

# MSC/PATRAN AS A PART OF A TOOL IN THE FIELD OF STATIC AND DYNAMIC AEROELASTICITY

Jari Hyvärinen and Per Kjellgren  
Anker-Zemer Engineering AB,  
Box 156, 691 23 Karlskoga, Sweden,  
e-mail: 100140.2132@compuserve.com

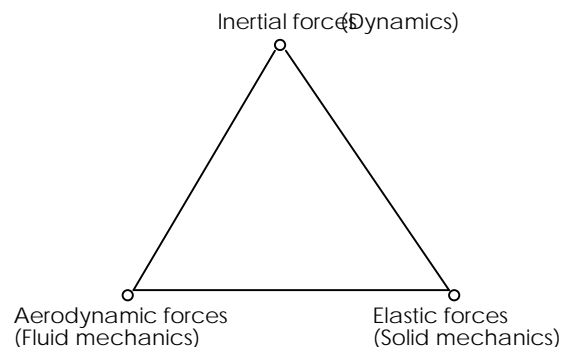
## Abstract.

This paper uses two measures for stability when studying aeroelastic behaviour in problems which require Non-linear fluid mechanics modelling for the solution. One measure is defined for static stability and the other for dynamic stability. The simulation performed on the fluid-structure interaction of a 2D ellipse in a sub critical Reynolds number flow field is shown in this paper.

## Introduction:

Since the initial introduction of the scientific field of aeroelasticity, the aerodynamic models used have either been based on experiments or on linear theories. During the last decade numerical methods for solving increasingly sophisticated fluid dynamic models have been developed world-wide to improve the predictions of unsteady aerodynamics in problems containing separated flow and/or transonic flow etc. The present state of art in aeroelasticity for simulation of flow of these categories is the use of full potential equation solvers and Euler equation solvers. The solution of Navier-Stokes equations with numerical methods such as the Finite Volume or Finite Element method adds an additional complication, namely time dependent nodal locations in the flow domain due to structural motion. The major drawback with having to introduce a dynamic mesh algorithm is the additional computer requirements for solving the equation system. The memory and CPU requirement for the solution of a 3D flow case is presently expensive even with a static mesh.

The definition of the field of aeroelasticity can be visualized through forming a triangle of disciplines, as was first done by Collar several years ago.



Aeroelasticity covers phenomena which involves significant mutual interaction among inertial, elastic and aerodynamic forces.

Although the technology around aeroelasticity has been growing from and developing in the area of aeronautical applications, applications are found at an increasing rate in other disciplines of engineering such as civil engineering (flow about bridges and tall buildings), mechanical engineering (flow around

turbomachinery blades and fluid flow in flexible pipes) and nuclear engineering (flow about fuel elements and heat exchanger vanes). The number of applications will with high probability increase in both absolute and relative numbers as the technological development in these and other areas demand lighter weight structures under more severe flow conditions.

The present state of the art in terms of tools used for predicting aeroelastic characteristics of systems in the area of fluid mechanics is the use of linear small perturbation theory for the modelling of the unsteady aerodynamics. During the last decade full potential and Euler approximation programs have also been developed for prediction of unsteady aerodynamic for transonic and supersonic flow cases.

The computer hardware and software performance have been increasing to a state where software starts to appear that makes it possible to study aeroelastic phenomena where full Navier-Stokes equations has to be used for simulation of the unsteady aerodynamics.

This paper presents the principals behind a tool that is under development for the prediction of static and dynamic aeroelastic characteristics with a non-linear unsteady fluid mechanics model.

## 2. MODEL DESCRIPTION

### 2.1 Fluid and structural modeling:

The MSC/PATRAN pre-processor was used for modeling of both the structural and fluid finite element models. The interface between the MSC/PATRAN program and the developed software was written as

subroutines reading from a MSC/PATRAN model file.

MSC/PATRAN was used for the results post-processing by creating a results file from the code that MSC/PATRAN reads. Both steady state and transient post-processing was performed.

### 2.2 Aeroelastic equations:

When developing a method for studies of fluid-structure interaction phenomena the tool has to solve the complete equation of motion for the system, either through direct integration in the time domain or through some approximate method. The equation of motion may be written as

$$[M]\ddot{u} + [C]\dot{u} + [K]u = F(t) \quad (1).$$

Where

$[M]$	-	Mass matrix.
$[C]$	-	Damping matrix.
$[K]$	-	Structural stiffness matrix.
$\ddot{u}$	-	Acceleration vector.
$\dot{u}$	-	Velocity vector.
$u$	-	Displacement vector.
$F(t)$	-	Time dependent load vector.

