NASOPT: A Flexible Optimization Capability for MSC/NASTRAN

Harold Thomas

Structural Optimization Specialists
1914 Santa Barbara St.
Santa Barbara, CA 93101-1057
Phone: (805) 682-7965
Fax: (805) 682-7965
email: sos@silcom.com

ABSTRACT

NASOPT, a software product that runs in conjunction with MSC/NASTRAN, provides a flexible optimization capability for analysis types not supported by Sol 200. The design variables can be any real number in the input data, such as element properties, material properties, and/or loads, as well as shape design variables. The objective function and constraint responses can be any quantity calculated by MSC/NASTRAN. NASOPT runs in conjunction with any structured or unstructured solution sequence, as well as with any DMAP run. In addition to structural optimization, NASOPT can perform parametric studies and system identification. NASOPT was developed under an Partner Interface Development Agreement with The MacNeal-Schwendler Corporation.

INTRODUCTION

Since version 66, MSC/NASTRAN has had an gradient based optimization capability [1]. This capability is available for linear analysis types supported by Sol 200. While sensitivity analysis can be performed for nonlinear finite element analysis, see for example Refs. [2] and [3], its implementation into a commercial finite element code is quite costly.

Only a few approaches exist to perform optimization of structures designed with nonlinear analysis. One approach is to calculate gradients using full finite difference techniques and then optimize using a nonlinear mathematical program such as DOT (Ref. [4]). However, this approach is too inefficient for the design of real structures due to the large number of analyses required. A better approach is to use zero order (non gradient based) techniques.

One zero order technique is to convert the constrained optimization problem into an unconstrained problem using penalty methods and then optimize using Powell's Method (Ref. [5]). A more efficient technique is to approximate the response surfaces for the objective function and constraints, and then optimize this approximate analysis problem (Refs. [6,5,7,9,10]). A full analysis of the approximate optimum provides new design information that is used to improve the response surface approximations. Optimization of this new design point again provides more information for the next iteration. As this process continues, the approximate optimum designs approach the actual optimum design. While this process is efficient for problems with a small number of design variables (less that 10), it becomes costly for larger design problems, due to the number of actual analysis needed to accurately approximate the response surfaces.

CAPABILITIES

Since the response surface approach is very general, any element property, material property, and/or load can be used as design variables. Any quantity calculated my MSC/NASTRAN and stored in the MSC/ACCESS [11] data base can be used as the objective function or be constrained. In addition to analysis results in the MSC/ACCESS data base, the total system mass can be used as the objective function or be constrained.

The program flow for NASOPT is:

1. Read in the MSC/NASTRAN analysis data and NASOPT design optimization data.
2. Update the analysis data and write out the MSC/NASTRAN input data file.
3. Run MSC/NASTRAN and recover the objective function and constrained analysis results using MSC/ACCESS.
4. Check for design convergence. If yes, STOP
5. Form the response surfaces.
6. Optimize the design using the approximate analysis problem defined by the response surfaces.
7. Go to Step 2.

The program flow for NASOPT is diagramed below:
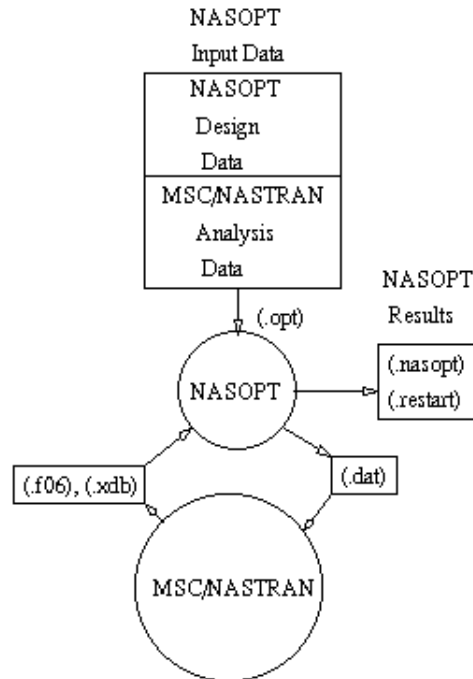
DESIGN OPTIMIZATION INPUT DATA

