COMBINING ACCELERATION AND DISPLACEMENT DEPENDENT MODAL FREQUENCY RESPONSES USING AN MSC/NASTRAN DMAP ALTER

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Abstract

Solving for dynamic responses of free-free launch vehicle / spacecraft systems acted upon by buffeting winds is commonly performed throughout the aerospace industry. Due to the unpredictable nature of this wind loading event, these problems are typically solved using frequency response random analysis techniques. To generate dynamic responses for spacecraft with statically-indeterminate interfaces, spacecraft contractors prefer to develop models which have response transformation matrices developed for mode acceleration data recovery. This method transforms spacecraft boundary accelerations and displacements into internal responses. Unfortunately, standard MSC/NASTRAN modal frequency response solution sequences cannot be used to combine acceleration- and displacement-dependent responses required for spacecraft mode acceleration data recovery. External user-written computer codes can be used with MSC/NASTRAN output to perform such combinations, but these methods can be labor and computer resource intensive. Taking advantage of the analytical and computer resource efficiencies inherent within MSC/NASTRAN, a DMAP Alter has been developed to combine acceleration- and displacement-dependent modal frequency responses for performing spacecraft mode acceleration data recovery. The Alter has been used successfully to efficiently solve a common aerospace buffeting wind analysis.

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Nomenclature

Abbreviations

DMAP	Direct Matrix Abstraction Program
DOF	Degree-of-freedom
ELV	Expendable Launch Vehicle
FRF	Frequency Response Function
OTM	Output Transformation Matrix
RMS	Root-mean-square

Matrices

a	Accelerations
B	Damping
G	Physical transformation
I	Identity
K	Stiffness
M	Mass
P	Applied loads
R	Physical responses
u	Displacements
v	Velocities
Φ	Modal transformation

Set Notation

a	a-set (assembled DOF)
A	Acceleration-dependent
b	b-set (physical boundary DOF)
C	Acceleration- & Displacement-dependent
D	Displacement-dependent
g	g-set (global DOF)
h	h-set (system modal DOF)
i	Applied loads index
j	Output frequency index

Introduction

Solving for dynamic responses of free-free launch vehicle / spacecraft systems acted upon by buffeting winds is commonly performed throughout the aerospace industry. Due to the unpredictable nature of this wind loading event, these problems are typically solved using frequency response random analysis techniques. Solving for coupled system frequency domain responses is commonly performed at the coupled system modal DOF level in order to take advantage of analytical efficiencies. Modal frequency response analysis is offered in MSC/NASTRAN via Solution Sequence 71 [1].

Once the coupled system frequency domain responses are solved for, spacecraft responses can be generated. To generate dynamic responses for spacecraft with statically-indeterminate interfaces, spacecraft contractors prefer to develop models which have response transformation matrices developed for the mode acceleration method of data recovery [2]; a method used to transform spacecraft boundary accelerations and displacements into internal responses. Unfortunately, while standard MSC/NASTRAN Solution Sequence 71 can be used to generate acceleration- or displacement-dependent responses, it cannot be used to combine acceleration- and displacement-dependent responses required for spacecraft mode acceleration data recovery.

One method for performing spacecraft mode acceleration data recovery during frequency domain buffeting wind analyses is to use external user-written computer codes that process MSC/NASTRAN output. Unfortunately, these methods can be labor and computer resource intensive. Given the analytical and computer resource efficiencies inherent within MSC/NASTRAN, it is advantageous to use the code directly. Hence, the objective of this work was to develop a methodology within MSC/NASTRAN for performing spacecraft mode acceleration data recovery during modal frequency response analyses. To this end, an MSC/NASTRAN DMAP Alter has been developed for Solution Sequence 71, and it has been used successfully to efficiently solve an ELV/spacecraft frequency domain buffeting wind analysis.

