

**CALCULATION WITHIN MSC/NASTRAN OF THE FORCES TRANSMITTED BY  
MULTIPOINT CONSTRAINTS (MPC) AND THE FORCES  
GENERATED IN SUPORT CONSTRAINTS**

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**SUMMARY**

This paper describes a procedure that permits the calculation of the forces generated in MPC and SUPORT constraints, in STATIC and DYNAMIC (Frequency response) load cases. These constraints are extensively used in MSC/NASTRAN. In the paper the theoretical bases of the procedure are given, as well as DMAP sequences that allow to calculate these forces, and demonstration examples.

## **1. INTRODUCTION**

MSC/NASTRAN provides two types of constraints that are very useful in a number of instances. First, Multipoint Constraints (MPC), that establishes geometric relationships that have to be met by the displacements of certain nodes of the structure. This is useful for modelling among others very stiff elements, without the numeric difficulties that would imply to model them as truly flexible bodies of high stiffness. MSC/NASTRAN has extended the old MPC's relationships by including rigid elements (RBAR, RBE, etc) that generate internally these relationships from the geometric data provided by the user. These forces are often essential for the correct dimensioning of the structure. This is the case of screwed joints, very stiff elements, etc.

On the other hand, the treatment of the properties of a structure as a free body, either in static or dynamic load cases, is facilitated by the employment of SUPORT constraints. Given that the stiffness matrix of a structure that can move as a free body is necessarily singular, the constraint is carried out by means of special support conditions.

It in the case of the MPC constraints, it is already possible to obtain the forces transmitted by these constraints in the version 68 of MSC/NASTRAN, but with some limitations in dynamic cases: it can not be obtained in XY format, nor in a RANDOM excitation case. These capabilities are incorporated by our procedure. Moreover the procedure proposed in this article for the dynamic case is advantageous with respect to the one used by MSC/NASTRAN, because it requires less operations for the computation.

With respect to the SUPORT constraints, it could be very useful for the dimensioning of the structure to compute the forces transmitted to the rest of the structure thorough the degrees of freedom at which the SUPORT is imposed. MSC/NASTRAN does not provide these forces in dynamic problems.

In this paper are explained the theoretical basis which allow to calculate the forces generated in both MPC, and SUPORT constraints in the load cases STATIC and DYNAMIC (frequency response, modal formulation). Also are given DMAP sequences that permit the recovery of these forces.

## **2. STATIC CASE**

### **2.1. Theoretical basis**

In a static load case, the equations that govern the equilibrium of the structure, are expressed in terms of the displacements of the nodes by means of a system of equations:

$$K u = P \quad (1)$$

where K is the stiffness matrix, u is the displacements vector, and P is the vector of the forces applied in the nodes.

Before beginning the successive elimination of degrees of freedom, the above equation is

expressed in MSC/NASTRAN notation as:

$$K_{gg} u_g = P_g + R_g + Q_g \quad (2)$$

where:

$P_g$ : External forces **directly applied** on the nodes.

$Q_g$ : Forces on the nodes originated by SPC constraints.

$R_g$ : Forces on the nodes originated by the MPC constraints.

The subindex -g- indicates that the system of equations is established to the complete set of structural degrees of freedom (set -g- in MSC/NASTRAN terminology).

Once MSC/NASTRAN has solved the static problem, the displacements  $u_g$  are known, so the first member of the above equation is known. The vector  $P_g$  is also known because it has been directly generated from the loads introduced by the user. We call  $F_g$  to the term  $K_{gg} u_g - P_g$ . Finally, the vector  $Q_g$  that contains the SPC reaction forces, is directly accessible to the user. The equation (2) may then be written as:

$$R_g = F_g - Q_g \quad (2b)$$

Partitioning these vectors according to the m set of displacements (dependent degrees of freedom), and set n (independent degrees of freedom):

$$\begin{pmatrix} R_n \\ R_m \end{pmatrix} = \begin{pmatrix} F_n \\ F_m \end{pmatrix} - \begin{pmatrix} Q_n \\ Q_m \end{pmatrix}$$

Since  $Q_m$  has to be a null vector (SPC constraints cannot be applied to the degrees of freedom -m-), it verifies:

$$R_m = F_m$$

expression that allows to calculate the forces originated by MPC constraints applied on the dependent degrees of freedom -m-.

For the computation of the rest of the MPC forces ( $R_n$ ), we proceed in the following way. The MPC constraints establish a relationship between the displacements  $u_m$  and  $u_n$ , that can be written as:

$$u_m = G \cdot u_n \quad (3)$$

where  $G$  is a matrix that MSC/NASTRAN generates and is accessible. Since the forces  $R_m$  and  $R_n$  have to be in equilibrium, the total work produced by them should be null for any displacements vector that complies the conditions established in (2). This means:

$$u_m^T R_m + u_n^T R_n = 0$$

Taking into account the equation (3):

$$u_n^T G^T R_m + u_n^T R_n = 0$$

Since this equation should be verified by any vector  $u_n$ , we finally find:

$$- G^T R_m = R_n$$

expression that allows the recovery of the forces  $R_n$ .

## 2.2. Operational Sequence

The steps needed to obtain the Multipoint Constraint Forces, are easily deduced from the procedure explained above. These are:

1. Resolution of the MSC/NASTRAN problem. This is equivalent to the explicit obtention of the vector  $u_g$ .

2. Calculate

$$F_g = K_{gg} u_g$$

3. Partition

$$F_g \rightarrow \begin{pmatrix} F_n \\ F_m \end{pmatrix}$$

4. Perform

$$R_m = F_m$$

5. Calculate

$$R_n = - G^T R_m$$

6. Rebuild the matrix to size -g-

$$R_g = \begin{pmatrix} R_n \\ R_m \end{pmatrix}$$

7. Printing of the results

$$R_g$$

### **NOTE**

In the event of that several load cases are included in the same run, the displacements vector  $u_g$  will be a matrix with a number of columns equal to the number of load cases defined. In this case, MSC/NASTRAN identifies the different columns with the set -v-. The displacements are contained in a matrix  $u_{gv}$ . Therefore the MPC forces are given in a matrix of -v- columns, that can be called

$$R_{gv}$$

### **2.3. DMAP Sequence**

It is enclosed the DMAP sequence for the calculation of the forces transmitted by MPC constraints, for the MSC/NASTRAN solution STATICS (SOL 24). The module OFP has been used for printing the computed forces. This module provides an output in which it is explicitly indicated the value of the load, as well as the number of identification of the gridpoint, the degree of freedom, and the load case. The forces are treated as directly applied loads for printing purposes. This requires to build up the vectors to the size -g- and to identify them with the vector  $P_g$ , once this is no longer necessary. If it has been requested the output of the applied forces with the CASE CONTROL command **OLOAD = n**, it will produce the correct printing of the results.

```

$.....
$
$
$   DMAP SEQUENCE FOR THE OBTENTION OF THE FORCES
$   TRANSMITTED BY MPC CONSTRAINTS IN A STATIC ANALYSIS,
$   MSC/NASTRAN VERSIONS 67.7 Y 68, AND SOL 24.
$
$
$