Equation (1) is generally non-linear and is sufficient for the solution of problems such as determining stalled and unstalled flutter speeds, forced response analysis and steady-state deflection calculations.

In the case of static stability analysis all the time dependent variables are zero in equation (1) and reduces to:

$$[K]u = F \quad (2).$$

To be able to solve either equation (1) or (2) in the case of a fluid-structure interaction problem the structural left hand side of these equations has to be solved simultaneously with the solution of the flow field for the calculation of the right hand side. The solution of the right hand side will in the most general case involve the solution of the unsteady Navier-Stokes equations.

When attempting to solve equation (1) with for example a finite element method the fluid dynamic element mesh has to move together with the structural finite element mesh. This motion of the fluid finite element model node locations can be performed with methods such as spring models [2] or a Arbitrary Lagrangian-Eulerian method [3] with structure nodal velocity linearly decaying out in to the fluid mesh. A third method that may be used is a blowing model [4], which is a boundary condition type of model.

### 2.3 Aeroelastic modelling:

The dynamics of the structure in the tool that has been developed is presently solved with a Euler predictor-corrector algorithm [5]. This algorithm finds the equilibrium of the left hand side of (1) at each time step through a sequence of equilibrium iterations. The algorithm reformulates (1) into two first order differential equations:

$$w = \frac{dw}{dt} \quad (3)$$

$$\frac{dw}{dt} = F(t, u) \quad (4)$$

$$x(t_0) = x_0 \quad (5)$$

$$w(t_0) = x'_0.$$

To define both a measure for static and dynamic stability the typical section airfoil is often used [6]. Fig. 1 shows the principles of the structurally two degree of freedom typical section airfoil in a wind tunnel.

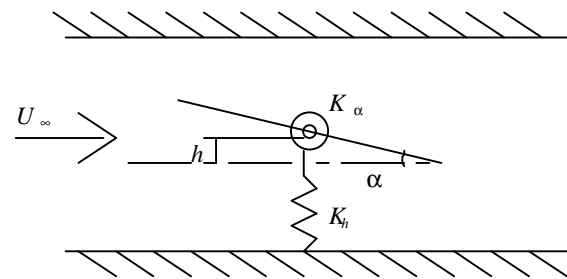


Fig. 1 Aeroelastic model with a typical airfoil.

## 3. STABILITY

### 3.1 Static aeroelastic stability.

In the case of static aeroelasticity analysis that has to be performed contains three steps.

1. Calculation of a steady state condition.
2. Disturbing the steady state condition with a small increment in the structural displacement field and then calculating the aerodynamic derivatives
3. Based on the assumption that the calculated fluid/structure derivatives of the flow does not changes in the range of interest, the velocity at which static instability occurs is calculated.

To illustrate this procedure and define a measure for static stability consider the system in Fig.1. with only one degree of freedom; the angle of attack

degree of freedom. The total angle of attack is the sum of some initial angle of attack,  $\alpha_0$  and an increment in the angle of attack due to the elastic twist of the spring,  $\alpha_e$ .

$$\alpha = \alpha_0 + \alpha_e \quad (5)$$

To be able to set up the equilibrium equation (2) for the typical section airfoil in Fig.1 we have to define the point called the aerodynamic centre. This is the point on the airfoil about which the aerodynamic moment is independent of the angle of attack. The equilibrium about the elastic axis may now be written as:

$$K_\alpha \alpha = M_{AC} + eL \quad (6)$$

Where

- $K_\alpha$  - Elastic torsional stiffness.
- $M_{AC}$  - Moment about the aerodynamic centre. Moment is defined positive nose up.
- $L$  - Lift or Vertical force, positive up.
- $e$  - Distance from aerodynamic centre to elastic axis, positive elastic axis aft.

