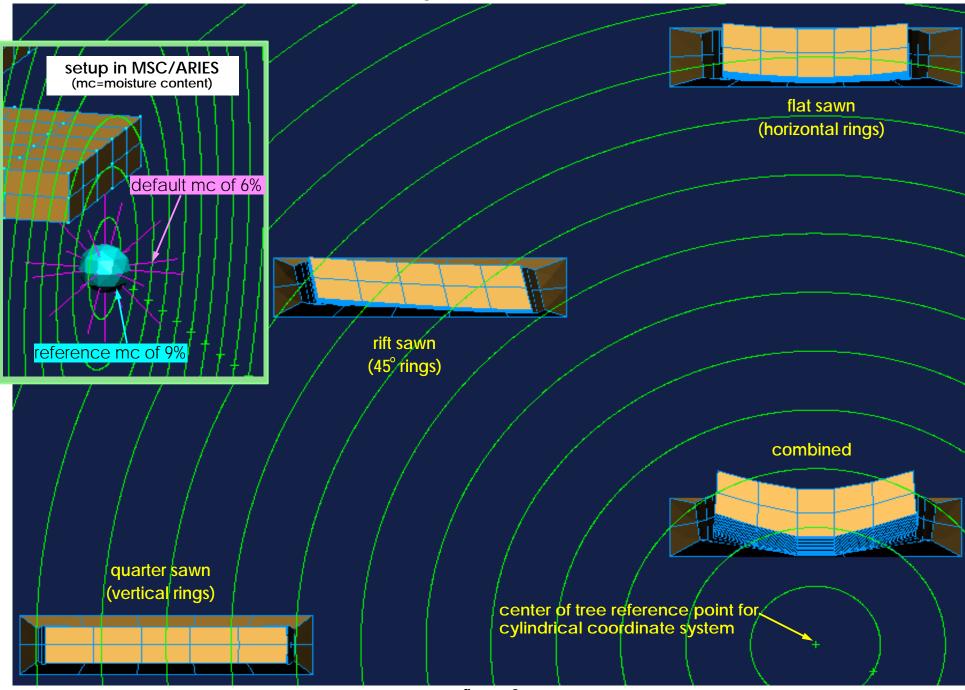
- (1)Backus, John, *The Acoustical Foundations of Music*, W.W. Norton Company Inc., 1969.
- (2)U.S. Forest Products Laboratory, *Wood Engineering Handbook*, Prentice-Hall Inc., 1974.
- (3)Roark, Raymond J., *Formulas for Stress and Strain*, 4th ed., McGraw-Hill, 1965.
- (4)Jahnel, Franz, *Manual of Guitar Technology*, 1st ed., Verlad Das Musikinstrument, 1981.
- (5) *MSC/NASTRAN Linear Static Analysis User's Guide, Version 68*, The MacNeal–Schwendler Corporation, Los Angeles, CA, March 1994.
- (6) *MSC/NASTRAN Basic Dynamic Analysis User's Guide, Version 68*, The MacNeal–Schwendler Corporation, Los Angeles, CA, December 1993.
- (8) *MSC/NASTRAN Version 68 Release Notes*, The MacNeal–Schwendler Corporation, Los Angeles, CA, September 1994.