There are three components of the design optimization input data: 1) Design variable definition; 2) objective function definition; and 3) design constraint definition. The NASOPT design variable input data format is:

```
|---1--||---2--||---3--||---4--||---5--|
$$DESVAR  DVID     VALUE    LOWER    UPPER
```

Where DVID is the design variable number, VALUE is the initial value of the design variable, LOWER is the lower bound on the design variable value, and UPPER is the upper bound on the design variable value. For example:

```
|---1--||---2--||---3--||---4--||---5--|
$$desvar        1     2.0      1.0     10.0
$$DESVAR,1,0.3,0.1,1.0
```

The design variable value updates the MSC/NASTRAN input data value flagged by placing the design variable number within curly brackets ("{" and "}") next to the input data value. For example, in the following input data is:

```
|---1--||---2--||---3--||---4--|
PROD            3        1  {2}1.0
PSHELL, 1, 1, 1.0{2}, 1, 1.0
```

the value of design variable 2, instead of 1.0, is used. The design variable number within curly brackets must be touching the value to be updated. Note that a design variable may be used to design more than one input data value.

The design variable number can be referenced by NASOPT $$DVGRID data, for shape optimization, and NASOPT $$DSTUDY data, for parametric studies.

Shape design basis vectors are specified using NASOPT $$DVGRID data with the format:

```
|---1--||---2--||---3--||---4--||---5--||---6--|
$$DVGRID  DVID    GID     X       Y       Z
```

where `DVID` is the design variable number, `GID` is the GRID ID, and `X`, `Y`, and `Z` are the GRID location components. For example:

```
|---1--||---2--||---3--||---4--||---5--||---6--|
$$DVGRID     3       2     1.0     0.0     0.0
```

The objective function is defined by the NASOPT $$OBJFUN data with the formats:

```
|---1--||---2--||---3--||---4--||---5--||---6--||---7--|
$$OBJFUN  RTYPE    SID      0      GID     ITEM   MINMAX
$$OBJFUN  RTYPE    SID    ETYPE    EID     ITEM   MINMAX
$$OBJFUN  RTYPE    SID      0      MODE    ITEM   MINMAX
$$OBJFUN  MASS      0       0       0              MINMAX
```

where `RTYPE` is the MSC/ACCESS response type, `SID` is the SUBCASE number, `GID` is a GRID number for grid responses, `ETYPE` and `EID` are the element type and element number for element responses, `MODE` is a mode number for eigenvalue responses, and `ITEM` is the MSC/ACCESS response item (WORD) number. Note the the `RTYPE` for mass responses is `MASS`. `MINMAX` is a parameter used to tell NASOPT whether to minimize or maximize the objective function. For example:

```
|---1--||---2--||---3--||---4--||---5--||---6--||---7--|
$$OBJFUN  DISPR     1       0      101      3        0
$$OBJFUN  SRODR     3     CROD      2       2        0
$$OBJFUN  LAMA     10       0       1       4        1
$$OBJFUN  MASS      0       0       0       0        0
```

Lower and upper bound constraint data have the same format as the objective function except that $$OBJFUN is replaced by $$LBCON or $$UBCON. In addition, `MINMAX` is replaced by the `BOUND`, the constraint bound value. For example, for an upper bound constraint on a grid response:

```
|---1--||---2--||---3--||---4--||---5--||---6--||---7--|
$$UBCON   RTYPE    SID      0      GID     ITEM    BOUND
$$UBCON   DISPR     1       0      101      3       1.3
```

In addition, constraints can be placed on all of the elements that reference a single property using data with the format:

```
|---1--||---2--||---3--||---4--||---5--||---6--||---7--|
$$UBCON   RTYPE    SID    PTYPE    PID     ITEM    BOUND
```

where `PTYPE` is the property type and `PID` is the property number. For example:

```
|---1--||---2--||---3--||---4--||---5--||---6--||---7--|
$$UBCON    SQD4R       3 PSHELL       2       9 3000.0
```

PARAMETRIC STUDY INPUT DATA

In parametric studies the analysis results are calculated for a series of designs generated by varying a single design variable over a specified range. The NASOPT parametric study data is $$DSTUDY and has the format:

```
|---1--||---2--||---3--||---4--||---5--|
$$DSTUDY  DVID    VALUE0     INC    NUMINC
```

Where DVID is the ID of the design variable to to be used in the parametric study, VALUE0 is the initial value of the design variable, INC is the increment value, and NUMINC is the number of increments to take in the parametric study. For example:

```
|---1--||---2--||---3--||---4--||---5--|
$$DSTUDY       2      6.     2.0      10
```

The total number of analyses to be performed is NUMINC. The analysis with the initial value of the design variable counts as the first increment. Only a single $$DSTUDY data is allowed.