The underlying theory of spacecraft mode acceleration data recovery during MSC/NASTRAN modal frequency response analysis is detailed in the next section. Implementation of the theory within an MSC/NASTRAN DMAP Alter is then explained in a subsequent section. Lastly, spacecraft mode acceleration data recovery is performed during an ELV/spacecraft frequency domain buffeting wind analysis to demonstrate the accuracy of using the new Alter versus using an external user-written computer code.

Theory

Let all physical DOF of a free-free coupled ELV/spacecraft system be defined as g-set DOF. Neglecting damping, the residual level coupled system equations are

$$[M_{gg}][a_g] + [K_{gg}][u_g] = [P_g] \tag{1}$$

After accounting for DOF defined via multi-point and single-point constraints, the system equations are reduced from g-set size to a-set size [3]; hence,

$$[M_{aa}][a_a] + [K_{aa}][u_a] = [P_a] \tag{2}$$

To solve the coupled system frequency response equations in modal coordinates, the coupled system mode shapes are used to transform the coupled system equations from physical space to modal space. As implemented within MSC/NASTRAN Solution Sequence 71 [3], the coupled system modal frequency response equations for a j^{th} output frequency, ω_j, for an i^{th} applied load are

$$[M_{hh}]\{a_h^i(\omega_j)\} + [B_{hh}]\{v_h^i(\omega_j)\} + [K_{hh}]\{u_h^i(\omega_j)\} = \{P_h^i(\omega_j)\} \tag{3}$$

where system damping has been included via $[B_{hh}]$. The modal applied loads $\{P_h^i(\omega_j)\}$ are "unit" loads so that the modal solutions are transfer functions between the actual applied loads and the modal responses. This facilitates MSC/NASTRAN data recovery operations. Assuming there are "n" number of output frequencies being requested for each applied load, the modal displacements for an i^{th} applied load are

$$[u_h^i] = \left[\{u_h^i(\omega_1)\} \ \{u_h^i(\omega_2)\} \ \{u_h^i(\omega_3)\} \ \cdots \ \{u_h^i(\omega_n)\}\right] \tag{4}$$

Note that the modal displacements are complex quantities and are commonly referred to as frequency response functions (FRFs). Next, assuming there are "m" number of loads being applied to the system, the modal displacement FRFs corresponding to each load are appended to form a matrix of all modal displacement FRFs

$$[u_h] = \left[[u_h^1] \ [u_h^2] \ [u_h^3] \ \cdots \ [u_h^m]\right] \tag{5}$$

Given the system modal displacement FRFs, the boundary displacement FRFs for any component of the system can then be generated. The component boundary displacement FRFs for a jth frequency for an ith applied load are

$$\{u_a^i(\omega_j)\} = [\Phi_{ah}]\{u_h^i(\omega_j)\} \tag{6}$$

where $[\Phi_{ah}]$ is the partition of the coupled system mode shapes corresponding to the component boundary (a-set) DOF. These DOF are generally comprised of both component physical and generalized DOF. Because the problem at hand is set in the frequency domain, solving for acceleration FRFs is simple once displacement FRFs are known due to the properties of the Fourier Transform [4]. Given the system modal displacement FRFs, component a-set DOF acceleration FRFs for a jth frequency for an ith applied load are

$$\{a_a^i(\omega_j)\} = [\Phi_{ah}]\{a_h^i(\omega_j)\} = -\omega_j^2[\Phi_{ah}]\{u_h^i(\omega_j)\} \tag{7}$$

Assuming there are "n" number of output frequencies being requested for each applied load, the component a-set DOF displacement FRFs and acceleration FRFs for an ith applied load are

$$[u_a^i] = [\Phi_{ah}][u_h^i] \tag{8}$$

$$[a_a^i] = -[\Phi_{ah}][u_h^i][{}^{\nwarrow}\bar{\omega}^2{}_{\searrow}] \tag{9}$$

where

$$[{}^{\nwarrow}\bar{\omega}^2{}_{\searrow}] = \begin{bmatrix} \omega_1^2 & & & & \\ & \omega_2^2 & & & \\ & & \omega_3^2 & & \\ & & & \ddots & \\ & & & & \omega_n^2 \end{bmatrix} \tag{10}$$