```

\$ THE OUTPUT IS OBTAINED AS THESE WERE DIRECTLY APPLIED LOADS  
\$ AND CAN BE REQUESTED BY MEANS OF THE INSTRUCTION:

\$ \*\*\*\*\*  
\$ \* OLOAD = ..... \*  
\$ \*\*\*\*\*

\$ IT WILL PRODUCE ALSO THE OUTPUT OF THE REAL EXTERNAL APPLIED LOADS.  
\$ FOR CLARIFICATION THE MPC FORCES ARE PRINTED BETWEEN MESSAGES:

\$ " FORCES IN RIGID ELEMENTS "  
\$ " END OF MPC FORCES "

\$ PREPARED BY E. DE LA FUENTE Y F.J. SAN MILLAN (MARCH 1994)

\$ I.N.T.A. STRUCTURES AND MECHANISMS DEPARTMENT.

\$.....  
\$ COMPILE SOL24 SOUIN = MSCSOU LIST NOREF  
\$ ALTER 190

\$.....  
\$ PARTITION PG AND KGG  
\$ EXTRACTING THE D.O.F. - M -

\$ UPARTN USET,PG/,PM,,/'G/'N/'M'/1 \$  
\$ UPARTN USET,KGG/,KMG,,/'G/'N/'M'/1 \$

\$.....  
\$ MAKES THE OPERATION  
\$ QM = KMG \* UGV - PM  
\$ QN = -(GM)T \* QM

\$ AND COMBINE THEM IN QMN (MPC FORCES SET -G-)

\$ MPYAD KMG,UGV,PM/QM///-1 \$  
\$ MPYAD GM,QM,/QN/1/-1 \$  
\$ UMERGE USET,QN,QM/QMN/'G/'N/'M' \$

\$.....  
\$ GENERATE A DATABLOCK WITH MPCFORCES  
\$ EMULATING THE CORRESPONDENT TO THE ORDINARY  
\$ OUTPUT OF THE APPLIED FORCES (OLOAD)

\$ QQMN1 = 'MPC FORCES'

\$ SDR2 CASECC,CSTM,MPT,DIT,EQEXIN,,ETT,EDT,BGPDT,QMN,,EST,  
\$ XYCDB,,/  
\$ OQMN1 ,,,,/'STATICS'/S,N,NP/V,Y,NOCOMP \$

\$.....  
\$ AND PRINT IT BETWEEN EXPLICIT MESSAGES.  
\$ THE OUTPUT IS GIVEN AS DIRECT APPLIED LOADS

\$ MESSAGE // ' '\$  
\$ MESSAGE // ' '\$  
\$ MESSAGE // ' '\$  
\$ 'FORCES IN RIGID ELEMENTS '\$  
\$ MESSAGE // ' '\$  
\$ MESSAGE // ' '\$  
\$ OFF OQMN1 // \$  
\$

```
MESSAGE // ' '$
MESSAGE // ' '$
MESSAGE // '
      'END OF MPC FORCES' '$
MESSAGE // ' '$
MESSAGE // ' '$
$
```

## 2.4. Demonstration examples

To verify the correctness of the results a number of small, easily calculated by hand, models were checked. The results proved to be successful in all cases. Finally, a complex FEM, namely the model of an Antenna Pointing Mechanism (APM) was submitted to verification. The FEM is shown in figure 1. This mechanism is made up of two motorgear units joined to each other through an L shape structure, and a set of auxiliary brackets.

### MODEL STATISTICS

845 nodes  
34 elements ELAS2  
727 elements QUAD4  
30 elements BAR  
6 elements QUAD8  
58 elements TRIA3  
12 elements TRIA6  
119 elements RBAR

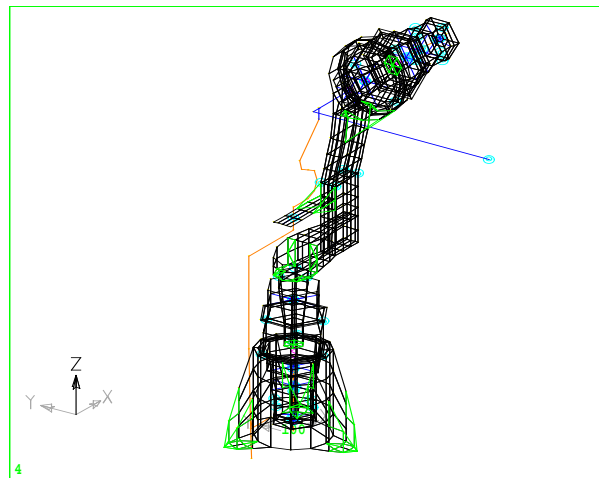


Figure 1: Finite element model of the APM

The load case used was an acceleration of 0.8 g acting in the three axes simultaneously. Table 2 shows the MPC forces transmitted to a set of nodes of the model, obtained with the ALTER DMAP implemented in the static solution SOL24. In this table it is indicated the number of node, together with the MPC forces transmitted to the node through each DOF. Table 3 shows the same forces obtained directly from MSC/NASTRAN within the structured solution 101, that provides these forces by means of the MPCFORCES command. As it can be seen the results of the two analysis are identical as expected, so that the efficiency of the DMAP sequence is proven.