SYSTEM IDENTIFICATION INPUT DATA

In system identification the design is optimized so that the analysis results match target values. These target values can be generated from experimental tests on the actual hardware. System identification is sometimes called model tuning. The NASOPT system identification data is $$SYSID and has the same format as the $$OBJFUN data, except that min/max parameter is replaced by the target value. The $$SYSID data has the format:

```
|---1--||---2--||---3--||---4--||---5--||---6--||---7--|
$$SYSID    RTYPE   SID        0      GID    ITEM    TARGET
$$SYSID    RTYPE   SID    ETYPE      EID    ITEM    TARGET
$$SYSID    RTYPE   SID        0     MODE    ITEM    TARGET
$$SYSID    MASS      0        0        0       0    TARGET
```

or

```
|---1--||---2--||---3--||---4--||---5--||---6--||---7--|
$$+        RTYPE   SID        0      GID    ITEM    TARGET
$$+        RTYPE   SID    ETYPE      EID    ITEM    TARGET
$$+        RTYPE   SID        0     MODE    ITEM    TARGET
$$+        MASS      0        0        0       0    TARGET
```

for continuation lines of data. TARGET is the target value for the response. The first response to be matched is specified with the $$SYSID data and additional responses are identified with the $$+ continuation data. There is no limit on the number of responses to be matched. For example:

```
|---1--||---2--||---3--||---4--||---5--||---6--||---7--|
$$SYSID     LAMA       1       0       1       4    4000.
$$+         DISPR      2       0    1001       3      6.0
$$+         MASS       0       0       0       0     80.0
```

can be used to make the optimized design match eigenvalue, displacement, and mass target values.

RESTART INPUT DATA

The optimization process can be restarted using the NASOPT $$RSTART data:

```
|---1--||---2--|
$$RSTART NUMDES
```

where `NUMDES` is the number of designs to read in from the '.restart' file. For example:

```
$$RSTART,6
```

The $$RSTART command causes NASOPT to read design variable and response values of previous analyses from the '.restart' file. Optimization is then resumed starting with analysis NUMDES+1.

RESPONSE SURFACE APPROACH

In the response surface approach used here the design objective and constraint functions (F) are approximated in terms of the design variables (X) using a second order polynomial of the form

$$Fk = A0k + BikXi + CijkXiXj$$
$$i,j = 1,2,...,NDV$$
$$k = 1,2,...,1+NCON$$

where NDV is the number of design variables, NCON is the number of constraints, and A, B, and C are the polynomial coefficients. The polynomial coefficients are determined using a least squares fit of the functions on to the previous design points (actual nonlinear FEM analysis results). Initially 1+NDV designs are analized and the A and B coefficients are determined. As each of the next NDV designs are analized an additional diagonal of the Hessian matrix [C] is determined. Additional designs are used to determine the off diagonals of the symmetric [C] matrix. After 1 + NDV + (NDV+1)(NDV)/2 designs, an algorithm is used to weight the designs. The optimization procedure is as follows:

1. Analize the initial design and NDV perturbed designs
2. Use a least squares fit to determine the polynomial coefficients for the objective function and each constraint function
   - If the number of designs (ND) is 1+NDV, then calculate then A0 and Bi coefficients
   - If 1+NDV < ND < 2(1+NDV), then calculate the A0, Bi, and Cjj(j=1,ND-1-NDV) coefficients
   - If 1+2*NDV < ND < 2 + NDV + (NDV+1)(NDV)/2, then calculate the A0, Bi, Cjj, and successive off diagonals of [C] coefficients
   - If ND > 1+NDV+(NDV+1)(NDV)/2, then weight the designs and calculate the A0, Bi, Cij coefficients
3. Solve for the approximate optimum design using mathematical programming
4. Analize the approximate optimum design
5. If the designs have converged, then stop
6. Go to step 2

EXAMPLES

The following examples will be used to demonstrate NASOPT [12] for linear and material nonlinear analysis optimization problems and for a geometrically nonlinear analysis system identification problem.