Assuming there are "m" number of loads being applied to the system, the component a-set DOF displacement FRFs corresponding to each load are appended to form a matrix of all component a-set DOF displacement FRFs

$$[u_a] = [\Phi_{ah}][u_h] \tag{11}$$

Similarly for the component a-set DOF acceleration FRFs,

$$[a_a] = -[\Phi_{ah}][u_h][{}^{\nwarrow}\omega^2{}_{\searrow}] \tag{12}$$

where

$$[{}^{\nwarrow}\omega^2{}_{\searrow}] = \begin{bmatrix} [{}^{\nwarrow}\bar{\omega}^2{}_{\searrow}] & & & & \\ & [{}^{\nwarrow}\bar{\omega}^2{}_{\searrow}] & & & \\ & & [{}^{\nwarrow}\bar{\omega}^2{}_{\searrow}] & & \\ & & & \ddots & \\ & & & & [{}^{\nwarrow}\bar{\omega}^2{}_{\searrow}] \end{bmatrix} \tag{13}$$

Given that the component boundary displacement and acceleration FRFs can be solved for, consider the recovery of component internal response FRFs and other boundary response FRFs. For mode displacement data recovery, let component internal responses, {R}, be recovered through a transformation, [G], of the component a-set DOF displacements or accelerations as

$$\{R_D(x)\} = [G_{Da}]\{u_a(x)\} \tag{14}$$

$$\{R_A(x)\} = [G_{Aa}]\{a_a(x)\} \tag{15}$$

where "x" refers to either the time or frequency domain. Examples of responses $\{R_D\}$ include internal displacements and loads, and examples of responses $\{R_A\}$ include internal accelerations. Substituting for the component a-set DOF displacement and acceleration FRFs shown by Eqs. (11) and (12), displacement-dependent response or acceleration-dependent response FRFs for all applied loads are generated as

$$[R_D] = [G_{Da}][\Phi_{ah}][u_h] = [\Phi_{Dh}][u_h] \tag{16}$$

$$[R_A] = -[G_{Aa}][\Phi_{ah}][u_h][{}^{\nwarrow}\omega^{2}{}_{\searrow}] = -[\Phi_{Ah}][u_h][{}^{\nwarrow}\omega^{2}{}_{\searrow}] \tag{17}$$

The component displacement-dependent response or acceleration-dependent response FRFs shown by Eqs. (16) and (17) can be generated within standard MSC/NASTRAN Solution Sequence 71. Once response FRFs are generated, post-processing operations, including random response RMS calculations, can commence.

Now consider component internal or other boundary responses which are functions of both the component a-set DOF displacements and accelerations. Such is the case when spacecraft loads are recovered using the mode acceleration data recovery method for a statically-indeterminate model. In general, let these types of responses be referred to as "combined responses," and let them be recovered through a transformation

$$\{R_C(x)\} = [G^{A}_{Ca}]\{a_a(x)\} + [G^{D}_{Ca}]\{u_a(x)\} \tag{18}$$

where, as before, "x" refers to either the time or frequency domain. Substituting for the component a-set DOF displacement and acceleration FRFs shown by Eqs. (11) and (12), combined response FRFs for all applied loads are generated as

$$\begin{aligned} [R_C] &= -[G^{A}_{Ca}][\Phi_{ah}][u_h][{}^{\nwarrow}\omega^{2}{}_{\searrow}] + [G^{D}_{Ca}][\Phi_{ah}][u_h] \\ &= -[\Phi^{A}_{Ch}][u_h][{}^{\nwarrow}\omega^{2}{}_{\searrow}] + [\Phi^{D}_{Ch}][u_h] \end{aligned} \tag{19}$$

Unfortunately, responses of the type shown by Eq. (19) cannot be solved for within standard MSC/NASTRAN Solution Sequence 71.