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^^
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1
0
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                                LOAD VECTOR

POINT ID.  TYPE    T1      T2      T3      R1      R2      R3
15071     G    7.861104E+01  1.609391E+01 -3.749458E+02 -1.110274E+00  1.656089E+00  2.396556E-08
15073     G    7.497629E+01  2.824913E+01 -3.441682E+02 -1.586540E+00  2.793936E-01  3.233611E-08
15075     G   -1.222863E+01 -6.207857E+01  8.863432E+01 -5.905173E-02  5.898988E-01 -6.817037E-08
15077     G    2.903225E+01  4.999120E+01  4.808663E+02 -1.248237E+00  1.811587E+00  1.701969E-08
15079     G   -2.952158E+01  7.198444E+01  3.749961E+02 -1.981523E+00  2.190986E-01  4.259578E-08
15081     G    5.478954E+01  9.243044E+01 -3.156163E+01 -3.122648E-01  3.511912E-01 -4.664496E-08
17001     G    8.416367E+01  9.102634E+00  3.686208E+01  7.573230E-08  2.729335E-01 -6.221956E-01
17003     G    8.036130E+01  2.514936E+01 -3.915869E+01 -1.835049E-06  2.630912E-01 -3.614500E-01
17005     G   -6.728338E+01 -3.638429E+01  1.296095E+01  2.624950E-06 -5.444998E-01 -2.454515E-01
17007     G    2.515929E+02  4.118267E+01  1.430866E+01 -6.221426E-08 -2.326826E-01  1.375764E+00
17009     G    1.119534E+02 -6.426996E+01  1.421729E+02 -4.562036E-06 -2.722465E-01  5.205440E-01
17011     G   -3.234877E+02  1.636058E+02 -5.504374E+01 -2.638067E-08  1.562415E+00  9.474190E-01
17014     G    2.604851E+02 -1.331447E+00  1.944620E+02  6.486856E-01 -2.306046E-01 -8.890024E-08
17015     G   -5.532692E+02  1.118232E+02 -6.988396E+02  2.674013E+00 -1.615698E-01 -3.672299E-07
17016     G   -8.190182E+01  2.449507E+02 -3.263564E+02  1.115318E+00 -7.357068E-01  6.630939E-09
17017     G    1.078855E+01  1.371116E+01 -1.187700E+02 -2.216154E-01 -1.698262E-01  3.676398E-08
17103     G    1.968053E+01  1.011927E+01  2.941002E+01 -1.470501E-02  0.0      9.840265E-03
17104     G    0.0      7.053346E+00 -2.178907E+01  0.0      0.0      0.0
17105     G    2.635023E+00 -8.062255E+00 -5.244826E+01 -2.622413E-02  0.0      -1.317511E-03
17113     G   -2.901401E+01  4.515302E+00  4.020326E-03 -2.010163E-06  0.0      -1.450701E-02
17114     G    0.0      1.412189E+01  6.565560E+01  0.0      0.0      0.0
17115     G    4.187728E+00 -3.033263E+01  1.711445E-02  8.557227E-06  0.0      -2.093864E-03
17288     G    3.872791E+02 -2.286237E+02  9.661671E+02  2.542740E-01 -4.425221E-01  4.136789E-02
17290     G   -1.776564E+02 -2.958162E+02 -1.691000E+02  4.033074E-01 -8.194732E-01 -3.489449E-01
20581     G   -2.752919E+02  3.085621E+01 -1.897531E+02 -1.373490E-01 -1.297396E-02  2.659997E-01
20587     G   -3.807363E+01  1.858684E+02 -9.051492E+01 -1.087853E-01 -5.314330E-02  6.375848E-02
20592     G    2.548622E+02  2.109193E+02  1.013062E+02 -1.179938E-01  1.580505E-01  1.310837E-01
20597     G    3.105802E+02  3.368734E+01  2.439337E+02 -1.557661E-01  1.270369E-03  2.946407E-01
20602     G    7.336211E+01 -1.883000E+02  1.287804E+02 -1.843301E-01 -7.082495E-02  9.041532E-02
20607     G   -2.195741E+02 -1.660803E+02 -1.130854E+02 -1.751215E-01  1.261244E-01  1.290995E-01
1
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0
^^
^^
^^
                                END OF MPC FORCES

```

Table 2 : MPC forces obtained with the DMAP sequence

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1
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0

                                FORCES OF MULTIPOINT CONSTRAINT

POINT ID.  TYPE    T1      T2      T3      R1      R2      R3
15071     G    7.861104E+01  1.609391E+01 -3.749458E+02 -1.110274E+00  1.656089E+00  2.396557E-08
15073     G    7.497629E+01  2.824913E+01 -3.441682E+02 -1.586540E+00  2.793936E-01  3.233611E-08
15075     G   -1.222863E+01 -6.207858E+01  8.863431E+01 -5.905177E-02  5.898988E-01 -6.817037E-08
15077     G    2.903225E+01  4.999120E+01  4.808663E+02 -1.248237E+00  1.811587E+00  1.701969E-08

```



15079	G	-2.952158E+01	7.198445E+01	3.749962E+02	-1.981523E+00	2.190986E-01	4.259578E-08
15081	G	5.478954E+01	9.243045E+01	-3.156162E+01	-3.122648E-01	3.511912E-01	-4.664496E-08
17001	G	8.416367E+01	9.102635E+00	3.686208E+01	7.573231E-08	2.729335E-01	-6.221956E-01
17003	G	8.036130E+01	2.514936E+01	-3.915869E+01	-1.835049E-06	2.630912E-01	-3.614500E-01
17005	G	-6.728338E+01	-3.638429E+01	1.296095E+01	2.624950E-06	-5.444998E-01	-2.454515E-01
17007	G	2.515929E+02	4.118267E+01	1.430865E+01	-6.221426E-08	-2.326826E-01	1.375764E+00
17009	G	1.119534E+02	-6.426996E+01	1.421729E+02	-4.562036E-06	-2.722465E-01	5.205440E-01
17011	G	-3.234877E+02	1.636058E+02	-5.504374E+01	-2.638067E-08	1.562415E+00	9.474190E-01
17014	G	2.604851E+02	-1.331444E+00	1.944620E+02	6.486857E-01	-2.306046E-01	-8.890025E-08
17015	G	-5.532693E+02	1.118232E+02	-6.988396E+02	2.674013E+00	-1.615698E-01	-3.672299E-07
17016	G	-8.190183E+01	2.449507E+02	-3.263565E+02	1.115318E+00	-7.357069E-01	6.630940E-09
17017	G	1.078855E+01	1.371117E+01	-1.187700E+02	-2.216153E-01	-1.698262E-01	3.676398E-08
17103	G	1.968053E+01	1.011927E+01	2.941002E+01	-1.470501E-02	0.0	9.840266E-03
17104	G	0.0	7.053346E+00	-2.178907E+01	0.0	0.0	0.0
17105	G	2.635022E+00	-8.062255E+00	-5.244827E+01	-2.622413E-02	0.0	-1.317511E-03
17113	G	-2.901401E+01	4.515302E+00	4.020325E-03	-2.010162E-06	0.0	-1.450701E-02
17114	G	0.0	1.412189E+01	6.565560E+01	0.0	0.0	0.0
17115	G	4.187728E+00	-3.033263E+01	1.711445E-02	8.557227E-06	0.0	-2.093864E-03
17288	G	3.872791E+02	-2.286237E+02	9.661672E+02	2.542740E-01	-4.425221E-01	4.136789E-02
17290	G	-1.776564E+02	-2.958162E+02	-1.691000E+02	4.033074E-01	-8.194733E-01	-3.489449E-01
20581	G	-2.752919E+02	3.085621E+01	-1.897531E+02	-1.373490E-01	-1.297396E-02	2.659997E-01
20587	G	-3.807363E+01	1.858684E+02	-9.051492E+01	-1.087853E-01	-5.314330E-02	6.375848E-02
20592	G	2.548622E+02	2.109193E+02	1.013062E+02	-1.179938E-01	1.580505E-01	1.310837E-01
20597	G	3.105802E+02	3.368734E+01	2.439337E+02	-1.557661E-01	1.270369E-03	2.946407E-01
20602	G	7.336211E+01	-1.883000E+02	1.287804E+02	-1.843301E-01	-7.082495E-02	9.041532E-02
20607	G	-2.195741E+02	-1.660803E+02	-1.130854E+02	-1.751215E-01	1.261244E-01	1.290995E-01