Three Bar Truss (D200X1): Linear Analysis

The first example is taken from the first example problem in the MSC/NASTRAN Design Sensitivity and Optimization User's Guide [1]. This is a three bar truss. The mass (weight) is minimized subject to lower and upper bound displacement constraints on two degrees of freedom in two SUBCASE's, for a total of 8 displacement constraints. In addition, there are lower and upper bound stress constraints on the three CROD elements in both SUBCASE's, for a total of 12 stress constraints. In all there are 20 design constraints. The NASOPT input data is shown below:

```
ID NASOPT, D200X1
TIME   10
SOL 101
CEND
TITLE = SYMMETRIC THREE BAR TRUSS DESIGN OPTIMIZATION   -        D200X1
SUBTITLE = BASELINE - 2 CROSS SECTIONAL AREAS AS DESIGN VARIABLES
ECHO   = SORT
SPC    = 100
DISP   = ALL
STRESS = ALL
SUBCASE 1
   LABEL = LOAD CONDITION 1
   LOAD  = 300
SUBCASE 2
   LABEL = LOAD CONDITION 2
   LOAD  = 310
BEGIN BULK
$ MSC/NASTRAN ANALYSIS MODEL DATA
GRID,   1,       ,        -10.0 ,   0.0,  0.0
GRID,   2,       ,          0.0 ,   0.0,  0.0
GRID,   3,       ,         10.0 ,   0.0,  0.0
GRID,   4,       ,          0.0 , -10.0,  0.0
SPC,    100,    1,       123456, ,        2,        123456
SPC,    100,    3,       123456, ,        4,        3456
CROD,   1,       11,      1,       4
CROD,   2,       12,      2,       4
CROD,   3,       13,      3,       4
PROD,   11,      1,    {1}1.0
PROD,   12,      1,    {2}2.0
PROD,   13,      1,    {1}1.0
MAT1,   1,       1.0E+7, ,        0.33,   0.1
FORCE,  300,    4,        ,       20000.,  0.8,   -0.6
FORCE,  310,    4,        ,       20000., -0.8,   -0.6
PARAM,GRDPNT,0
PARAM,POST,0
$ NASOPT OPTIMIZATION DATA
$$DESVAR 1       1.0      0.1      100.0
$$DESVAR 2       2.0      0.1      100.0
$$OBJFUN mass   0        0        0         0         0
$$LBCON   DISPR 1        0        4         2         -0.20
$$LBCON   DISPR 1        0        4         3         -0.20
$$LBCON   DISPR 2        0        4         2         -0.20
$$LBCON   DISPR 2        0        4         3         -0.20
```

7

```
$$UBCON  DISPR  1       0       4       2       0.20
$$UBCON  DISPR  1       0       4       3       0.20
$$UBCON  DISPR  2       0       4       2       0.20
$$UBCON  DISPR  2       0       4       3       0.20
$$LBCON  SRODR  1     PROD     11       2    -15000.
$$LBCON  SRODR  1     PROD     12       2    -15000.
$$LBCON  SRODR  1     PROD     13       2    -15000.
$$LBCON  SRODR  2     PROD     11       2    -15000.
$$LBCON  SRODR  2     PROD     12       2    -15000.
$$LBCON  SRODR  2     PROD     13       2    -15000.
$$UBCON  SRODR  1     PROD     11       2     20000.
$$UBCON  SRODR  1     PROD     12       2     20000.
$$UBCON  SRODR  1     PROD     13       2     20000.
$$UBCON  SRODR  2     PROD     11       2     20000.
$$UBCON  SRODR  2     PROD     12       2     20000.
$$UBCON  SRODR  2     PROD     13       2     20000.
$$PARAM,MOVE,0.5
ENDDATA
```

In the optimum design there are only two active constraints. These are the upper bound stress constraints in CROD 1 in SUBCASE 1 and CROD 3 in SUBCASE 2. The NASOPT and MSC/NASTRAN Sol 200 design cycles histories are shown in Table 1. These runs were made on a SUN Sparc1 with MSC/NASTRAN Version 68.2 using mem=10Mb. The CPU time for the Sol 200 run was 358.3. The total CPU time for the 10 Sol 101 runs was 309.1.