**Fretboard Definitions** 

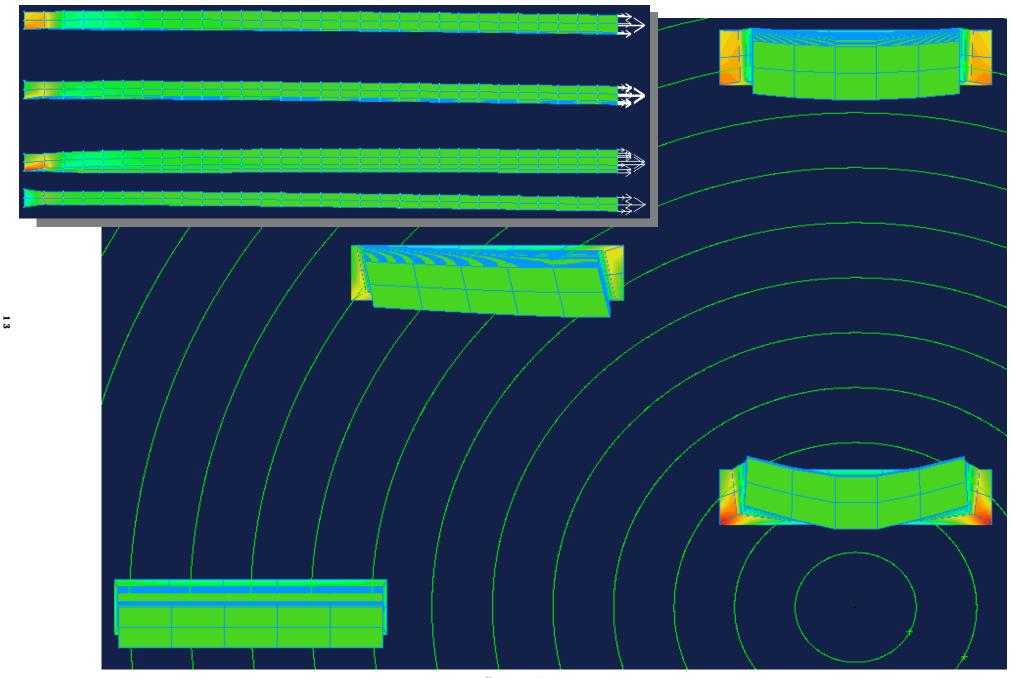


11

# Various Cut Neck Blanks Exhibiting Deformation Due to a 3% loss in moisture content



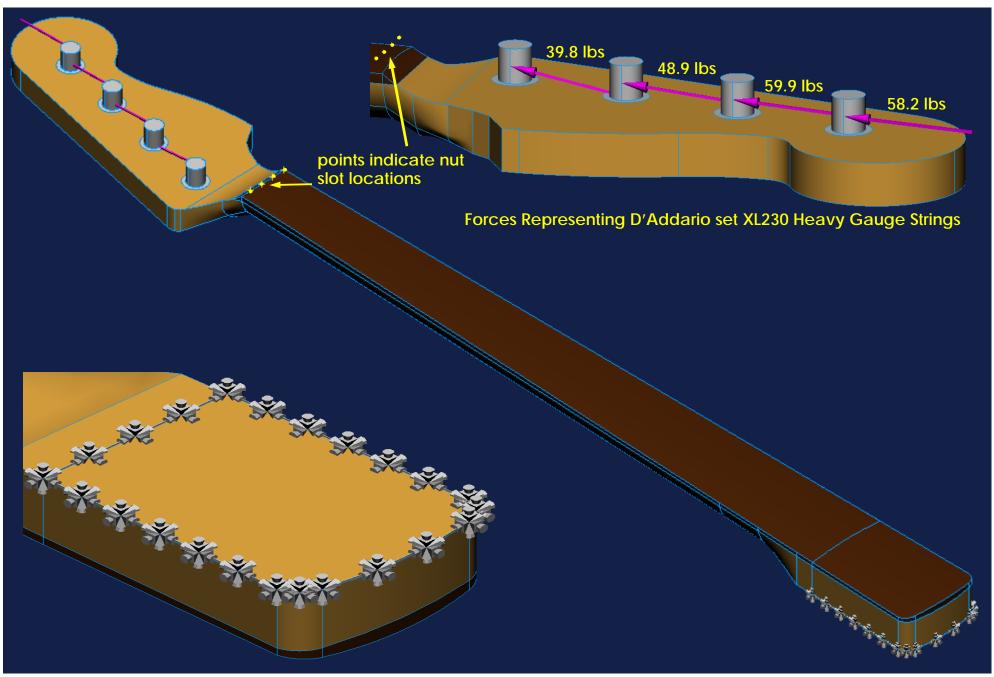
12



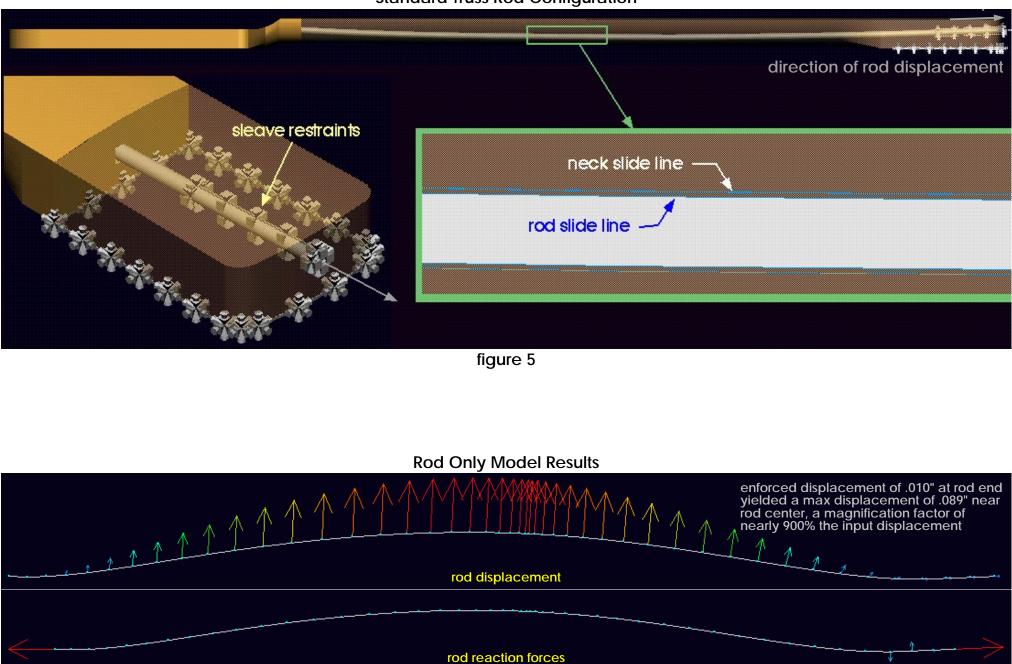
# Orthotropic Twisting & Stresses Due to Force in Longitudinal Direction