Table 3 : MPC forces obtained directly from MSC/NASTRAN in SOL 101

## 2.5. Conclusions and comments

As it has been indicated in section 2.3 the DMAP sequence provided is applicable in the unstructured static solution STATICS (SOL 24). Within this solution MSC/NASTRAN provides a tool for the calculation of the forces produced by MPC constraints. This tool is a RF ALTER that can be invoked directly from the input file: RF ALTER24D34. This RF ALTER calculates the MPC forces ( $R_g$ ), as it was indicated in the equation (2b):

$$R_g = K_{gg} u_g - Q_g$$

This procedure is less efficient than the one exposed in this article, given that uses for the calculation matrixes of higher size, so the process is more expensive.

As it was commented before, the static structured solution SOL 101 allows the recovering of these forces by means of the MPCFORCES command. The sequence of this solution follows the same steps that the procedure proposed in this article. Anyway, this procedure for the static case is useful as an explanatory introduction to the dynamic problem, in wich the procedure proposed in the paper presents fundamental differences to the one implemented in the MSC/NASTRAN sequences.

### 3. DYNAMIC CASE

#### 3.1. Theoretical Basis

In a Frequency Response case, the equations of movement are written as

$$(-\omega^2 M_{gg} + i\omega B_{gg} + K_{gg}) u_g = P_g + Q_g + R_g$$

where:

$M_{gg}$  : Mass Matrix

$B_{gg}$  : Damping Matrix

$\omega$  : Frequencies at which the user has requested outputs of the analysis.

Once MSC/NASTRAN has solved the equations, a vector of displacements is obtained for each output frequency requested. All these vectors are grouped in a matrix  $u_{gv}$  which has as many columns as output frequencies exist. Accordingly, the force vectors are grouped into similar matrices of -v-columns.

The first member of the above equation is known once the solution is completed. We could proceed in the same way as we did for the static case, by performing the operation

$$F_{gv} = (-\omega^2 M_{gg} + i\omega B_{gg} + K_{gg}) u_{gv} \quad (4)$$

and computing the second member for each one of the output frequencies. This is the way in which MSC/NASTRAN computes the MPC forces in the structured solution SEMFREQ or SOL 111.

However, these frequencies are stored in a data block from which it is not easy to recover their values for further processing. Therefore we will proceed in a different way. First, let's assume the structure vibrating free (i.e. with no external forces) along one of its natural modal vectors  $\Phi_{gi}$ , at its corresponding natural frequency  $\Omega_i$ . In absence of damping, the equations of equilibrium are written:

$$(-\Omega_i^2 M_{gg} + K_{gg}) \Phi_{gi} = Q_{gi} + R_{gi} \quad (5)$$

where  $Q_{gi}$  and  $R_{gi}$  are the SPC and MPC forces respectively, that act on the structure when this is vibrating along one of its natural modes.

Writing this equation for each of the natural modes computed in the problem, and lumping all the equations together, we find:

$$- M_{gg} \Phi_{gh} \Lambda_{hh}^2 + K_{gg} \Phi_{gh} = Q_{gh} + R_{gh}$$

where  $\Phi_{gh}$  is the Modal Matrix, having the natural modes of the structure as columns, and that is accessible. The force vectors have become matrixes of h columns (number of modes computed)

The matrix  $\Lambda_{hh}$  is a diagonal matrix whose terms are the natural frequencies of the structure. These frequencies are stored in a vector called LAMA, that it is obtained as an output of the Module READ. This vector is reshaped into matrix form by means of the Module DIAGONAL.

Then, the first member of the above equation can be calculated once MSC/NASTRAN has extracted the natural frequencies and modalshapes, and the force matrixes  $Q_{gh}$  and  $R_{gh}$  can be obtained following a procedure completely similar to that used for the Static Case.

In a modal frequency response analysis MSC/NASTRAN has to calculate the modal coordinates  $u_{hv}$ , that will be a matrix with so many columns as output frequencies have been requested. The displacements in the global set -g- are calculated as:

$$u_{gv} = \Phi_{gh} u_{hv}$$

and the constraint forces will be:

$$R_{gv} = R_{gh} u_{hv}$$

so the problem is then solved. The output of these forces is performed similarly as explained for the Static Case.

The above procedure remains still valid for the case of Random Vibration, provided that the user requests the Power Spectral Density of the desired external forces. To this purpose it is enough to input the Matrix  $R_{gv}$  in the external applied forces datablock before the Module RANDOM is called.

With respect to the SUPORT Forces, it should be clear that the SUPORT Constraints do not transmit any force to the structure. However, it is interesting to calculate the force transmitted by the -r- degrees of freedom (degrees of freedom constrained by a SUPORT), to the rest of the structure. The equation (5) may be written for the -r- set as (having in mind that the SPC forces have to be zero at the set -r-):

$$(-\Omega_i^2 M_{rg} + K_{rg}) \Phi_{gi} = R_{ri}$$

This equation may be split into:

$$(-\Omega_i^2 M_{rr} + K_{rr}) \Phi_{ri} = R_{ri} + (\Omega_i^2 M_{rg2} - K_{rg2}) \Phi_{g2i}$$

where SET  $g_2$  is (SET  $g$  - SET  $r$ ). The term  $R_{ri} + (\Omega_i^2 M_{rg2} - K_{rg2}) \Phi_{g2i}$  which will be called  $S_{ri}$  is a vector that represents the forces transmitted by the rest of the structure (set  $g_2$ ), to the degrees of freedom  $-r$ , when the structure is vibrating along its modal vector  $-i$ .