To enable the solution of Eq. (19) within MSC/NASTRAN Solution Sequence 71 and thus enable spacecraft mode acceleration data recovery for a model with a statically-indeterminate interface, Eq. (19) is first rewritten as

$$[R_C] = -[\Phi^{A}_{Ch}][u_h][{}^{\nwarrow}\omega^{2}{}_{\searrow}] + [\Phi^{D}_{Ch}][u_h][{}^{\nwarrow}\omega^{-2}{}_{\searrow}][{}^{\nwarrow}\omega^{2}{}_{\searrow}] \tag{20}$$

where

$$[\nwarrow \omega^{-2} \searrow] = \begin{bmatrix} [\nwarrow \bar{\omega}^2 \searrow]^{-1} & & & & \\ & [\nwarrow \bar{\omega}^2 \searrow]^{-1} & & & \\ & & [\nwarrow \bar{\omega}^2 \searrow]^{-1} & & \\ & & & \ddots & \\ & & & & [\nwarrow \bar{\omega}^2 \searrow]^{-1} \end{bmatrix} \tag{21}$$

Rewriting Eq. (20) in matrix form and defining terms, combined response FRFs for all applied loads can be generated as

$$\begin{aligned} [R_C] &= -\begin{bmatrix} [\Phi^A_{Ch}] & [\Phi^D_{Ch}] \end{bmatrix} \begin{bmatrix} [u_h][\nwarrow \omega^2 \searrow] \\ -[u_h][\nwarrow \omega^{-2} \searrow][\nwarrow \omega^2 \searrow] \end{bmatrix} \\ &= -\begin{bmatrix} [\Phi^A_{Ch}] & [\Phi^D_{Ch}] \end{bmatrix} \begin{bmatrix} [u_h] \\ [\hat{u}_h] \end{bmatrix} [\nwarrow \omega^2 \searrow] \\ &= -[\Phi^{/}_{Ch}][u^{/}_h][\nwarrow \omega^2 \searrow] \end{aligned} \tag{22}$$

Note that the form of combined response FRFs shown by the last line of Eq. (22) is the same as that for acceleration-dependent FRFs shown by Eq. (17). Since Eq. (17) can be solved within standard MSC/NASTRAN Solution Sequence 71, it then follows that Eq. (22) can also be solved within the solution sequence. An MSC/NASTRAN DMAP Alter has been written to implement Eq. (22) within MSC/NASTRAN Solution Sequence 71 for the solution of combined response FRFs, thus enabling spacecraft mode acceleration data recovery for models with statically-indeterminate interfaces.

MSC/NASTRAN DMAP Alter

In the preceding section it was shown that the solution of combined response FRFs can take the same form as the solution of acceleration-dependent response FRFs. A solution technique was developed for spacecraft mode acceleration data recovery for models with statically-indeterminate interfaces. To implement this solution technique within MSC/NASTRAN Solution Sequence 71, a DMAP Alter was written.

Before the Alter is described, it is important to note what assumptions regarding spacecraft model processing are made within the Alter.

1. First, it is assumed that the spacecraft model and associated OTMs are saved as an MSC/NASTRAN external superelement database.

2. Second, it is assumed that the spacecraft OTM rows are stored on the database as the rows of the MSC/NASTRAN datablock [GOAT]. This can be accomplished using a simple DMAP routine and user-defined DOF. In terms of the transformation matrices defined in the preceding section,

$$[GOAT] = \begin{bmatrix} [G_{Aa}] \\ [G_{Da}] \\ [G^A_{Ca}] \\ [G^D_{Ca}] \end{bmatrix} \tag{23}$$

Note that for any particular spacecraft model, one or more of the submatrices of Eq. (23) need not be present. Also note that for OTM rows corresponding to spacecraft mode acceleration data recovery, the acceleration-dependent terms, $[G^{A}_{Ca}]$, and displacement-dependent terms, $[G^{D}_{Ca}]$, are stored as separate rows within [GOAT].