figure 3

# Forces and Restraints Applied to Simulate String Tension



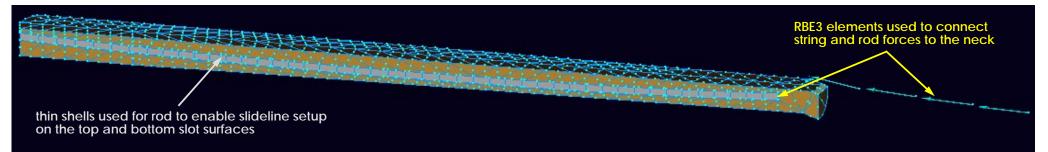
## Standard Truss Rod Configuration



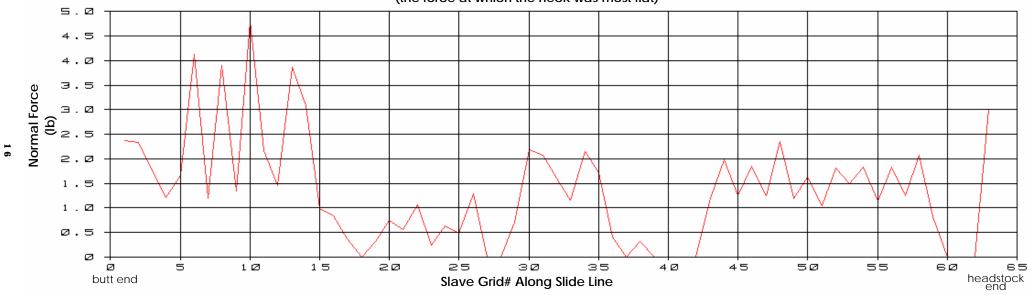
Ċī,

figure 6

## Solid Model Setup of Neck for Study of Slideline Contact Details



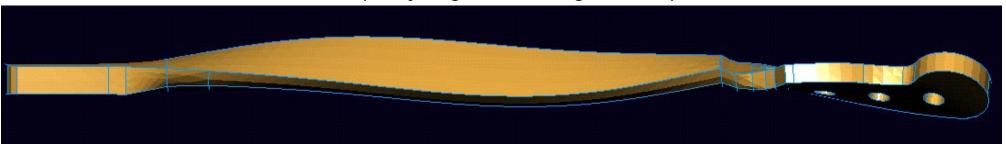
#### Slideline Contact Forces at Applied Rod Force of 825 lbs. (the force at which the neck was most flat)



Example of information generated with BOUTPUT case control command. This data was used to construct the graph above

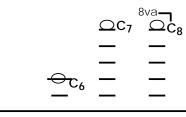
LOA	AD STEP =	1.0000	0E+00 R E S U	LTS F	ORSL	IDE LIN	EELEI	MENTS	(IN ELEMENT	SYSTEM)		
SLAVE	CONTAC	MAS	TER	SURFACE	NORMAL	SHEAR	NORMAL	SHEAR	NORMAL	SLIP	SLIP	SLIP
GRID	ID	GRID1	GRID2	COORDINATE	FORCE	FORCE	STRESS	STRESS	GAP		RATIO	CODE
40133	19	20119	20120	0.306	0.0	0.0	0.0	0.0	-1.4039E-05	5 -4.0515E-03	0.0	OPEN
40134	19	20118	20119	0.081	0.0	0.0	0.0	0.0	-1.6012E-05	5 -3.7525E-03	0.0	OPEN
40135	19	20116	20117	0.858	1.0029E+00	-2.5072E-01	5.5326E+00	-1.3832E+00	4.8109E-08	3 -3.4297E-03	-1.00	SLIP
40136	19	20115	20116	0.639	6.8728E-01	-1.7182E-01	3.8597E+00	-9.6492E-01	3.2250E-08	3 -3.1137E-03	-1.00	SLIP