Collecting all the equations corresponding to each of the modal vectors in a single matrix equation, it is obtained:

$$S_{rh} = R_{rh} + M_{rg2} \phi_{g2h} \Lambda_{hh}^2 - K_{rg2} \phi_{g2h}$$

Now, the constraint forces can be obtained similarly as done for the MPC forces:

$$S_{rv} = S_{rh} u_{hv}$$

As it is clear this procedure is valid for frequency response problems, solved with the modal formulation. The modal solutions are anyway extensively used when dealing with models of large size, because of the numeric problems of the direct formulation solutions.

### 3.2. Operating Procedure

The following steps will lead to the obtention of all the constraint forces

1. Computation of natural frequencies and modes of the undamped system: matrixes  $\Lambda_{hh}$  and  $\Phi_{gh}$ .

2. Compute

$$F_{gh} = -M_{gg} \phi_{gh} \Lambda_{hh}^2 + K_{gg} \phi_{gh}$$

3. Partition

$$F_{gh} \rightarrow \begin{pmatrix} F_{nh} \\ F_{mh} \end{pmatrix}$$

4. Perform

$$R_{mh} = F_{mh}$$

5. Compute

6. Partition  $R_{nh} = -G^T R_{mh}$

$$\phi_{gh} \rightarrow \begin{pmatrix} \phi_{rh} \\ \phi_{g2h} \end{pmatrix}$$

7. Partition

$$M_{gg} = \begin{pmatrix} M_{rr} & M_{rg2} \\ M_{g2r} & M_{g2g2} \end{pmatrix} \quad K_{gg} = \begin{pmatrix} K_{rr} & K_{rg2} \\ K_{g2r} & K_{g2g2} \end{pmatrix}$$

8. Partition

$$R_{gh} \rightarrow \begin{pmatrix} R_{rh} \\ R_{g2h} \end{pmatrix}$$

9. Compute

$$S_{rh} = R_{rh} + M_{rg2} \phi_{g2h} \Lambda_{hh}^2 - K_{rg2} \phi_{g2h}$$

10. Once MSC/NASTRAN has obtained the modal coordinates ( $u_{hv}$ ) for the output frequencies, compute

$$R_{nv} = R_{nh} u_{hv}$$

$$R_{mv} = R_{mh} u_{hv}$$

$$S_{rv} = S_{rh} u_{hv}$$

11. Rebuild the matrices up to size -g-

$$R_{gv} = \begin{pmatrix} R_{nv} \\ R_{mv} \end{pmatrix}$$
$$S_{gv} = \begin{pmatrix} S_{rv} \\ 0_{g2v} \end{pmatrix}$$

12. Write the results

$$R_{gv}, S_{gv}$$

### 3.3. DMAP Sequence

It is enclosed a DMAP Sequence which allows the calculation of the forces transmitted by both MPC and SUPORT constraints. This sequence is valid for the MSC/NASTRAN solution MFREQ (SOL 30). The forces are dealt with as if they would be directly applied loads, and are given in the standard output formats available within MSC/NASTRAN, XY format included. For the case of RANDOM excitation, the PSD of the constraint forces may also be recovered if the user requests them with the Case Control Command XYPRINT OLOAD PSDF....

```
$.....  
$  
$ DMAP SEQUENCE TO OBTAIN THE CONSTRAINT FORCES PRODUCED  
$ BY MPC AND SUPORT CONSTRAINTS IN FREQUENCY RESPONSE PROBLEMS.  
$ THIS SEQUENCE IS VALID FOR MSC/NASTRAN VERSIONS 67.7 AND 68,  
$ AND SOL 30 (MFREQ).  
$  
$ OUTPUT IS OBTAINED AS IF THE FORCES WERE DIRECTLY APPLIED  
$ EXTERNAL LOADS. FOR THIS, THE USER MUST REQUEST THEM BY THE  
$ COMMAND:  
$  
$ *****  
$ * OLOAD (SORT1,PHASE) = ALL *  
$ *****  
$  
$ OR ELSE :  
$  
$ *****  
$ * XYPRINT OLOAD RESPONSE .... *  
$ *****  
$  
$ OR, FOR RANDOM CASE,  
$  
$ *****  
$ * XYPRINT OLOAD PSDF ..... *  
$ *****  
$  
$ THE ACTUAL EXTERNAL LOADS WILL BE OUTPUT AS WELL, TOGETHER  
$ WITH THE CONSTRAINT FORCES. TO DISTINGUISH BOTH, THE  
$ CONSTRAINT FORCES ARE PRINTED BETWEEN EXPLANATION MESSAGES:  
$  
$ " FORCES IN RIGID ELEMENTS "  
$ " END OF MPC FORCES "  
$  
$ " FORCES IN SUPPORTS "  
$ " END OF SUPPORT FORCES "  
$  
$ PREPARED BY E. FUENTE Y F.J. SAN MILLAN (MARCH, 1994)  
$
```

```

$
$ I.N.T.A. STRUCTURES AND MECHANISMS DEPARTMENT.
$.....
$ COMPILE SOL30 SOUIN = MSCSOU LIST NOREF
$ ALTER 417
$.....
$
$ PARTITION MGG AND KGG
$ SELECTING THE - M -
$ DEGREES OF FREEDOM
$ UPARTN USET,MGG,MMG,,'G'/'N'/'M'/1 $
$ UPARTN USET,KGG,KMG,,'G'/'N'/'M'/1 $
$.....
$
$ PARTITION MGG AND KGG
$ SELECTING THE - R - DEGREES OF FREEDOM
$ (SET G2 = SET G - SET R)
$
$ UPARTN USET,MGG,,'MRG2'/'G'/'R'/'O' $
$ UPARTN USET,KGG/KRG,,'G'/'R'/'O'/1 $
$.....
$
$ GENERATE A DIAGONAL MATRIX (OM2) CONTAINING
$ THE SQUARE OF THE NATURAL FREQUENCIES
$ LAMX ,,LAMA/LMAT/-1 $
$ MATMOD LMAT,,,,/OM2,/28 $
$ ALTER 507
$.....
$
$ PERFORMS THE OPERATION
$
$ - MMG * PHIGH * OM2 + KMG * PHIGH
$
$ THAT GIVES THE MODAL MPC FORCES (SET M)
$
$ (QMPCM)
$ MPYAD MMG,PHIGH, /MMG1 $
$ MPYAD KMG,PHIGH, /KMG1 $
$ MPYAD MMG1,OM2,KMG1/QMPCM// -1 $
$.....
$
$ SELECT FROM PHIGH THE -G2- D.O.F. SET
$ : PHIG2H
$
$ UPARTN USET,PHIGH,PHIG2H,,'G'/'R'/'O'/1 $
$.....
$
$ COMPUTES QM = QMPCM * UHV
$ QN = -(GM)T * QM
$
$ AND MERGE UP TO QMN (MPC FORCES, SET G)
$
$ MPYAD QMPCM,UHV,/QM $
$ MPYAD GM,QM,/QN/1/-1 $
$ UMERGE USET,QN,QM/QMN/'G'/'N'/'M' $
$.....
$
$ PERFORMS
$
$ FRH = MRG2 * PHIG2H * OM2 - KRG * PHIGH
$ FRV = FRH * UHV
$
$ FRV ARE FORCES IN SET -R-
$
$ MPYAD MRG2,PHIG2H, /MMRH $
$ MPYAD KRG,PHIGH, /KMRH $
$ MPYAD MMRH,OM2,KMRH/FRH///-1 $