The aerodynamic lift and moment are defined as:

$$L = C_L qS \quad (7)$$

$$M_{AC} = C_{mAC} qSc \quad (8)$$

where

$$C_L = C_{L_0} + \frac{\partial C_L}{\partial \alpha} \alpha \quad , \text{Lift coefficient}$$

$C_{mAC} = C_{mAC_0}$  , Constant moment coefficient about aerodynamic centre due to airfoil centre line camber.

$$q = \frac{\rho U^2}{2}, \quad \text{Dynamic pressure}$$

$\rho$  - Air density

$U$  - Air velocity

$c$  - Airfoil cord

$l$  - Airfoil span

$S$  - Airfoil area,  $c \times l$

By inserting (7) and (8) into (6), the equilibrium equation takes the following form

$$K_\alpha \alpha = eqS \left[ \frac{\partial C_L}{\partial \alpha} (\alpha_0 + \alpha_e) \right] + qSc C_{mAC_0} \quad (9)$$

This may be rewritten to the form of a eigenvalue problem ( assuming  $C_{mAC_0} = 0$  and  $\alpha_0 = 0$  for simplicity)

$$\left[ 1 - q \left[ S e \frac{\partial C_L}{\partial \alpha} / K_\alpha \right] \right] \alpha_e = 0 \quad (10)$$

The eigenvalue of (10) is the dynamic pressure at which static instability occurs for the structure. The velocity corresponding to  $q_D$  is called the divergence velocity.

$$q_D = \frac{K_\alpha}{S e \frac{\partial C_L}{\partial \alpha}} \quad (11)$$

Linear aerodynamic models are sufficient when calculating static aeroelastic characteristics for an airfoil flying at subsonic speeds and subjected to small angles of attack. The methods for solution of this category of problems is usually based on potential flow theory. When

studying airfoils subjecting to high angle of attack or other applications with separated flows a non-linear fluid flow model is required.

### 3.2 Dynamic aeroelastic stability.

To define a measure of dynamic stability consider the typical section airfoil in Fig. 1 and study the equation of motion for the  $h$  degree of freedom.

$$m\ddot{h} + K_h h = -L \quad (12)$$

Where:

- $m$  - Mass of the airfoil
- $K_h$  - Translational spring stiffness

By assuming quasi-steady aerodynamics with small changes in angle of attack per time step equation (12) takes the following form.

$$m\ddot{h} + qS \frac{\partial C_L}{\partial \alpha} \frac{\dot{h}}{U} + K_h h = 0 \quad (13).$$

In standard theory of vibration the equation of motion for a one degree of freedom system is often written as

$$\ddot{x} + 2\xi\dot{x} + \omega_0^2 x = F(t) \quad (14)$$

By studying the homogenous part of (14) where  $F(t)=0$  the damping coefficient in (13) may be identified as

$$\xi = \frac{qS \frac{\partial C_L}{\partial \alpha} \frac{1}{U}}{m} \quad (15)$$

and the systems eigenfrequency as

$$\omega_0^2 = K_h / m \quad (16)$$

By assuming harmonic motion one may set up the characteristic equation for the homogenous part of (14) as

$$\lambda^2 + 2\xi\lambda + \omega_0^2 = 0 \quad (17)$$

This has the following roots

$$\lambda_{1,2} = -\xi \pm \sqrt{\xi^2 - \omega_0^2} \quad (18)$$

Finally, this gives a general solution of (14) as

$$x = e^{-\xi t} (Ae^{\sqrt{\xi^2 - \omega_0^2} t} + Be^{-\sqrt{\xi^2 - \omega_0^2} t}) \quad (19)$$

By using (18) and (19) on (14) an effective aerodynamic damping constant may be defined as

$$\zeta_A = \frac{\xi}{\omega_0} = \frac{qS \frac{\partial C_L}{\partial \alpha} \frac{1}{U}}{2\sqrt{mK_h}} \quad (20)$$

A total effective damping constant is defined as

$$\zeta = \zeta_A + \zeta_S \quad (21)$$

where  $\zeta_S$  is the effective structural damping constant.

The value of  $\zeta$  may cause five different physical phenomena which may be characterised as a system in motion with

$$\left\{ \begin{array}{lll} \zeta > 1.0 & \text{Strong} & \text{Damping} \\ \zeta = 1.0 & \text{Critical} & \text{Damping} \\ 0 < \zeta < 1.0 & \text{Weak} & \text{Damping} \\ \zeta = 0 & & \text{Undamped} \\ \zeta < 0.0 & \text{Negative} & \text{Damping} \end{array} \right.$$

When  $\zeta$  becomes negative the system is locally or globally unstable and flutter occurs. The system is also said to have an insufficient level of structural damping to overcome the contribution negative aerodynamic influence.