3. Third, it is assumed that for recovering combined response FRFs using spacecraft mode acceleration data recovery, additional DOF have been defined during spacecraft processing and assigned to the OTM rows used for mode acceleration data recovery. The OTM rows corresponding to the acceleration-dependent terms have been defined as u_1-set DOF, and the OTM rows corresponding to the displacement-dependent terms have been defined as u_2-set DOF during spacecraft processing. These DOF definitions are made using MSC/NASTRAN USETi,U1 and USETi,U2 Bulk Data cards. If combined response FRF calculations are desired during an analysis and the u_1-set and u_2-set DOF are not defined, execution stops.

Given the assumptions stated in the preceding paragraph, the alterations to MSC/NASTRAN Solution Sequence 71 for generating combined response FRFs and performing spacecraft mode acceleration data recovery are as follows:

1. Matrix [UGVS], used to recover component responses, is redefined according to Eq. (22). Originally,

$$[UGVS] = \begin{bmatrix} [I_{aa}] \\ [GOAT] \end{bmatrix} [\Phi_{ah}] = \begin{bmatrix} [\Phi_{ah}] \\ [G_{Aa}][\Phi_{ah}] \\ [G_{Da}][\Phi_{ah}] \\ [G^{A}_{Ca}][\Phi_{ah}] \\ [G^{D}_{Ca}][\Phi_{ah}] \end{bmatrix} = \begin{bmatrix} [\Phi_{ah}] \\ [\Phi_{Ah}] \\ [\Phi_{Dh}] \\ [\Phi^{A}_{Ch}] \\ [\Phi^{D}_{Ch}] \end{bmatrix} \tag{24}$$

After redefining, [UGVS] becomes [UGVS2] as

$$[UGVS2] = \begin{bmatrix} [\Phi_{ah}] & [0_{ah}] \\ [\Phi_{Ah}] & [0_{Ah}] \\ [\Phi_{Dh}] & [0_{Dh}] \\ [\Phi^{A}_{Ch}] & [\Phi^{D}_{Ch}] \\ [0^{A}_{Ch}] & [0^{D}_{Ch}] \end{bmatrix} \tag{25}$$

2. Matrix [UHVF], containing the system modal displacement FRFs for all applied loads, is redefined according to Eq. (22). Originally,

$$[UHVF] = [u_h] \tag{26}$$

After redefining, [UHVF] becomes [UHVF2] as

$$[UHVF2] = \begin{bmatrix} [u_h] \\ -[u_h][\diagdown \omega^{-2} \diagdown] \end{bmatrix} \tag{27}$$

By changing the appropriate data recovery DMAP module calls to make use of the new data block names described above, standard MSC/NASTRAN Solution Sequence 71 data recovery operations, including random response RMS calculations, can proceed as usual. In general, response FRFs are calculated as

$$[R] = [UGVS2][UHVF2] \tag{28}$$

To request MSC/NASTRAN XY-OUTPUT data for any of the original rows of the OTMs for which combined response FRFs are being calculated, the analyst must request acceleration type data recovery for the corresponding acceleration-dependent (u_1-set) DOF.

Numerical Example

An MSC/NASTRAN Solution Sequence 71 DMAP Alter for performing spacecraft mode acceleration data recovery for models with statically-indeterminate interfaces was developed to analyze ELV/spacecraft buffeting wind events. A typical frequency domain buffeting wind analysis is illustrated in Figure 1. The free-free ELV/spacecraft system is assumed to be flying through the atmosphere and subjected to buffeting wind loads. These types of loads typically build up in areas of differing system geometry. Two such locations are at the intersection of a bulbous Nose Fairing and the core vehicle and at the intersection of Solid Rocket Motors and the core vehicle. These areas are identified in Figure 1. Due to their random nature, the wind loads are represented as applied load power spectral density functions. The objective of such analyses is to generate spacecraft response RMS values.

For this numerical example, the ELV/spacecraft system is comprised of seven external superelements and residual structure bulk data. Approximately 3,200 MSC/NASTRAN g-set DOF are within the residual structure. Eighteen applied loads, each defined by a power spectral density curve, act on the coupled system perpendicular to the long axis of the system.