```

```

MPYAD FRH,UHV,/FRV $
$.....
$
$          SELECT THE MPC FORCES CORRESPONDENT TO THE -R-
$          DOF FORCES (QR) FROM QMN AND COMPUTES
$
$          SRV = FRV + QR   SUPORT FORCES (SET -R-)
$          SNORV = 0
$
$          AND MERGES THEM UP IN SGV (SUPORT FORCES, SET -G-)
$
UPARTN USET,QMN/QR,,,/'G/'R/'O'/1 $
ADD    FRV,QR/SRV $
UMERGE      USET,SRV,/SGV/'G/'R/'O' $
$.....
$
$          GENERATES TWO DATA BLOCKS WITH MPC AND
$          SUPORT FORCES EMULATING THE CONVENTIONAL APPLIED
$          LOADS OUTPUT (OLOAD)
$
$          OQMN2 = ' MPC FORCES '
$          RSG2 = ' SUPORT FORCES '
$
SDR2 CASEXX,CSTM,MPT,DIT,EQEXIN,,ETT,OLST,BGPD,T,QMN,,,EST,
      XYCDB,,/
      OQMN1 ,,,,/SOLTYPE/S,N,NOSORT2 $
SDR3 OQMN1,,,,/OQMN2, ,,,,/ $
$
SDR2 CASEXX,CSTM,MPT,DIT,EQEXIN,,ETT,OLST,BGPD,T,SGV,,,EST,
      XYCDB,,/
      RSG1 ,,,,/SOLTYPE/S,N,NOSORT2 $
SDR3 RSG1,,,,/RSG2, ,,,,/ $
$.....
$
$          AND PRINT THEM BETWEEN EXPLANATION MESSAGES
$
MESSAGE // ' '$
MESSAGE // ' '$
MESSAGE // ' '
          ' FORCES IN RIGID ELEMENTS '$
MESSAGE // ' '$
MESSAGE // ' '$
OFF OQMN2 //S,N,CARDNO $
$
XYTRAN XYCDB,OQMN2,,,,/XYPLTF/SOLTYPE/'PSET'/S,N,PFILE/
      S,N,CARDNO/S,N,NOQMN2 $
COND LABELQM,NOQMN2 $
XYPLOT XYPLTF // $
LABEL LABELQM $
$
RANDOM      XYCDB,DIT,PSDL,,OQMN2,,,,CASECC/PSDF1,AUTO1/
      S,N,NORAND1 $
XYTRAN XYCDB,PSDF1,AUTO1,,,/XYPLTTA/'RAND/'PSET'/S,N,PFILE/
      S,N,CARDNO/S,N,NOPLTR1 $
COND LABELRD,NOPLTR1 $
XYPLOT XYPLTTA // $
LABEL LABELRD $
$
MESSAGE // ' '$
MESSAGE // ' '$
MESSAGE // ' '
          ' END OF MPC FORCES '$
MESSAGE // ' '$
MESSAGE // ' '$
$
MESSAGE // ' '
          ' FORCES IN SUPPORTS '$

```



```

MESSAGE // ' '$
MESSAGE // ' '$
OFF      RSG2 //S,N,CARDNO $
$
XYTRAN  XYCDB,RSG2,,/XYPLTFF/SOLTYPE/'PSET'/S,N,PFILE/
        S,N,CARDNO/S,N,NOSUP $
COND    LABELSP,NOSUP $
XYPLOT  XYPLTFF // $
LABEL   LABELSP $
$
RANDOM   XYCDB,DIT,PSDL,,RSG2,,CASECC/PSDF2,AUTO2/
        S,N,NORAND2 $
XYTRAN  XYCDB,PSDF2,AUTO2,,/XYPLTAA/RAND/'PSET'/S,N,PFILE/
        S,N,CARDNO/S,N,NOPLTR2 $
COND    LABELRD2,NOPLTR2 $
XYPLOT  XYPLTAA // $
LABEL   LABELRD2 $
$
MESSAGE // ' '$
MESSAGE // ' '$
MESSAGE // ' '
        'END OF SUPPORT FORCES'/$
MESSAGE // ' '$
MESSAGE // ' '$
$

```

### 3.4. Demonstration examples

For the dynamic case, the same procedure as for the static one has been used, that is we started with simple examples, calculated by hand, to demonstrate that the equilibrium was maintained with the constraint forces, as well as for debugging the DMAP sequence. The results were correct in all instances.

To assess the performance of the DMAP alter for an engineering sized problem, we used the same FEM as for the static case, modified for dynamic loading. The model was submitted to a random vibration loading case, with a PSD of 0.125 g<sup>2</sup>/Hz within a frequency range of 0-2000 Hz. The forces at the node 15077, DOF 3, were obtained by means of the DMAP sequence included in para 3.3. This gridpoint is connected to node 20519, by means of a rigid element RBAR (ID 19001). Table 4 shows the PSD of the above mentioned MPC force at node 15077 (for convenience, the results are given up to a frequency value of 151 hz).

To assess the validity of the results, the FEM was modified by changing the RBAR 19001, so DOF 3 of node 15077 is not connected. Then a very stiff ELAS2 element (ID 998877, K=1e14 N/m) is added to the model, connecting gridpoints 15077 and 25019 through the DOF number 3. The force transmitted by this new element was obtained directly from MSC/NASTRAN. This force should be significantly similar to that given by the DMAP sequence. Table 5 shows that this is indeed the case.