#### 4. Computational results:

The dynamic aeroelastic behaviour of a 2D elliptic structure subjected to fluid loads at sub-critical Reynolds number have been studied. The goal of the analysis was to study the stability characteristics of the system if the ellipse was held steady with an initial elastic torque and then released. Fig. 2 shows the principles of the system that has been studied. The ellipse is allowed to move as a rigid body with two degrees of freedom  $\alpha$  and  $h$ . The structural stiffness corresponding to these degrees of freedoms are  $K_\alpha$  and  $K_h$ . The damping of each degree of freedom is given by  $C_\alpha$  and  $C_h$ .

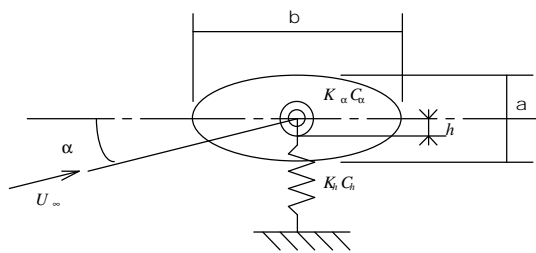


Fig. 2 Principles for ellipse model.

Where:

$$\begin{aligned} a/b &= 2.0 & C_\alpha &= 5.0 \\ \text{Re} &= 800 & C_h &= 0.93 \\ K_\alpha &= 700 \text{ N/m} \\ K_h &= 5.5 \text{ N/m} \\ m &= 10.0 \text{ Kg} \\ J_z &= 3500 \text{ Kgm}^2 \\ h(t=0) &= 0.0 \\ \alpha(t=0) &= 3.0 \text{ (Deg.)} \end{aligned}$$

Figure 3. shows the finite element model around the ellipse at zero degree angle of attack in the upper part and the same model with some - 10 degree angle of attack and a h displacement of  $0.2 \times a$ .

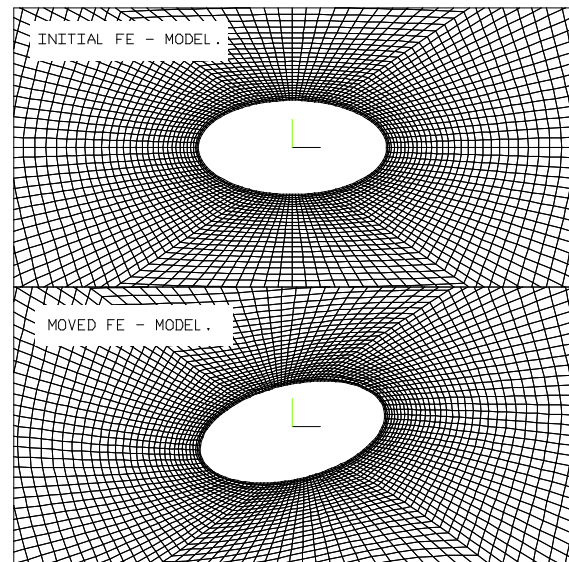


Fig. 3.

- a) Top model, undeformed mesh
- b) Lower model, deformed mesh

The flow field characteristics before time  $t=0.0$  sec. show as expected a vortex shedding behind the ellipse. The drag, lift and pitching moment loads on the ellipse are harmonically alternating as a function of time as shown in figure 4,

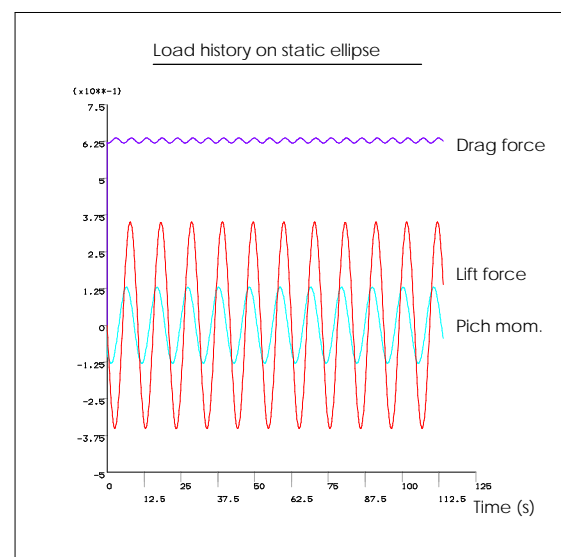


Fig. 4 Load history on static ellipse

When the ellipse is released at time  $t=0.0$  internally stored elastic energy starts to interact with the external fluid load.

Figure 5 shows the response in  $\alpha$  motion. The motion shows weak damping behaviour due to structural and aerodynamic damping of this mode. The motion reaches a more or less steady harmonic motion after about 120 seconds.