Table 1. Three Bar Truss Design Cycle Histories

Design Cycle	NASOPT Mass	NASOPT Maximum Constraint	MSC/NASTRAN Sol 200 Mass	MSC/NASTRAN Sol 200 Maximum Constraint
1	4.83	32.3% Feasible	4.83	32.3% Feasible
2	6.38	53.8% Feasible	3.01	0.0% Feasible
3	5.93	35.1% Feasible	2.82	1.9% Feasible
4	2.46	34.5% Violated	2.73	0.1% Feasible
5	3.03	1.4% Feasible	2.71	0.0% Feasible
6	2.71	1.4% Violated	2.70	0.0% Feasible
7	2.73	0.2% Feasible	2.70	0.0% Feasible
8	2.39	13.1% Violated	2.70	0.0% Feasible
9	2.60	4.1% Violated	-----------	-----------
10	2.72	0.6% Feasible	-----------	-----------

The optimum values of the design variables, for each design are shown in Table 2.

Table 2. Three Bar Truss Optimum Designs

Item	NASOPT	MSC/NASTRAN Sol 200
Design Variable 1	0.868	0.838
Design Variable 2	0.263	0.325
Mass	2.72	2.70

It can be seen in Table 2 that NASOPT produces a design that is similar to the one produced by MSC/NASTRAN Sol 200.

Three Bar Truss: Nonlinear Analysis

This example is the same as the first except that the material is nonlinear elastic. The stress-strain curve for this material is shown in Figure 1.
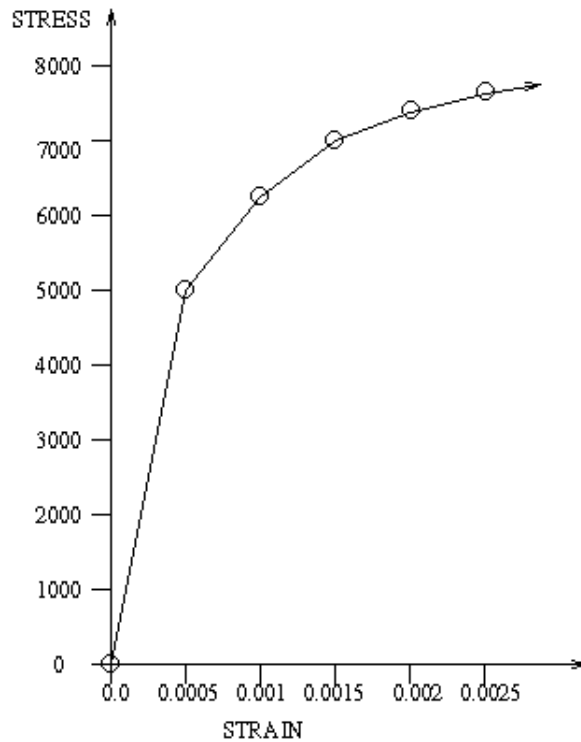

Figure 1. Stress-Strain Curve for Three Bar Material

The modifications for the NASOPT input data file for the nonlinear material are:

1. Change `SOL 101` to `SOL 106`.
2. Add `NLPARM` data to apply the loads in one increment.
3. Add the following nonlinear elastic material data:

```
$ MSC/NASTRAN NONLINEAR MATERIAL DATA
NLPARM  1        1
```

```
MATS1    1        1          NLELAST
TABLES1 1
         0.0       0.0      .0005    5000.    .001     6250.     .0015    7000.
         .002     7500.     .0025    7750.    ENDT
```

4. Change the response type from SRODR to NRODR to constrain the nonlinear stress in the elements.

The design cycle history for the nonlinear three bar truss is shown in Table 3.