For the example spacecraft, the physical connection between its model and the ELV model is made at six grid points and corresponds to thirty-six MSC/NASTRAN b-set DOF; hence, it is statically-indeterminate. To recover spacecraft loads, mode acceleration method OTMs are used. The spacecraft physical and generalized boundary (a-set) DOF accelerations and physical boundary (b-set) DOF displacements are transformed into loads. For the thirty-six spacecraft interface DOF, interface loads are generated as

$$\{R_b(x)\} = [G_{ba}]\{a_a(x)\} + [G_{bb}]\{u_b(x)\} \tag{29}$$

which has the form of the combined responses shown by Eq. (18).

To demonstrate the accuracy of the MSC/NASTRAN Solution Sequence 71 Alter, spacecraft interface load RMS values were generated using two methods. The first method involved MSC/NASTRAN and an independently verified and validated external user-written computer code. MSC/NASTRAN Solution Sequence 71 was used to generate XY-PUNCH files containing the applied load power spectral density functions, acceleration-dependent term FRFs, and displacement-dependent term FRFs. The acceleration- and displacement-dependent term FRFs were combined within the external code, and the response RMS values were then calculated. The second method involved only MSC/NASTRAN Solution Sequence 71 and the new Alter to generate the response RMS values.

Spacecraft interface load RMS responses generated using the two methods are listed in Table 1. The first and second columns are for identifying the spacecraft connection and recovered load, respectively. Listed in the third and fourth columns are the spacecraft interface load RMS responses generated using MSC/NASTRAN and the external code and using MSC/NASTRAN with the new Alter, respectively. In column five are listed the spacecraft interface load RMS response ratios which are equal to the value in column four divided by the value in column three. It is clear from this numerical example that the MSC/NASTRAN Solution Sequence 71 Alter for performing spacecraft mode acceleration data recovery for models with statically-indeterminate interfaces executes correctly. It is important to note that excellent agreement was also achieved when responses internal to the spacecraft were compared.

Summary

An MSC/NASTRAN Solution Sequence 71 DMAP Alter has been written to combine acceleration- and displacement-dependent modal frequency responses. The new Alter was developed to perform spacecraft mode acceleration data recovery for models with statically-indeterminate interfaces during modal frequency response analyses. Through the combined use of DMAP and user-defined sets, combined frequency response functions are automatically generated. The Alter was written to replace an external user-written computer code used to solve a typical aerospace engineering problem. It has been shown via a numerical example that the new Alter allows for accurate solutions without the added user-interfaces and computer resources typically associated with external solution methodologies.

References

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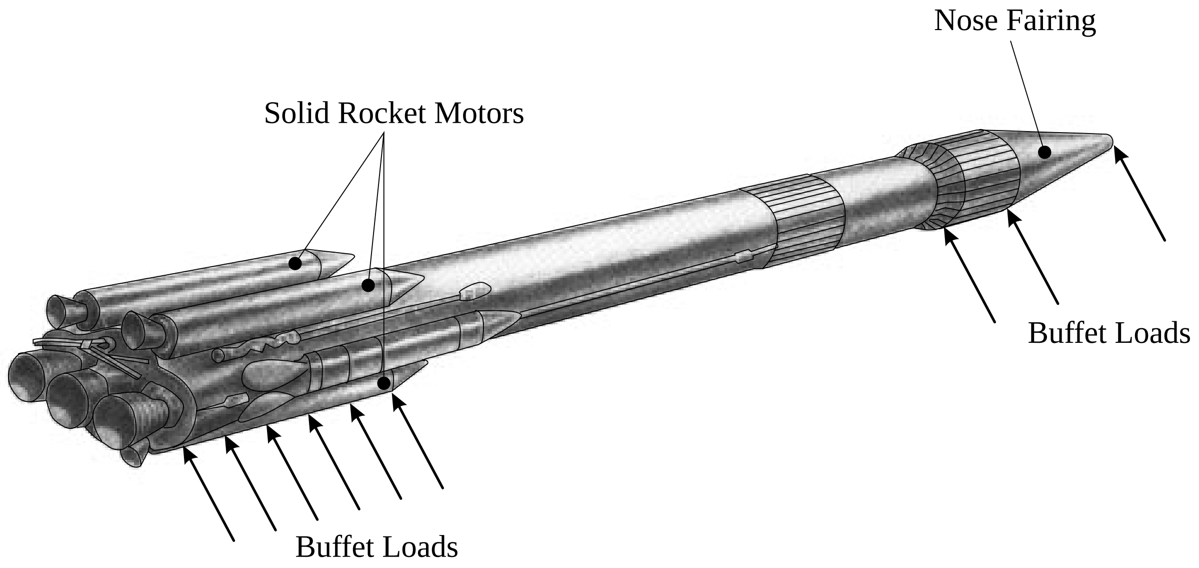