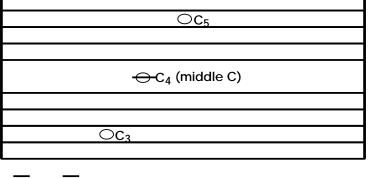
 $\label{eq:lowFrequencyLongitudinal Twisting Mode Shap e$ 



## Frequencies of Notes in the Tempered Scale (Hz)

(indented frequencies represent sharp and flat notes)

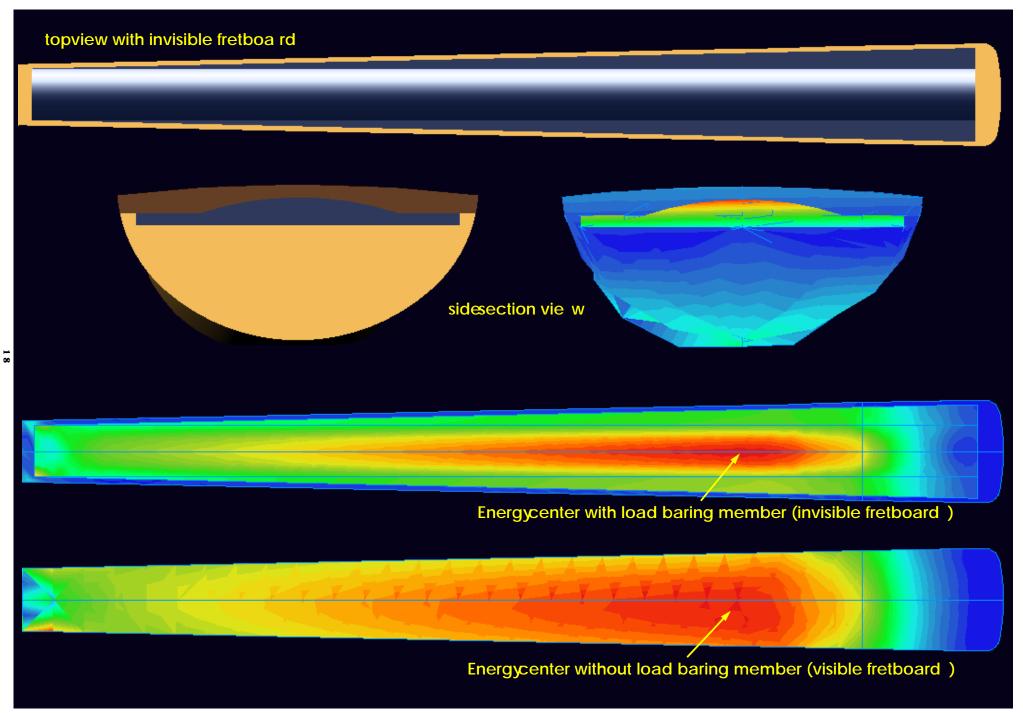




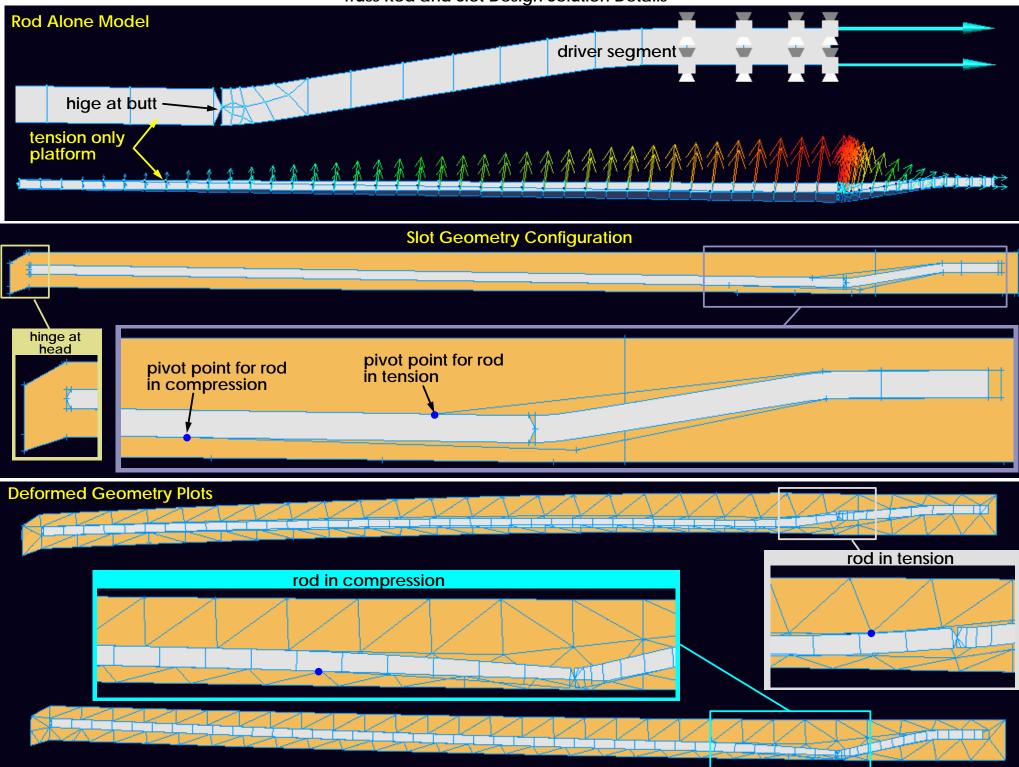
 $\ominus c_2$ 

 $\overline{\bigcirc}_{C_1}$ 

C <sub>0</sub>	16.3515 17.3239	C <sub>3</sub>	130.8127	C <sub>6</sub>	1046.5022
D <sub>0</sub>	18.3540	D3	138.5913 146.8323	D <sub>6</sub>	1108.7305 1174.6590
Ŭ	19.4454	, in the second s	155.5634	Ū	1244.5079
E <sub>0</sub>	20.6017 21.8267		164.8137	E <sub>6</sub>	1318.5101
F <sub>0</sub>	23.1246	F <sub>3</sub>	174.6141 184.9972	F <sub>6</sub>	1396.9128 1479.9776
G <sub>0</sub>	24.4997	G <sub>3</sub>	195.9977	G <sub>6</sub>	1567.9816
Ŭ	25.9565	Ŭ	207.6523	Ŭ	1661.2187
A <sub>0</sub>	27.5000	A <sub>3</sub>	220.0000	A <sub>6</sub>	1760.0000
Ρ.	29.1352 30.8677	Р.	233.0818 246.9416	D.	1864.6549 1975.5331
В <sub>0</sub>	30.0077	B <sub>3</sub>	240.9410	B <sub>6</sub>	1975.5551
$C_1$	32.7031	C₄	261.6255	C <sub>7</sub>	2093.0044
	34.6478		277.1826		2217.4609
D <sub>1</sub>	36.7080 38.8908	D <sub>4</sub>	293.6647	D <sub>7</sub>	2349.3180