Table 3. Nonlinear Three Bar Truss Design Cycle History

Design Cycle	Mass	Maximum Constraint
1	4.83	3.7% Feasible
2	6.38	53.5% Feasible
3	5.93	3.7% Feasible
4	3.80	2.4% Violated
5	3.78	0.0% Feasible
6	3.42	0.0% Feasible
7	3.10	5.8% Violated
8	3.30	1.7% Feasible
9	3.10	6.4% Violated
10	3.16	2.9% Violated
11	3.24	2.1% Feasible
12	3.18	0.9% Violated
13	3.21	0.6% Feasible
14	3.19	0.8% Violated
15	3.21	0.4% Feasible

In the optimum design, design variable 1 has a value of 1.01 and design variable 2 has a value of 0.35. The lower bound displacement constraint on GRID 4, direction T1, in SUBCASE 2 and the upper bound displacement constraint on GRID 4, direction T1, in SUBCASE 1 are active in the optimum design.

Cantilever Beam: Geometric Nonlinear Analysis

In nonlinear finite element analysis the final displacement field is determined by incrementally applying the loading and calculating the incremental displacements using the Newton-Raphson iteration solution technique (Ref. [13]). If the analysis is conservative, ie. geometric nonlinearity only, then the analysis results from a previous design may provide a good starting point for the analysis of the present design. This is especially true as the designs approach the optimum, as there is usually only a small change in the design at this point.

In this example the optimum load required to produce a specified deflection of a cantilevered beam will be determined. The beam has a length of 10.0, area of 1.0, bending inertia of 0.01, and is composed of a material with E=10.0E6. It is modeled with 10 equal length finite elements. The objective function in

this example is to find a tip load on the beam that will cause a tip deflection of 7.5.

Initially the beam is loaded in four steps to 2000.0, 4000.0, 6000.0 and 8000.0. These loadings result in a geometrically nonlinear deflection of the beam. The input data for this initial analysis is shown below:

```
ID ANALYSIS,EX3
SOL 106
TIME 10
CEND
$$analysis
$$desvar,1,.8,.5,.9
$$sysid,dispr,40,0,11,3,7.5
TITLE=TRACE LARGE DEFLECTION OF A CANTILEVERED BEAM
SUBTITLE=REF.: BISSHOPP AND DRUCKER; QAM 3(1):272-275; 1945
SPC=1
DISP=ALL
SPCF=ALL
NLPARM=10
SUBCASE 10
LOAD=200
SUBCASE 20
LOAD=400
SUBCASE 30
LOAD=600
SUBCASE 40
LOAD=800
BEGIN BULK
GRID,1,,0.,0.,0.,,345
=,*(1),=,*(1.),==$
=(9)
GRID,100,,0.,0.,1.,,123456
CBEAM,101,1,1,2,100
=,*(1),=,*(1),*(1),==$
=(8)
PBEAM,1,1,1.,,1.-2,1.-2
MAT1,1,10.+6,,.0
SPC,1,1,123456
FORCE,11,11,,1.+4,0.,1.,0.
LOAD,200,.2,1.,11
LOAD,400,.4,1.,11
LOAD,600,.6,1.,11
LOAD,800,.8{1},1.,11
PARAM,POST,0
PARAM,DBDRNL,-1
NLPARM,10,10
PARAM,LGDISP,1
ENDDATA
```

The resulting deflections are 4.94, 6.71, 7.46, and 7.87. When solved with 10 loading increments per load step, a total of 289 iterations are required. A summary of the number of iterations required for each load step is shown in Table 4.