To check the sequence for SUPORT loads, a similar procedure was used. The FEM was submitted to a base acceleration input (a sinusoidal excitation of 1.5g amplitude, within the range 140-200 Hz), by using the "large mass" procedure and the SUPORT constraint, as usual in this kind of problems.

The SUPORT is applied at node 120, DOF 3. As already mentioned, the DMAP sequence gives the force transmitted by the SUPORT dof to the rest of the structure. This force is shown in Table 6. Now an extra node 121 is added to the model. By using the "very stiff ELAS2" procedure similar as explained above, the node 120 is connected to node 121, through DOF 3, by means of a very stiff spring element (ELAS2 778899), while the boundary nodes of the structure are connected to the node

121. The force transmitted by this element should be significantly similar to that obtained above by the DMAP sequence. Table 6 (DMAP sequence results) and Table 7 (very stiff ELAS2 element results) indicate that the DMAP sequence gave the correct results.

```

^^
^^
^^
1
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0
                                XY-OUTPUT SUMMARY (AUTO OR PSDF)
0 PLOT CURVE FRAME              RMS    NO. POSITIVE XMIN FOR XMAX FOR YMIN FOR X FOR YMAX FOR X FOR*
  TYPE TYPE NO. CURVE ID.      VALUE  CROSSINGS ALL DATA ALL DATA ALL DATA YMIN ALL DATA YMAX
0 PSDF LOAD 0 15077( 5) 4.496820E+02 4.132293E+02 1.900E+01 1.999E+03 0.000E+00 1.900E+01 1.362E+04 1.810E+02
0 PRINTED DATA FOR THIS CURVE FOLLOWS
1
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0 LOAD CURVE ID = 15077 COMPONENT = 5 WHOLE FRAME
0 PRINT NUMBER X-VALUE Y-VALUE CARD NUMBER
  1 1.900000E+01 0.000000E+00
  2 2.200000E+01 3.573300E+00
  3 2.500000E+01 1.636390E-01
  4 2.800000E+01 2.594984E-01
  5 3.100000E+01 3.254265E-01
  6 3.400000E+01 3.856506E-01
  7 3.700000E+01 4.461532E-01
  8 4.000000E+01 5.094480E-01
  9 4.300000E+01 5.770567E-01
 10 4.600000E+01 6.501754E-01
 11 4.900000E+01 7.299132E-01
 12 5.200000E+01 8.165697E-01
 13 5.500000E+01 9.116578E-01
 14 5.800000E+01 1.016895E+00
 15 6.100000E+01 1.133771E+00
 16 6.400000E+01 1.263975E+00
 17 6.700000E+01 1.409446E+00
 18 7.000000E+01 1.572414E+00
 19 7.300000E+01 1.755464E+00
 20 7.600000E+01 1.961599E+00
 21 7.900000E+01 2.194321E+00
 22 8.200000E+01 2.459489E+00
 23 8.500000E+01 2.761399E+00
 24 8.800000E+01 3.104967E+00
 25 9.100000E+01 3.496937E+00
 26 9.400000E+01 3.945205E+00
 27 9.700000E+01 4.458937E+00
 28 1.000000E+02 5.048488E+00
 29 1.030000E+02 5.555976E+00
 30 1.060000E+02 6.124782E+00
 31 1.090000E+02 6.748999E+00
 32 1.120000E+02 7.386213E+00
 33 1.150000E+02 7.832779E+00
 34 1.180000E+02 8.648759E+00
 35 1.210000E+02 1.514771E+01
 36 1.240000E+02 1.641804E+01
 37 1.270000E+02 1.990584E+01
 38 1.300000E+02 2.836097E+01
 39 1.330000E+02 2.069602E+01
 40 1.360000E+02 1.218285E+01
 41 1.390000E+02 1.816655E+01
 42 1.420000E+02 2.373202E+01
 43 1.450000E+02 2.986516E+01
 44 1.480000E+02 3.728381E+01
 45 1.510000E+02 4.673955E+01

^^
^^
^^
                                END OF MPC FORCES
^^

```

Table 4 : PSD of MPC forces at gridpoint 15077, DOF 3

```

0
0
                                XY-OUTPUT SUMMARY (AUTO OR PSDF)
0 PLOT CURVE FRAME              RMS    NO. POSITIVE XMIN FOR XMAX FOR YMIN FOR X FOR YMAX FOR X FOR*
  TYPE TYPE NO. CURVE ID.      VALUE  CROSSINGS ALL DATA ALL DATA ALL DATA YMIN ALL DATA YMAX

```

0 PSDF EL FOR 0 998877( 2) 4.496712E+02 4.132271E+02 1.900E+01 1.999E+03 0.000E+00 1.900E+01 1.362E+04 1.810E+02  
 0 PRINTED DATA FOR THIS CURVE FOLLOWS  
 1 AUGUST 25, 1995 MSC/NASTRAN 6/24/94 PAGE 16

0  
 0 ELEMENT-FORCE CURVE ID = 998877 COMPONENT = 2 WHOLE FRAME  
 0

PRINT NUMBER	X-VALUE	Y-VALUE	CARD NUMBER
1	1.900000E+01	0.000000E+00	
2	2.200000E+01	3.573152E+00	
3	2.500000E+01	1.636310E-01	
4	2.800000E+01	2.594859E-01	
5	3.100000E+01	3.254110E-01	
6	3.400000E+01	3.856323E-01	
7	3.700000E+01	4.461321E-01	
8	4.000000E+01	5.094240E-01	
9	4.300000E+01	5.770297E-01	
10	4.600000E+01	6.501450E-01	
11	4.900000E+01	7.298792E-01	
12	5.200000E+01	8.165317E-01	
13	5.500000E+01	9.116155E-01	
14	5.800000E+01	1.016849E+00	
15	6.100000E+01	1.133719E+00	
16	6.400000E+01	1.263917E+00	
17	6.700000E+01	1.409381E+00	
18	7.000000E+01	1.572342E+00	
19	7.300000E+01	1.755384E+00	
20	7.600000E+01	1.961510E+00	
21	7.900000E+01	2.194222E+00	
22	8.200000E+01	2.459379E+00	
23	8.500000E+01	2.761275E+00	
24	8.800000E+01	3.104829E+00	
25	9.100000E+01	3.496781E+00	
26	9.400000E+01	3.945031E+00	
27	9.700000E+01	4.458740E+00	
28	1.000000E+02	5.048267E+00	
29	1.030000E+02	5.555734E+00	
30	1.060000E+02	6.124516E+00	
31	1.090000E+02	6.748707E+00	
32	1.120000E+02	7.385897E+00	
33	1.150000E+02	7.832451E+00	
34	1.180000E+02	8.648396E+00	
35	1.210000E+02	1.514701E+01	
36	1.240000E+02	1.641729E+01	
37	1.270000E+02	1.990494E+01	
38	1.300000E+02	2.835965E+01	
39	1.330000E+02	2.069504E+01	
40	1.360000E+02	1.218241E+01	
41	1.390000E+02	1.816585E+01	
42	1.420000E+02	2.373108E+01	
43	1.450000E+02	2.986397E+01	
44	1.480000E+02	3.728233E+01	
45	1.510000E+02	4.673770E+01	