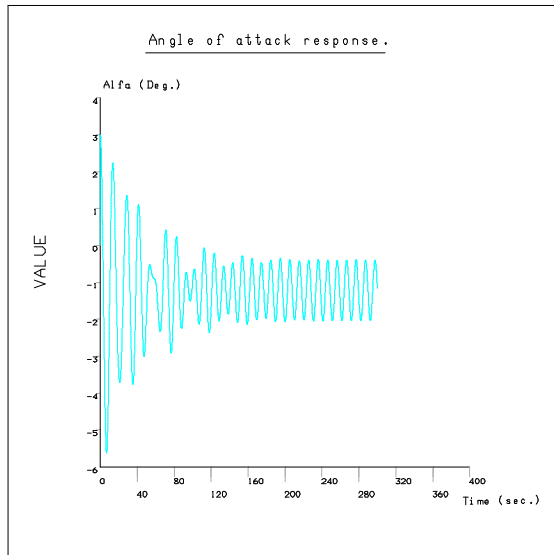


Figure 5. Response in alfa motion.

Figure 6 shows the response in  $h$  here the motion is driven from a steady elastically unloaded state to a final steady harmonic state. The solution reaching steady harmonic motion after about 80 seconds.

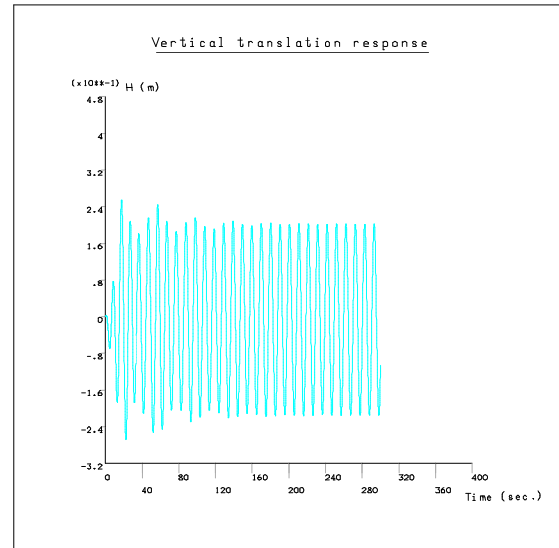


Fig. 6 response in h motion.

The frequency at which the both degree of freedoms are in motion after the initial structural mode motions have been damped out can be referred to the harmonically varying load due to the vortex shedding. The frequency for the dynamically moving ellipse is about two times higher than the corresponding frequency for the steady ellipse.

Figure 7 shows velocity contours around the moving ellipse. The plot is taken from a time when the ellipse is moving upwards and with a pitch-up change in angle of attack.

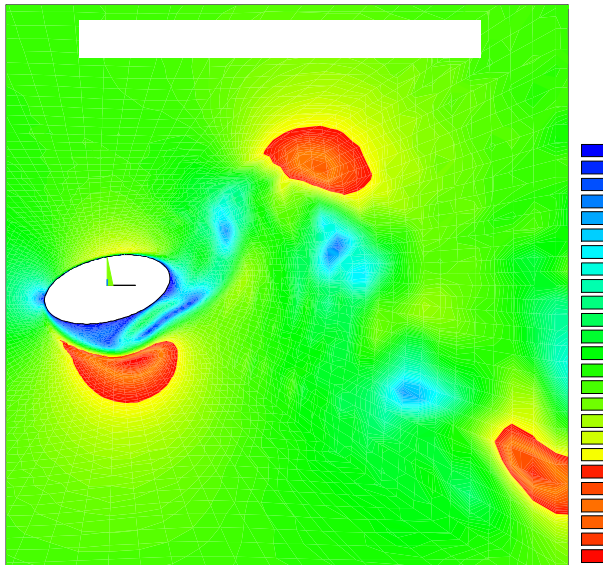


Fig. 7 Velocity field plot.

### 5. Conclusions.

This paper has shown that it is beginning to be possible to solve and study phenomena in the aeroelastic area which are dependent on the solution of unsteady non-linear fluid mechanics. A program that is developed for this type of problems should at least contain tools such as a Navier-Stokes solver with a dynamic mesh algorithm, an eigenvalue solver for the structural equation and a transient structural dynamic solver.

This paper has shown an example of a rigid ellipse supported by two elastic springs in a sub critical Reynolds number flow field. Both the structural behaviour and the characteristics are highly dependent on the fluid/structure interaction.

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