E1	41.2034	-	311.1269 329.6275	E7	2489.0157 2637.0203
F <sub>1</sub>	43.6535	E4 F4	349.2282	E7 F7	2793.8257
	46.2493	• 4	369.9944	• /	2959.9552
G <sub>1</sub>	48.9994	G4	391.9954	G <sub>7</sub>	3135.9633
	51.9130	·	415.3046		3322.4374
A <sub>1</sub>	55.0000 58.2704	A4	440.0000	A <sub>7</sub>	3520.0000
р.	61.7354	<b>_</b>	466.1637	<b>D</b> _	3729.3099 3951.0662
В <sub>1</sub>	01.7354	B4	493.8832	B <sub>7</sub>	3931.0002
C <sub>2</sub>	65.4063	C <sub>5</sub>	523.2511	C <sub>8</sub>	4186.0088
P	69.2956	J	554.3652	_	4434.9218
D <sub>2</sub>	73.4161 77.7817	D5	587.3295	D <sub>8</sub>	4698.6360
E <sub>2</sub>	82.4068	_	622.2539 659.2551	F	4978.0314 5274.0406
$F_2$	87.3070	E5 F5	698.4564	E <sub>8</sub> F <sub>8</sub>	5587.6513
_	92.4986	г5	739.9888	18	5919.9104
G <sub>2</sub>	97.9988	<b>G</b> 5	783.9908	G8	6271.9266
	103.8261		830.6093		6644.8747
A <sub>2</sub>	110.0000 116.5409	A <sub>5</sub>	880.0000	A <sub>8</sub>	7040.0000
B <sub>2</sub>	123.4708		932.3275 987.7665	Р	7458.6197 7902.1323
02	120.4700	В <sub>5</sub>	907.7005	B <sub>8</sub>	7702.1323



**Truss Rod and Slot Design Solution Details** 



19

figure10