Figure 1.—ELV/spacecraft system for buffeting wind analysis.

Table 1. Results Comparisons for ELV/spacecraft Buffeting Wind Analysis

Spacecraft Connection	Recovered Load	RMS from External Code	RMS from DMAP Alter	Ratio[a]
1	Shear (lb)	4.318435E+02	4.318439E+02	1.00
	Shear (lb)	6.076196E+02	6.076197E+02	1.00
	Axial (lb)	1.563176E+03	1.563176E+03	1.00
	Bend. Mom. (in-lb)	1.985372E+03	1.985372E+03	1.00
	Bend. Mom. (in-lb)	1.943986E+03	1.943989E+03	1.00
	Torque (in-lb)	4.726503E+02	4.726504E+02	1.00
2	Shear (lb)	4.464110E+02	4.464114E+02	1.00
	Shear (lb)	6.420402E+02	6.420404E+02	1.00
	Axial (lb)	1.571105E+03	1.571105E+03	1.00
	Bend. Mom. (in-lb)	2.125543E+03	2.125543E+03	1.00
	Bend. Mom. (in-lb)	1.911408E+03	1.911411E+03	1.00
	Torque (in-lb)	6.202775E+02	6.202775E+02	1.00
3	Shear (lb)	4.349328E+02	4.349332E+02	1.00
	Shear (lb)	6.021744E+02	6.021746E+02	1.00
	Axial (lb)	1.573512E+03	1.573512E+03	1.00
	Bend. Mom. (in-lb)	1.892175E+03	1.892176E+03	1.00
	Bend. Mom. (in-lb)	1.897608E+03	1.897611E+03	1.00
	Torque (in-lb)	4.395778E+02	4.395779E+02	1.00
4	Shear (lb)	4.045292E+02	4.045296E+02	1.00
	Shear (lb)	6.100565E+02	6.100567E+02	1.00
	Axial (lb)	1.508558E+03	1.508558E+03	1.00
	Bend. Mom. (in-lb)	1.924192E+03	1.924193E+03	1.00
	Bend. Mom. (in-lb)	1.834566E+03	1.834568E+03	1.00
	Torque (in-lb)	4.728231E+02	4.728232E+02	1.00
5	Shear (lb)	2.173483E+02	2.173484E+02	1.00
	Shear (lb)	6.608356E+02	6.608364E+02	1.00
	Axial (lb)	1.321302E+03	1.321302E+03	1.00
	Bend. Mom. (in-lb)	2.298343E+03	2.298345E+03	1.00
	Bend. Mom. (in-lb)	1.519133E+03	1.519134E+03	1.00
	Torque (in-lb)	2.323901E+02	2.323903E+02	1.00
6	Shear (lb)	2.203653E+02	2.203654E+02	1.00
	Shear (lb)	6.055359E+02	6.055367E+02	1.00
	Axial (lb)	1.331196E+03	1.331196E+03	1.00
	Bend. Mom. (in-lb)	2.108409E+03	2.108411E+03	1.00
	Bend. Mom. (in-lb)	1.551676E+03	1.551676E+03	1.00
	Torque (in-lb)	2.300419E+02	2.300421E+02	1.00

a - Ratio = (RMS from DMAP Alter) / (RMS from External Code)