Table 5 : PSD of force transmitted by the very stiff ELAS element (998877)

^^ FORCES IN SUPPORTS  
 ^^  
 1 AUGUST 25, 1995 MSC/NASTRAN 6/24/94 PAGE 37  
 0  
 0 XY - OUTPUT SUMMARY (RESPONSE)  
 0 SUBCASE CURVE FRAME XMIN-FRAME/ XMAX-FRAME/ YMIN-FRAME/ X FOR YMAX-FRAME/ X FOR  
 ID TYPE NO. CURVE ID. ALL DATA ALL DATA ALL DATA YMIN ALL DATA YMAX  
 0 1 LOAD 0 120( 5) 1.400000E+02 2.000000E+02 -6.670229E+02 1.700000E+02 5.599005E+02 1.400000E+02  
 1.400000E+02 2.000000E+02 -6.670229E+02 1.700000E+02 5.599005E+02 1.400000E+02  
 0 PRINTED DATA FOR THIS CURVE FOLLOWS  
 1 AUGUST 25, 1995 MSC/NASTRAN 6/24/94 PAGE 38  
 0  
 0 LOAD CURVE ID = 120 COMPONENT = 5 WHOLE FRAME  
 0

PRINT NUMBER	X-VALUE	Y-VALUE	CARD NUMBER
1	1.400000E+02	5.599005E+02	
2	1.430000E+02	3.481693E+02	
3	1.460000E+02	2.207250E+02	

4	1.490000E+02	1.313132E+02
5	1.520000E+02	6.024091E+01
6	1.550000E+02	-3.321212E+00
7	1.580000E+02	-6.764494E+01
8	1.610000E+02	-1.424559E+02
9	1.640000E+02	-2.452420E+02
10	1.670000E+02	-4.186168E+02
11	1.700000E+02	-6.670229E+02
12	1.730000E+02	4.908903E+02
13	1.760000E+02	2.094991E+02
14	1.790000E+02	-9.173552E+01
15	1.820000E+02	4.996120E+02
16	1.850000E+02	4.810296E+02
17	1.880000E+02	3.273380E+02
18	1.910000E+02	2.377301E+02
19	1.940000E+02	1.763675E+02
20	1.970000E+02	1.428260E+02
21	2.000000E+02	1.019104E+02

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1

0

Table 6 : DMAP sequence output of SUPORT force at node 120, DOF 3

```

0 SUBCASE CURVE FRAME          XMIN-FRAME/ XMAX-FRAME/ YMIN-FRAME/ X FOR YMAX-FRAME/ X FOR
  ID TYPE NO. CURVE ID.      ALL DATA ALL DATA ALL DATA YMIN ALL DATA YMAX
0  1 EL FOR 0 778899( 2)  1.400000E+02 2.000000E+02 -5.599006E+02 1.400000E+02 6.670243E+02 1.700000E+02
                                1.400000E+02 2.000000E+02 -5.599006E+02 1.400000E+02 6.670243E+02 1.700000E+02
0          PRINTED DATA FOR THIS CURVE FOLLOWS
1
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```

```

0 ELEMENT-FORCE CURVE ID = 778899 COMPONENT = 2 WHOLE FRAME
0          PRINT NUMBER      X-VALUE      Y-VALUE      CARD NUMBER
1          1.400000E+02      -5.599006E+02
2          1.430000E+02      -3.481693E+02
3          1.460000E+02      -2.207250E+02
4          1.490000E+02      -1.313132E+02
5          1.520000E+02      -6.024082E+01
6          1.550000E+02      3.321363E+00
7          1.580000E+02      6.764517E+01
8          1.610000E+02      1.424562E+02
9          1.640000E+02      2.452425E+02
10         1.670000E+02      4.186178E+02
11         1.700000E+02      6.670243E+02
12         1.730000E+02      -4.908930E+02
13         1.760000E+02      -2.095012E+02
14         1.790000E+02      9.173331E+01
15         1.820000E+02      -4.996119E+02
16         1.850000E+02      -4.810290E+02
17         1.880000E+02      -3.273378E+02
18         1.910000E+02      -2.377301E+02
19         1.940000E+02      -1.763676E+02
20         1.970000E+02      -1.428253E+02
21         2.000000E+02      -1.019097E+02

```

Table 7 : Forces transmitted by the very stiff ELAS2 element 778899

### 3.5. Conclusions and final comments

As it is well known, the forces originated by MPC can in fact be obtained in version 68 of MSC/NASTRAN in the structured solutions, by means of the command MPCFORCES. However, the results can not be obtained in XY format, nor for Random excitation cases. The DMAP sequence just developed does indeed produce this information.

Moreover, the procedure used by MSC/NASTRAN for the structured solution of frequency response SEMFREQ (SOL 111), is similar to the one used for the static solution. To this purpose, MSC/NASTRAN performs the operation (see section 3.1):

$$F_{gv} = (-\omega^2 M_{gg} + i \omega B_{gg} + K_{gg}) u_{gv}$$

which is computed for each of the output frequency. The matrix  $F_{gv}$  is then partitioned to give the corresponding forces at m and n sets, and it is proceeded as in the static solution. These operations are performed by module DSAP within SUBDMAP CFORCE.

The procedure followed by the DMAP sequence explained herein, seems to be advantageous since the number of floating point operations is lower, given that the matrix

$$R_{gh} = -M_{gg} \phi_{gh} \Lambda_{hh}^2 + K_{gg} \phi_{gh}$$

is computed just once and it is not necessary to repeat the operation for each output frequency. Finally, the SUPORT forces are in no way given by MSC/NASTRAN in dynamic cases, so that this DMAP sequence or any other produced by the user must be used to recover these forces.