Table 4. Iterations per Loading Step

Load	Iterations
2000	103
4000	74
6000	62
8000	50
Total	289

Since this analysis is conservative, it can be restarted from any analysis step and still achieve the same result. For this example, the design optimization analysis will be started from the end of the third step where the applied load is 6000. Note that it took 249 iterations to get to this load. This design optimization problem is solved with NASOPT using the data shown below:

```
RESTART,VERSION=1,KEEP
ASSIGN MASTER='ex3.MASTER'
ID RSTART,EX3
SOL 106
TIME 10
CEND
$$rstart,1
$$desvar,1,.8,.5,.9
$$sysid,dispr,40,0,11,3,7.5
TITLE=TRACE LARGE DEFLECTION OF A CANTILEVERED BEAM
SUBTITLE=REF.: BISSHOPP AND DRUCKER; QAM 3(1):272-275; 1945
SPC=1
DISP=ALL
SPCF=ALL
NLPARM=10
PARAM,LOOPID,30
PARAM,SUBID,4
SUBCASE 10
LOAD=200
SUBCASE 20
LOAD=400
SUBCASE 30
LOAD=600
SUBCASE 40
LOAD=801
BEGIN BULK
LOAD,801,.8{1},1.,11
ENDDATA
```

The applied load, tip deflection, and number of analysis iterations required for each design cycle are shown in Table 5.

Table 5. Design Cycle History

Design	Deflection	Load	Iterations
1	7.87	8000	289
2	7.15	5000	43
3	7.57	6456	29
4	7.50	6150	20
5	7.50	6149	20
6	7.50	6149	20
Total	-------	---	421

Table 5 show that this restart approach can be used to reduce the total analysis time dramatically. If the analysis was started from the unloaded state for each design cycle, the total number of iterations would be approximately 6x249=1494. When the restart approach is used the total number of iterations required to find the optimum design was only 421.

CONCLUSIONS

An response surface methodology for optimization of structures designed using MSC/NASTRAN has been presented. The design optimization input data has been described. Examples have been presented that show the optimization of structures analized with nonlinear analysis. These examples show the ease of use and power of NASOPT.

ACKNOWLEDGEMENTS

REFERENCES

1. *MSC/NASTRAN Design Sensitivity and Optimization User's Guide, Version 68,* MacNeal-Schwendler Corp., Los Angeles, CA, 1994.

2. Cardoso, J. B. and Arora, J. S., "Variational Method for Design Sensitivity Analysis in Nonlinear Structural Mechanics," AIAA J., Vol. 26, No. 5, 1988, pp. 595-603.

3. Yamazaki, K. and Shibuya, K., " Sensitivity Analysis of Nonlinear Material and its Application to Shape Optimization," Proceedings of the 36th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conf., New Orleans, LA, April 10-12, 1995, pp. 1706-1713.

4. *DOT User's Manual,* Vanderplaats Research & Development , Colorado Springs, CO, 1995.

5. Vanderplaats, G. N., *Numerical Optimization Techniques for Engineering Design: With Applications,* McGraw Hill Book Co., New York, 1984.

6. Vanderplaats, G. N., "Effective Use of Numerical Optimization in Structural Design," Finite Elements in Analysis and Design, Vol. 6, 1989, pp. 97-112.

7. Toropov, V. V., Filatov, A. A., and Polynkin, A. A., "Multiparameter Structural Optimization using FEM and Multipoint Explicit Approximations," Structural Optimization, Vol. 6., 1993, pp. 7-14.

8. Harrison, P. N., Le Riche, R., and Haftka, R. T., "Design of Stiffened Composite Panels by Genetic Algorithm and Response Surface Approximations," Proceedings of the 36th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conf., New Orleans, LA, April 10-12, 1995, pp. 58-68. AIAA-95-1163-CP.

9. Muller, G. and Tiefenthaler, P., "Design Optimization with a Commercial Finite Element Program," *Engineering Optimization in Design Processes,* Eshenauer, E. H., Mattheck, C., and Olhoff, N. (Eds.), Springer, Berlin, pp. 201-210.

10. Canfield, R. A., "A Rank Two Hessian Matrix Update for Sequential Quadratic Approximations," Proceedings of the 36th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conf., New Orleans, LA, April 10-12, 1995, pp. 1486-1493. AIAA-95-1334-CP.

11. *MSC/ACCESS User's Manual, Version 6.0,* MacNeal-Schwendler Corp., Los Angeles, CA, 1991.

12. *NASOPT User's Manual, Version 1.0,* Structural Optimization Specialists, Santa Barbara, CA, 1996.

13. *MSC/NASTRAN Handbook for Nonlinear Analysis, Version 67,* MacNeal-Schwendler Corp., Los Angeles, CA, 1992.